REPUTATION, PREDATION AND MARKET STRUCTURE*†

By

Dario G. De Maio**

In this paper I introduce a duopoly game that aims at describing the firms' decision of competing for acquiring the reputation of high-quality goods seller when an exogenous probability of exiting the market is introduced and when, for a firm, reputation is profitable only after the rival's exit. The reached conclusion indicates that the firm which is more patient towards future profits receive from gaining reputation the right incentives to predate on its competitor and thus becomes the monopolist. Furthermore reputation involves a relational contracts between the buyers and the firm with high-quality goods to be supplied, but at the same time, the lack of competition generated by the underlying market transition causes a worsening of consumer's welfare. (JEL- L 14; L 13)

Keywords: Relational Contracts, Reputation, Duopolistic competition, Predation strategy.

1. Introduction

"It is not enough to succeed. Others must fail."

Gore Vidal

Reputation is, of course, part of a firm's success. Recognized quality and brand name can be considered as assets that allow firms to create new strategies on the product market. On the one hand, being viewed as a high quality producer gives the opportunity to charge a higher price for the product sold and

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† * Extract from the MSc thesis *"Relational Contracts and Reputational Games"* discussed at University of Siena. Particular thanks to Nicola Dimitri (University of Siena) and Giacomo Calzolari (University of Bologna) for their capital supervision.

** IMT Institute for Advanced Studies Lucca.

consequentially to increase profits, at least in the short run[1]. On the other hand, the threat of dissipating such an asset induces firms to exert high effort, reducing the risks of the non-contractibility of the product's quality. While these mechanisms have been widely analyzed and discussed in competitive backgrounds, less has been said about the extent of these forces in non-competitive scenarios.

When is it convenient to get a reputation and how much does it cost in a non-cooperative market? Which strategies can be developed when reputation counts? Should we expect society to be better off after a firm invests in quality in such an environment, or will we just see a strengthening of its market share?

It can be imagined that the answers to these questions, and thus the sustainability of reputation, are strictly linked with the specific market structure considered. Intuitively it seems reasonable to consider reputation in a duopoly as more difficult to build and maintain than in a monopoly, so we expect firms in a duopoly to invest more in quality. However, this reasoning leads to a non certain result: the benefits in terms of increased profits due to reputation must be compared with the cost to sustain it, which depends on the degree of competition in the market. In this way, more than one result is possible in a duopoly: we can have situations where just one firm invests in quality and others in which both of them mix their strategies or alternatively situations in which no quality-investment is advanced.

With this paper I introduce a repeated non-cooperative duopoly game with the purpose of deriving the conditions under which one firm invests in quality and obtains positive payoffs while the best strategy its rival can employ is to quit the market. In doing so, I show what is the cost to bear by such an investor firm and the role that the threat of losing future payoffs has on its behavior.

The constraints for the firms are introduced by the possibility of market failures that will occur when the price requests by the firms for a given product, independently of quality, overwhelm the customer's willingness to pay or when the quality offered is lower than a given level, indicated by $\bar{q}$. In this context, a monopolist therefore produces a high-quality good as long as it can profitably impose a price at most equal to the maximum price payable by consumers. A firm in a duopoly faces an additional constraint when compared to the monopolist, i.e. the competition of its rival. In this last case I focus on a particular strategy of competition, predation, which implies the exit of a firm after a finite period of time due to the impossibility of recovering the losses suffered during the game. Considering predation strategy relies upon a specific assumption of the model, i.e. consumers prefer to pay a lower price than be satisfied with a higher quality product as long as the quality is at least equal to the minimum level under which the market fails. This assumption, although strong, seems implausible especially in those markets where the

[1] Klein-Laffler derives an active role for nonsalvageable productive assets and advertising in a competitive market, which in the long run erode a portion of the extra profits but are necessary for firms to distinguish themselves as high-quality producers.

goods are fungible with each other or when innovations don't improve substantially on the products traded[2]. The results obtained indicate the importance of the firms' discount factors for reaching an equilibrium where investing in high quality is a dominant strategy for one firm. As long as the prices of higher quality goods can't increase because of the previous assumption on the market demand form, the investment in quality is profitable only if it furnishes enough incentives to a firm after the latter drives its rival out the market via a predation strategy. However, the potency of such a predation strategy will increase the higher the firm's monopolistic expected return on investment, that is the lower the rate with which such future profits are discounted. Intuitively if a firm has more patience than its rival, it can force the latter to exit the market after a finite period of time beyond which it will raise the price of the traded good and will earn the expected monopolistic revenue. According to the validity of the conditions proposed, several equilibria are also possible in this game, included the cases where no one invests in quality or where firms randomize their choices.

A welfare analysis will be further developed in order to extract useful information on economic policy. In particular, the fact that customers' wealth decreases while the operating firm receives a reputational rent after a firm invests in quality and causes its rival to quit the market will be derived. I finally analyze some comparative statics to predict how the model responds to the variation of parameters. Specifically, what happens when a firm's discount factor changes and then when $\bar{q}$ increases will be shown.

This paper is organized as follows: in Section 3 I introduce the model and the underlying assumptions, the main equilibria of the game and some welfare analysis are derived in Section 4 while comparative statics are discussed in Sections 5 and in Section 6 a brief conclusion and some considerations are reported.

2. Related Literature

The majority of the literature that considers reputation as an incentive for firms in producing high quality and sustaining agreement in an environment with moral hazard and adverse selection lays its foundation on a perfect competition market structure. A duopoly market has been largely studied, in non-cooperative games, as a competition in prices or in quantities. Introducing reputation effects in a duopoly means allow firms to compete in quality too. Quality and price are interrelated and both operate to determine an equilibrium solution of the game, where the kind of solution found relies upon the specific assumptions introduced in the model.

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[2] Assuming this implicitly means that for some products there are no customers' preferences for luxury, which seems reasonable for a broad variety of the goods traded. Alternatively we can image that liquidity constraints hold, so that clients care more about saving than quality.

In a general competitive framework, *Klein and Leffler [1981]* focus on market incentives that help firms to introduce high quality goods when quality characteristics cannot be specified cheaply and measured by a third party. The authors allow the price for such goods to be higher than their marginal cost, so that bad type producers have no incentive to offer low quality products for a sufficiently higher discount factor and price above marginal cost. The authors discuss the existence of such a solution without stating specific functional forms for the variable involved, but exhibit some evidence that '*a quality-assuring price premium can generally be expected to exist*'. Such a perpetual price premium stream involves positive profits for the firms, but in a competitive framework this can only be valid in the short-run. Indeed, in the long-term, new firms will enter the market and this will lead profits to the competitive equilibrium level involving zero profits. However, in order to protect high quality production such an equilibrium cannot lead to a lower quality-guaranteeing price and so competition will involve 'firm-specific capital' expenditure or give an active role to nonsalvageable productive assets. When quality characteristics can however be specified cheaply and measured by a third party, and hence contract enforcement costs are anticipated to be low, then explicit contractual solutions with governmentally enforced penalties may be a less costly solution to assure that high quality is introduced onto the product market.

Remaining in a perfect competition market structure, *Horner [2002]* shows how competition generates reputation-building behaviors in repeated interactions when the product quality observed by consumers is a noisy signal of a firm's effort level. Horner finds that, allowing for the free entry and exit of firms in the market, reputation gives rise to equilibria where only high effort is provided and that the incentives pass through higher prices for young firms and higher growth rates for older ones.

The main insight of this analysis is that competition endogenously generates an outside option for consumers, given by the possibility for the latter of breaking off their current relationships with a firm as soon as it disappoints them with a low-quality good and starting new relationships with new other firms. This consumers' strategy is precisely what is required for consumers' behavior to exert effective discipline over sellers and causes opportunistic firms to always exert high effort.

Tadelis [2002] investigates the conditions that guarantee long-term incentives through an active market for reputations. The author provides a model where the possibility for an overlapping generation of agents to sell the brand name before they retire gives them the right incentive to continue to produce high quality goods in the last period too. In particular he shows that if a firm's name, or entity, is separated from its owner's identity, then incentives can survive throughout the owner's career as long as firms' clients do not observe this brand name trade. The presence of a market for reputation can thus alleviate the problems associated with moral hazard and adverse selection even with short-lived agents, but as the welfare analysis further developed by the author indicates, there are no reasonable conclusions as to whether the trade of names is better or worse for the society. However, more information about name transfers can cause the market for names to collapse and eliminate incentives for older agents. In many cases of the model, this can be detrimental for social surplus.

Moving from competitive markets toward non-perfectly-competitive ones, *Rob and Sekiguchi [2006]* studies the incentives in acquiring the reputation of high-quality good producer generated by competition in duopoly markets. Their paper introduces a duopoly game with turnover equilibrium where a firm alone exerts high effort, while the other momentarily abandons the market and returns after the former fails to produce quality above some fixed level. The authors exhibit some evidence on behalf of such an equilibrium but they don't explain why a firm starts as a "leader" on the market and the rival stays out. *Dana Jr and Fong [2008]* sustain that firms in an oligopoly market can more easily maintain a reputation for a high quality good than a monopolist or firms in a competitive market because of the wider range of discount factors at their disposal. In general, monopolists have higher margins but face less of a threat from competition, while firms in competitive markets have lower margins and face a greater threat from competition. Oligopoly markets are in between. On one hand, firms in such markets enjoy high margins (because of tacit collusion) but the fear of wasting this extra profit, on the other hand, causes firms to concentrate more on producing high quality and preserving reputation. Moreover, they find that even if oligopoly interactions can increase quality, there are no definitive conclusions to relax anti-trust scrutiny. The reason for this is that the higher price required for high quality goods can reduce the social surplus and consumers may prefer low quality equilibria to high quality ones.

The model introduced in this paper abandons the hypothesis of perfectly competitive markets to analyze a duopolistic competition where the quality of the product is a firm strategic variable. Even though the competitive environment differs from that in Klein-Leffler and Horner's works, some derived results again highlight the role played by the good's price and the personal discount factor on the firm's decision regarding the attainment of a reputation. The novelty here is that such variables are analyzed in relation to the competitive dynamics they generate in the product market and the consequences on its structure. In particular, the model tries to explain when a firm decides to invest in reputation and what are the sacrifices it must sustain to obtain it. In this manner, the model can be seen as something that precedes and integrates Rob and Sekiguchi's turnover analysis without replacing it. Further, the welfare analysis shows that the higher effort exerted by a producer in order to acquire a reputation can create a reputational rent that impoverishes the mass of consumers, even though the traded quality is higher. On one hand, this analysis reinforces the concerns already shown in Dana and Fong's work, but on the other hand it clarifies that such impoverishment originates from the lack of competition after a firm's predation causes a success. For the purpose of balancing the larger wealth introduced with high quality goods among buyers and sellers, an active role for policy makers is required in order to avoid that competition on the market vanishes completely.

A different but important approach to reputation that is involved here originated in the classical *Kreps and Wilson [1982]*. The authors show that a reputation of being "tough" can bring about higher payoffs in the long run along with reduced losses in the short term. In the context of the model described here, this means that the company that invests first in quality sends a clear signal to his competitors, that he is "ready to fight if entrance occurs".

3. The Model

I consider two risk-neutral firms, denoted by 'i' and 'j', operating in the same product market which is closed to new entrants. Firms compete on the price and quality of the good traded and the quantity sold depends on the realization of these two variables. The prices charged vary over time among firms and are denoted respectively by p_{it} and p_{jt} with p_{it} , p_{jt} $\in [0, +\infty)$. The qualities q_{it} and q_{jt} also vary and are included in the interval $[0,q_M]$ where q_M is the maximum quality that can be achieved with the existing technologies. The technology is shared between firms and is invariant over time. Let $\bar{q}$ be a positive and finite value denoting the minimum level of quality below which the market demand is null[3]. The cost of production is the same for both firms and is a continuous function C= C(x_t^s,e_t), that is, the cost at time t depends on the quantity produced x_t^s and the level of effort e_t , both at time t. Let also $C_e > 0, C_x$ = C/x_t^s for every e $\in [0, e_M]$ and any x_t^s $\in [0, x_t^d]$, where x_t^d is the total market demand[4]. Effort e_t is private seller information and varies over the interval $[0, e_M]$, where e_M represents, as for quality, the maximum level of effort available for the firms. Having the same cost and technology, the profit function $\Pi = \Pi(p_t, e_t, x^s)$ is the same for both the firms and is continuous in its dependent variables. I also assume that the profit function is directly proportional to the sold quantity, that is $\Pi(p_t, e_t, x^s/k) = \Pi(p_t, e_t, x^s)/k$; this simplification will be very helpful in determining the game's equilibria. Firms face a personal discount rate denoted by δ_i for firm 'i' and δ_j for firm 'j', with $\delta_i, \delta_j \in [0,1]$. The personal discount factor has the exponential form, so that firm 'i' (or 'j') discounts future incomes and costs with $e^{-\delta_i t}$ (and $e^{-\delta_j t}$

[3] One can consider such a level as a standard imposed by law or alternatively the minimum level below which consumers prefer not to buy. In this model, however, no penalties are inflicted on firms if the produced quality is lower than $\bar{q}$; it will emerge through the passage of the punishment through customer base losses.

[4] In other words the offer never exceeds the market demand, but can at most equal it. In this way no unsold stock is admitted.

for 'j'). The time is continuous and is denoted by t ∈ [0, +∞); this means that I consider an infinite horizon game. Consumers are homogeneous and have all the same preferences denoted by:

$$u^i(p_t, q_t, x_t^i) \equiv u^i(p_t, q_t)\ x_t^i$$

with $u_p < 0$ and $u_q > 0$ for every p and q varying in their respective intervals and for any t, and x_t^i representing the individual demand of customer 'i'. In every instant of time a single consumer buys just a unit of the product supplied, that is $x_t^i = 1$, so that the market is at equilibrium when the quantity offered is divided in equal parts among all the consumers and $x_t^s = x_t^d$. Furthermore there are no costs for consumers to pay to move from one firm to the other, i.e. each firm is fully reachable by consumers at any time[5]. For the remainder of the paper I assume that the product market is always at equilibrium so that the quantity demanded will always be satisfied by the firms. Let p* denote the maximum price payable by consumers, beyond which they will not demand any goods. In this way the same limit to the customers availability to pay for a given product is inserted in the model[6]. The buyers observe the price fixed by the firms but cannot verify the quality of the good until they purchase it and all of them share equal beliefs represented by $\beta\ (q_t)$; these are the conjectures about the quality of the product firms are going to introduce onto the market at every instant of time[7]. When a firm introduces no product onto the market, I let q_t offered by that firm equal zero and the customer utility for that firm $u(p_t, 0) = 0$.

The quality of the product is not fully under firm control, which is why no contract can be enforced through reliance on quality. I now introduce a noise term η_t that affects each firm at any instant of time, so that the realized quality for 'i' (or 'j') is expressed by:

(1) $q_{it} = \eta_{it} + e_{it}$

I denote the c.d.f of η_{it} by $F(\eta_{it})$ and let $f(\eta_{it})$ be its p.d.f. To simplify the analysis, let $f(\eta_{it}) \sim N(0,1)$ so that $E(q_{it}) = e_{it}$. This means that the expected value of the quality is exactly the invested effort level at every instant of time.

I now introduce the first six hypotheses of the model:

(i) $\forall\ p \leq p^*, q \geq \bar{q}$ *and* $\forall\ t$, $\left| u_p(p, \cdot) \right| > \left[u(\cdot, q_M) - u(\cdot, \bar{q}) \right]$ *and* $u(p, q) > 0$;

This first hypothesis tell us that consumers always prefer to spend less than have higher quality, as long as the latter is not lower than the minimum level required and the former is higher than p*. Furthermore, if

[5] Moving costs are controlled and geographical position advantages are thus eliminated.

[6] Formally $\forall$ p > p* $x_t^s(p) = x_t^d(p) = 0$.

[7] Assuming equal beliefs means allowing for perfect information among customers, i.e. the entire market is aware of the quality introduced by each firm even if a single consumer doesn't buy the same product from the two different sellers.

these conditions hold, consumers prefer to enjoy the use of the product rather than not purchase it. As discussed above, if the price of the good is higher than the maximum level payable by customers or the quality is too low, no product market exists[8]. As a consequence, a monopolist can satisfy the whole market with a price equal to p* and a quality at least equal to $\bar{q}$.

(ii) $\forall\, t \;\; C(x_t^s, e_t)/\, x_t^s < p^*$;

Whatever the product's average unit cost, it will always be less than p*. This implies that it is always profitable for a monopolist to produce high quality, satisfy the whole market, and charge a price equal to p* for the good, that is $\pi\left(p^*, e_M, x^s = x^d\right) > 0$.

(iii) $\forall\, t \quad x_{it}^s\ (or\ x_{jt}^s) \;\leq x_t^d$;

The third hypothesis is a consequence of the anytime equilibrium condition in the product market. In other words, the quantity demanded will always be satisfied by the firms. As a consequence of this, no production constraints are imposed on firms, which means that a single firm can satisfy the whole market, as a monopolist does, if the other exits.

(iv) $\beta\,(q_t) = q_{t-s}$ *with* $s \rightarrow 0$;

Customer's beliefs are of the adaptive kind. This means that customers expect a quality that is exactly equal to that of the previous instant of time for each firm. The employment of adaptive beliefs instead of rational expectations performs two important functions. First of all, adaptive beliefs don't require that consumers possess a sophisticated ability to make inferences regarding the qualities that firms are going to introduce onto the market, and thus permit the reduction of the hypothesis regarding consumers' purchase decisions: consumers can't predict future quality and so they refer to their previous purchases. The second aspect concerns the competition that such a consumer's behavior generates. If, on equal terms, at a certain instant of time a firm provides a good whose quality is lower than its rival then, independently of subsequent investment, its market share will be reduced. Indeed the customers' purchase option fuels the competition via punishing the firm that invests in quality less than its competitor. In order to avoid losing their customer base, each firm is held to supply a quality at least equal to its rival at any instant of time. This constraint will be rendered binding, and consequently the role of adaptive beliefs dramatically enhanced, by the next three hypotheses that bear the name of *Exit Hypotheses* and indicate how competition causes the exit of a firm from the market.

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[8] At difference with *Klein and Leffler [1981]*, it is still possible to impose a higher price and offer higher quality, but now the price set by firms can cause market collapse if it is too high. As a result, tighter price constraints are introduced here compared to that model.

(v) *If* $p_{it} = p_{jt} = p_t \leq p^*$ *and* $q_{it} > q_{jt} \geq \overline{q}$ *then* $u(p_t, q_{it}) > u(p_t, q_{jt})$ *and* $\forall$ *h>0:* $q_{it+h} \geq \overline{q}$ *and* $p_{it+h} \leq p* \rightarrow x^s_{it+h} = x^d_{t+h}$ *and* $x^s_{jt+h}=0$*;*

This is the model's first Exit Hypothesis: higher quality is strictly preferred by consumers *ceteris paribus* over price, so that at any instant of time following the introduction of the higher quality good its producer will satisfy the entire market as long as the quality offered is at least equal to the standard and the price doesn't exceed the limit value payable by consumers. If a firm fails to supply a good that is at least as good as the other firm, then it will see its customer base reset to zero and will leave the monopoly to the latter. Hypothesis (v) thus makes predation an extremely efficacious tool to force a firm out of the market, thus restricting the analysis to the study of how such a strategy must be conducted and which conditions make it successful; this, as will be shown, will address the expected monopolistic profits. It is however important to notice that the monopolist, after predation has taken place, has no other constraints on the good's quality apart from maintaining at least the standard level required by the market. In this case, the quality offered will be determined by the problem of the maximization of the monopolistic expected income conditioned to the exerted effort level and to the supply of the entire market demand. The objective function is introduced in paragraph 4.1 while in Appendix 1 the condition that guarantees the highest effort to be selected by the monopolist is derived.

(vi) *If* $p_{it} = p_{jt} = p_t \leq p^*$ $q_{it} < \overline{q}$ *and* $q_{jt} \geq \overline{q}$ *then* $\forall$ *h>o :* $q_{jt+h} \geq \overline{q}$ *and* $p_{jt+h} \leq p* \rightarrow x^s_{jt+h} = x^d_{t+h}$ *and* $x^s_{it+h} = 0$*;*

Here we have the second Exit Hypothesis: a firm is able to sell its production as long as the quality is not lower than $\overline{q}$. If this happens, the other firm will operate as a monopolist for the entire time that the quality offered is at least equal to $\overline{q}$. We will therefore see the exit of a firm from the product market if it fails to produce the minimum quality requested by the customers: the remaining firm will satisfy the entire market alone. If the latter also fails to produce the minimum quality, the game will restart in the initial equilibrium described in the next section. This last hypothesis does not impact the model's results, which instead focus on the role played by reputation in the competitive strategies feasible by firms. One can image that the market requires cost conditions that cause entry to be permitted to few firms; in this way, after the operating firms fail to satisfy the market demand, two new competitors (not necessarily two single companies) are ready to enter the market and play the same game. Conclusively this model analyzes a type of market characterized by non-cooperation and elevated entry barriers that make it accessible simultaneously to a limited number of players.

(vii) If $p_{it} < p_{jt} \leq p^*$ *then* $\forall h \geq 0$: $q_{it+h} \geq \bar{q}$ *and* $p_{it+h} \leq p* \rightarrow x^{s}_{it+h} = x^{d}_{t+h}$ and $x^{s}_{jt+h}=0$;

The third Exit Hypothesis is complementary to the first hypothesis of the model and expresses the higher demand elasticity of the price compared to the quality. If the price fixed by a firm in a certain instant of time is lower than its rival than, both now (because consumers observe the price) as well as in the future, it will provide the entire demand as long as the quality is not less than $\bar{q}$, while the more expensive firm will exit the market. Also in this case it is important to notice that hypothesis (vii) does not establish the price of the good after a firm acquires the monopoly, i.e. the only constraint imposed to the monopolist is fixing a price that is not higher than p*. If this is so, as in the previous case, the game will restart at its initial equilibrium.

Given that the quality offered is not fully under the control of the firm, the exit of firm 'i' (or 'j') occurs $\forall e_{it} > 0$ with a probability given by F($\bar{q}$ -e_{it}). After a rival's exit occurs, firm 'i' (or 'j') expects to do business as a monopolist for a period equal to T= 1/ F($\bar{q}$ - e_{it})[9]. As F($\bar{q}$ - e_{it}) increases with respect to $\bar{q}$ and decreases with respect to e_{it} , that is $F_{e_{it}}(\bar{q} - e_{it}) < 0$, we expect that increasing effort will protract the monopoly period of a firm after a rival exits the market. Furthermore, two limit points must be analyzed:

1) When the effort is null, so that e_{it} = 0, this means that firm 'i' (or 'j') will not introduce a product onto the market, so *leadership* will last just an instant and firm 'i' (or 'j') will exit as well. Formally this means that F($\bar{q}$) = 1 independently of the value of $\bar{q}$ and T → 0;

2) When the effort is extremely high and the difference ($\bar{q} - e_{it}$) is sufficiently large, we expect leadership to remain over the long-run and turnover to never occur, that is $\lim_{eit \rightarrow eM} T \cong \infty$.

Summarizing the probability that exit occurs, we have these two cases:

$$\begin{cases} F(\bar{q} - \mathrm{e_{it}}) = 1 \;\; and\; T \;\rightarrow 0 \; if\; e_{it} = 0 \\ F(\bar{q} - \mathrm{e_{it}}) \text{ and T} = \dfrac{1}{\mathrm{F}\,(\bar{q} - \mathrm{e_{it}})} \; if\; \mathrm{e_{it}} > 0 \end{cases}$$

[9] One can consider, alternatively, such a period as that one in which all the customers become loyal to a specific firm considered of the "good type". So if a firm fails in producing the minimum quality, this will result in a reputational loss implying a shift of the customers base to the other firm.

4. Game Equilibria

In the beginning of the game I consider the equilibrium where both firms produce the minimum quality $\bar{q}$, and fix the price equal to the marginal cost of producing an additional unit of such a quality, denoted by $c_{\bar{e}}$, in a general Bertrand competition structure.

One can easily recognize that in the one-shot game, such an equilibrium would be the unique equilibrium of the game. Indeed no firm could enhance its situation by deviating from the strategy on the equilibrium path. Introducing high quality and letting p = $c_{\bar{e}}$ is not profitable for a single firm because of increasing marginal cost, while setting a higher price for a superior quality is not profitable because of assumption *(i)*. The results derived in the one-shot game thus reconverge to those of the Bertrand competition[10].

I am therefore interested to discover if other equilibria in the infinitive horizon game are possible. First of all, let's note that in the infinite horizon there is an incentive to the firm to invest in quality higher than that of his rival and bear short-run losses because of the exit hypothesis. The key now is that the reputation as a high quality producer causes a shift of the customer base from one firm to the other. The perspective of becoming a monopolist for a given period of time in the future causes the firm to care about the quality of the good introduced onto the market. The reason why I focus on the predation strategy is now clear: given customers' preferences and the opportunity to operate as a future leader in the market, a firm will invest in quality and secure a price at least equal to his rival, forcing the latter to exit. Naturally, any predation attempts by a firm can be fought by the other, so I cannot select a particular equilibrium *a priori*. The conditions assuring successful predation thus need to be derived in order to define a specific equilibrium of the game and as I will show, several equilibria are available depending on which conditions are respected. The analysis will proceed as follows: at first I compute the payoff a monopolist expects to earn during the game, and then I compare such a result with the cost of sustaining reputation using a predation strategy. Finally, I derive the conditions that must be honored in order for predation to be successful.

[10] According to *Tadelis [2002]*, reputational concerns could still be possible allowing for a market for reputation to exist. If the game was played every time by two new firms for an infinite horizon, a "good type" firm could charge a higher price to sell its brand name to new entrants who want to buy it, making an investment in quality still possible. However, this further solution is discounted here by the assumptions at the base of the model.

4.1 Monopolist's income

Assume that the exit of a firm has occurred. Firm 'i' (or 'j') will therefore operate as a monopolist for the period of time equal to T_i which, as discussed above, will be given by T_i = 1/F($\bar{q}$ - e_{it}). A utility maximizer monopolist will therefore ask for the price p*, which will be paid by all the consumers because of hypothesis *(i)*, and choose e_{it} in a way to maximize:

(2) $max_{e_{is}} \int_{s=0}^{Ti} e^{-\delta_i s} \;\; \pi_{is}(p^*, e_{is}, x^s = x^d)\, ds$

s.t.

$e_{is} \in [\bar{e}, e_M]$;

p*= $\bar{p}$* (i.e. p* is constant and exogenous)

where $\bar{e}$ is the level of effort necessary to produce the quality $\bar{q}$.

The value of e_{is} chosen by firm 'i' (or firm 'j') will take into account the disutility of the increasing marginal cost of producing higher quality and the impact that a higher effort has on reducing the probability of failure. As will be discussed in Appendix 1, if the second effect overpowers the first one, we will see a firm that always provides high effort, although effort will not fall below $\bar{e}$ in order to avoid exit. Given the stationarity of the game, the value maximizing (2) will be the same throughout its course and for assumption *(ii)*, (2) is always positive. From now on, I assume that condition (4) in Appendix 1 is satisfied for both the firms, which means that the solution of (2) is always given by e_M. Further, with the same profit functions for the two firms, this implies that:

$$T_i = T_j = T$$

and

$$\pi_{is}(p^*, e_M, x^s = x^d) = \pi_{js}(p^*, e_{Ms}, x^s = x^d) = \pi(p^*, e_{\mathrm{M}}, x^s = x^d)$$ [11]

In this manner, it is assured that any firm achieving success via predation will exert high-effort and will offer high-quality products. The two firms are therefore symmetric except for their personal discount factors.

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[11] That is $T$and $\pi(p^*, e_{\mathrm{M}}, x^s = x^d)$ become the same for the two firms and constants.

4.2 The cost of reputation

In the beginning of the game, both firms produce good quality equal to $\bar{q}$ and fix a price equal to $c_{\bar{e}}$, receiving zero profit. Increasing quality and charging a higher price is not profitable for both firms given that customers strictly prefer to pay less than benefit from higher quality. This means that any firm investment entails losses, so that $\pi < 0$ for the entire time that investment continues. In this model a firm's investment consists of lowering the price of the good (maintaining the same quality) or introducing higher quality (maintaining the same price) onto the market, or even both of them. With the intent of achieving a general result, a specific functional form of cost is not introduced here. The minimal instantaneous investment made by a firm will therefore be given for firm 'i' (or 'j') by:

(3) $(-1) \cdot Max\ \{\pi\ (p_0 - \varepsilon, \bar{e}, x_i^s = x^d)\ ;\ \pi\ (p_0, \bar{e} + \varepsilon\ , x_i^s = x^d)\}$;

s.t. $p_0 = c_{\bar{e}}$ and with $\varepsilon \rightarrow 0$

In this way, (3) indicates the minimal investment (the values in bracket are negatives) that a firm must support if it wants to drive its competitor out of the market and capture the entire market ($x_i^s = x^d$). A firm will therefore carry out a predation strategy only if the expected payoff of being a monopolist is at least equal to the cost of the minimal investment necessary to drive its rival out of the market. That is, the first necessary condition for firm 'i' (or 'j') to plunder at a generic time t[12] is:

(4) $$e^{-\delta_i h} \int_{s=0}^{T} e^{-\delta_i s}\ \pi(p^*, e_M, x^s = x^d)\, ds\ +\ \int_{v=0}^{h} e^{-\delta_i v}\ \max \left\{ \begin{array}{c} \pi\ (p_0 - \varepsilon\ , \bar{e}\ , x_i^s = x^d) \\ \pi\ (p_0, \bar{e} + \varepsilon, x_i^s = x^d) \end{array} \right\} dv \equiv z_1 \geq 0$$

(*or* z_2 *for 'j'*) with $h \rightarrow 0$;

Condition (4) indicates a *necessary but not sufficient* condition for predation to be a successful strategy that the firm must comply with at time t. The first term of equation (4) represents the monopolistic income gained by 'i' if it was the only firm on the market, while the second term is the minimal initial loss that such a firm must sustain if it wants to play a predation strategy. This loss is set by the cost of offering a quality slightly higher than the standard level (i.e. the equilibrium quality at the beginning of the game) at the same equilibrium initial price, or in offering the same quality at a lower-than-initial price. If the value z_1 is positive for firm 'i' but not for 'j', the latter will never begin a predation strategy towards 'i' or alternatively it will never respond to i's predation. If 'i' decides to undertake a predation strategy, on the basis of the exit hypothesis, in the instant following such a decision, firm j will see its

[12] Time 't' is considered as the time at which the predation game starts. This implies that I don't need to discount future income and costs to this day, that is of t periods until today. Instead, the stated conditions require future values to be discounted to the time at which the predation starts. The values obtained are therefore the same for any period t in which the predation takes place and consequently fit together perfectly with the case t = 0.

market demand reset to zero and will thus quit the market leaving the monopoly to 'i'. The predation in this case will therefore last just an instant of time (h→0) and will allow 'i' to gain its expected monopoly income, finally receiving a payoff equal to z_1. Therefore, if condition (4) holds for one firm only, we expect it to plunder its rival and become the leader on the market, while if (4) is verified for no firms then we expect the game staying in its initial equilibrium with no high effort provided. Given that firms share the same technology and have the same cost function, the minimal instantaneous cost (3) will be the same for both of them. What therefore determines whether condition (4) is verified or not must be found in the higher value a firm expects to receive when it works as a monopolist.

It is possible to rewrite condition (4) solving the equation for the discount factor and thus obtaining a condition for firm 'i':

$$\int_{s=0}^{T} e^{-\delta_i s}\ \pi(p^*, e_{\mathrm{M}}, x^s = x^d)\, ds \ > -Max\ \{ \pi\,(p_0-\varepsilon, \bar{e}, x_i^s = x^d)\,; \pi(p_0\,, \bar{e} + \varepsilon\ , x_i^s = x^d)\,\}$$

which implies:

$$\frac{1-e^{-\delta_i \mathrm{T}}}{\delta_i} > -\ max\ \left\{ \frac{\pi\ (p_0-\varepsilon, \bar{e}, x_i^s = x^d)}{\pi(p^*, e_{\mathrm{M}}, x^s = x^d)}\ ;\ \frac{\pi\,(p_0\,, \bar{e}+\varepsilon\ , x_i^s = x^d)}{\pi(p^*, e_{\mathrm{M}}, x^s = x^d)} \right\}.$$

Opposed to the minimal instantaneous investment, I introduce the maximal one which is given by:

(5) $\pi(\mathrm{o}, e_M, x^s = x^d)$

That is, providing the maximal effort possible at a null price and satisfying the entire market.

If both firms satisfy condition (4) I expect predation of a firm to be fought by its rival, so that I can't predict which firm will capture the market without further conditions. Intuitively if a firm receives a higher payoff from its status as monopolist, it seems reasonable that it is more willing to invest to reach such a status relative to its rival. Formally, the differential of utility is denoted as:

(6) $$\Delta \equiv \int_{s=0}^{Ti} e^{-\delta_i s}\ \pi_i(p^*, e_{\mathrm{M}}, x^s = x^d)\, ds\ -\ \int_{s=0}^{Tj} e^{-\delta_j s}\ \pi_j\,(p^*, e_{\mathrm{M}}, x^s = x^d)\, ds \quad \leftrightarrow$$

$$\leftrightarrow \quad \pi(p^*, e_{\mathrm{M}}, x^s = x^d) \left[\frac{1-e^{-\delta_i \mathrm{T}}}{\delta_i} - \frac{1-e^{-\delta_i \mathrm{T}}}{\delta_j} \right]$$

The last passage is again derived thanks to the equality profit condition stated in the previous paragraph. We will then see firm 'i' (or alternatively firm 'j') plunder firm 'j' (or 'i') if the second necessary condition is verified, that is:

(7) $\Delta > 0$ (or < 0 for firm 'j' to invest) $\leftrightarrow \frac{1-e^{-\delta_i \mathrm{T}}}{\delta_i} > \frac{1-e^{-\delta_j \mathrm{T}}}{\delta_j}$

Like condition (4), (7) is a *necessary but not sufficient* condition that must occur to see a firm invest in quality and plunder its rival. Both conditions (4) and (7) must be verified to see a transition from a duopoly market to a monopoly. Indeed, if a firm satisfies condition (4), it means that it deems forcing its rival to quit the market profitable, and at the same time if (7) holds for the same firm, it will sacrifice more to achieve its goal. If (7) does not hold for either firm, that is Δ=0, there will be no prevalence of one firm upon the other, so if predation starts both firm will suffer losses[13].

The cost of sustaining reputation for firm 'i' (or 'j') then varies with the capability of its rival to combat the predation strategy, which in turn depends on the payoff expected to be earned as the only seller. A first conclusion can therefore already be pointed out: in a duopoly with reputation, the cost a firm must sustain to invest in quality and receive higher profits is always equal or larger than the cost for a monopolist that wants to do the same. The reason for this is that in a non-cooperative game, competition, which makes it likely that predation will be fought, causes reputation to be less desirable than monopoly, where reputation is not contrastable (and thus verifiable[14]) by anyone. Once having showed the condition, I can solve the game in the next section but, before doing so, I need to introduce three further assumptions necessary to define a complete plan of action (i.e. the strategies) for both the firms and restrict the range of feasible strategies.

4.3 Several equilibria of the game

In this section I derive the main equilibria of the game according to the validity of conditions (4) and/or (7) for the firms. Before doing so, in order to simplify the analysis and derive all of the game's pay-offs, I need to introduce three further assumptions concerning the information available to firms, the behaviors of the latter outside of the equilibrium path, and the particular type of strategies they will employ.

Note that if there is not perfect knowledge about firms' payoffs, multiple equilibria of the game can be selected according to the beliefs of a firm on their capability to fight their rival[15]. I do not consider lack of

[13] In other words, the case in which operating firms are perfectly symmetric with one another.

[14] One important consequence of this reasoning is that if predation is contrasted, we will see the introduction of higher quality more rapidly onto the market than in a monopoly. The continuous launch of higher quality products onto the market requires that customers' beliefs are updated quickly and this can be valued by them in terms of difficulty in creating a reputation. Instead, in a monopoly, the slowness in updating beliefs can bring about two different conclusions: the monopolist is not investing in quality (or at least the investments are managed over the course of time) or it is difficult for product innovations to be realized. This ambiguous consideration makes it impossible to assess the price of reputation in a monopoly.

[15] In other words, the game is that of incomplete information and equilibria rely upon firms' beliefs computed via the Bayesian rule.

information in this model, so the game involves perfect information among firms. This can be expressed by the following *no-bluff* assumption:

(viii) Let H be the set: {$e_{is}, \delta_i, e_{js}, \delta_j$ | (2) and Δ are well-defined}, so H is well-known to firm 'i' and firm 'j'}.

Hypothesis *(vii)* tells us that each firm exactly knows its payoff as well as that of its rival, so that no firm can bluff during the game. With this further assumption, the best a firm weaker than its rival can do is to exit the market immediately after predation begins, because it knows that it can in no way survive the predation of its rival. However, this is not enough to define a complete plan of action. Indeed, now consider the weak firm fighting its rival's predation. Given that predation leads to losses for the firm carrying out such a strategy, nothing until now has been said about the bounds of the losses such a firm can support. Precisely, it is still possible for a weak firm to threaten its rival and incur infinite negative profits in order to avoid being plundered and exiting the market. Running the risk of suffering infinite losses could be a deterrent for the stronger firm to invest in quality and conquer the market[16]. In order to avoid such a situation and thus define an exit-rule of the game for the 'weak' firm and make predation feasible, I introduce this further hypothesis of the *no-Ponzi scheme*, which is a debt limit given by:

(ix) Firm 'i' (or 'j') continues to play a predation strategy, until with α ≥ 0 :

$$e^{-\delta_i \alpha} \int_{s=0}^{T} e^{-\delta_i s}\, \pi(p^*, e_M, x^s = x^d)\, ds \geq - \int_{s=0}^{\alpha} e^{-\delta_i s}\, \pi_{is}(p_{is}, e_{is}, x_{is}^s)\, ds$$

s.t.

$p_{is} \in [\, 0\, ,\ c\bar{e}\ ]$

$e_{is} \in [\, \bar{e}\, ,\ e_M\,]$

$x_{is}^s \in \left[\frac{x^d}{2},\ x^d\right]$

After which 'i' (or 'j') :

a) will quit the market if 'j' (or alternatively 'i)' remains; or

b) will gain its monopolistic profit if 'j'(or 'i') quits.

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[16] Actually, such a threat could be defined by the literature as a non-credible threat. Indeed, a firm that decides to fight predation in a non-profitable way questions the rationality of the same, but a further remark is involved. In other words, I am looking for a complete plan of action that must also consider what happens outside of the equilibrium-path, and in doing so I can obtain a result of the game in those cases where the "trembling-hand" move occurs.

With hypothesis *(viii)*, a firm will decide to quit the market (or not fending off predation) when it observes that the value of the realized losses is higher than the value it can obtain operating as a monopolist. At that point such a firm will suffer losses given by the right-hand side of equation *(ix)* and will exit from the market, letting its rival work as a monopolist. In practice, a firm deviating from the equilibrium strategy will suffer losses equal to the profits expected to be earned being a monopolist. This is therefore the general predation strategy's exit-rule for a firm[17]. The left-hand side is the firm's expected income in the event that it works alone on the market discounted for α periods, that is for the predation's entire duration. The right-hand side of equation in (ix) is instead the cost of reputation to be sustained by the stronger firm to capture the whole market after α periods and depends on the realizations of prices and qualities and whether the predation is fought ($x_i^s = x^d/2$) by the rival or not ($x_i^s = x^d$). The price and the quality will then depend on the particular form of the function cost and on the competition existing on the market. I am now able to find several equilibria of the game where two firms compete in quality as well as in price according to whether one firm or both respects one or more of the conditions illustrated. One important thing must yet be noted. If a firm has expected future monopolist income that is higher than that of its rival, this is due to the higher discount factor it utilizes, but this also implies that a higher weight must be assigned to future costs today. This in turn means that in order to force the weak firm to quit the market, the leader needs not only to have a future monopolist income higher than its rival but also a *sufficiently* higher income. Indeed when the predation lasts for multiple instants of time, a firm will force the other out the market when it has an expected income that is high enough to assure that the period beyond which such gains are lost is farther away than its rival's.

The simultaneous predation strategies that can be played by firms in the game introduced here are manifold. The description of all of these strategies is tedious and may not contribute to the ultimate purpose of this analysis, i.e. investigating how reputation increases competition, bringing about higher quality to be introduced onto the market and the modification that it conveys to the latter's structure. The work can thus be restricted to focus on a particular class of strategies in order to obtain generalized results that are easier to argue. Consider the following definition:

Definition 1. *A highly competitive successful predation strategy (c.s.p.) for firm 'i' (or 'j') of the dynamic game* $\Gamma(p_{it}, p_{jt}, e_{it}, e_{jt}, \bar{q}, p^*, \Phi_{it}, \Phi_{jt}, \beta(q_t))$ *is a function that assigns to any amount K a string of product prices and exerted efforts that if played cause:*

[17] This exit-rule is somewhat arbitrary, but it could be interpreted, off the equilibrium-path, as a firm that erroneously *read* the rival's payoff. That is, a firm deviates from the equilibrium strategy, subsequently realizes to be weaker than its rival, and quits the market after losses (to fight predation) are realized and future incomes are surely not to be gained.

a) *the rival's market exit after a finite period of time $\alpha \geq 0$;*

b) *the expected payoff, given by the difference between the expected monopolistic income and such an amount, to be positive;*

and so that:

c) *at any instant of time p during predation the rival cannot unilaterally improve its instantaneous (or per period) expected income given by the residual expected income after p predation periods.*

That is:

$\Phi_{it}: \forall K \in R \rightarrow (p_0, e_0) \in R^{\alpha} \times R^{\alpha}$ *with* $\alpha \in R^{+} + \{0\}, p_0 = \{p_v \in [0, p*] \ \forall \ v \in [t, t+\alpha]\}$ *and* $e_0 = \{e_v \in [0, \bar{e}] \ \forall \ v \in [t, t+\alpha]\}$;

so that if (p_0, e_0) *is played then:*

a) $\forall \alpha \geq 0 \quad x^s_{it+\alpha} = x^d$ *and* $x^s_{jt+\alpha} = 0$,

b) $e^{-\delta_i \alpha} \int_{s=0}^{T} e^{-\delta_i s} \pi(p^*, e_M, x^s = x^d) - K > 0$;

c) *for firm 'j' (or alternatively 'i')*

$$\int_{p=0}^{\alpha} e^{-\delta p} \left[G - \int_{s=0}^{p} \pi_{js}(p_{js}, e_{js}, x^s_{js})ds \right] dp$$

where $G = e^{-\delta_j \alpha} \int_{s=0}^{T} e^{-\delta_j s} \pi(p^*, e_M, x^s = x^d)\, ds = \frac{\pi(p^*, e_M, x^s = x^d)}{\delta_j} \left[e^{-\delta_j \alpha} - e^{-\delta_j (T+\alpha)} \right]$

cannot be unilaterally enhanced.

A firm's successful predation strategy thus gives it a string of prices and efforts that can be invested in order to force the rival out of the market. It is however still possible that such a string cannot exist for firm 'i' and/or firm 'j'. In the case that multiple predation strategies exist for a firm, I focus the analysis on a particular subset in order to specifying some of the game's equilibria and draw useful conclusions. The strategies that I consider for this game are that ones that minimize the length of predation, i.e. minimize α, the period beyond which the expected monopoly profits of one firm are lost. The reason why I consider this particular class relies upon its specific characteristic: these strategies are, among all the highly competitive successful predation strategies, the easiest to derive and imply the existence of a c.s.p. (and vice versa) so that deriving the conditions under which such strategies can be successfully played means that any other highly competitive predation can always be played and a structural market change is always expected to occur. To see how this occurs, consider the case that in a particular instant of time the rival expected monopoly income of a 'leader' firm is positive. In order to exclude its rival from the market, the leader can choose either to invest an amount for more periods that those for which its rival has no convenience to deviate from, for example half of the rival per period expected income in order to satisfy half-market demand (because of competition and hypothesis (ix)), or invest an amount that causes the instantaneous exit of a firm, that is investing at least the rival's total expected income to provide goods, yet for half-market demand. A lower expected income for a firm means that the latter's discount factor is lower than its rival and so, for more predation periods, the instantaneous changeable amount that can be sustained by such a firm becomes smaller and less expensive for the leader. However, in order to minimize α, the more patient firm cannot take advantage of this property and will therefore pay today an amount higher than the discounted string of half of all the per period rival's expected income. The subset of all of the strategies that minimize α is composed by all of the amounts that imply a cost to be sustained at least equal to the per period rival's expected income bounded by the maximal loss endurable in any instant of time in any period. The following definition can thus be considered:

Definition 2. *The highest competitive successful predation strategy (h.c.s.p) for firm 'i' (or 'j') of the game* $\Gamma(p_{it}, p_{jt}, e_{it}, e_{jt}, \bar{q}, p^*, \Phi_{it}, \Phi_{jt}, \beta(q_t))$ *is the competitive successful predation strategy that minimizes α and from which firm 'i' (or 'j') has no convenience to deviate in any instant during predation.*

In Appendix 2 it is shown by induction that the h.c.s.p. is unique and thus from a non-empty subset of all the α minimizer strategies, it is always possible to elicit a single function that assigns a h.c.s.p. to any amount K satisfying the c.s.p. definition that indicates a string of prices and efforts to play during predation. It is also noted that if a h.c.s.p. does not exist, there is no c.s.p. that can be successfully played.

Once the class of strategies I look for has been described, the equilibria of the game will be drawn by considering the following final hypothesis:

(x) If the highest competitive successful predation exists for a firm, then that firm will always play it.

The problem now is of deriving the conditions under which a h.c.s.p. strategy exists. In line with the analysis that has been developed so far, the necessary and sufficient condition that performs this function is outlined here:

Theorem 1. *In the dynamic game $\Gamma(p_{it}, p_{jt}, e_{it}, e_{jt}, \bar{q}, p^*, \Phi_{it}, \Phi_{jt}, \beta(q_t))$ where assumptions (i)-(ix) hold, the highest competitive successful predation strategy exist for firm 'i' (or 'j') if and only if it is valid that, for finite values of monopolist income :*

If $\int_{s=0}^{T} e^{-\delta_j s}\ \pi(p^*, e_M, x^s = x^d)\, ds \leq -\pi(o, e_M, x^s = x^d/2)$:

$$\frac{1-e^{-\delta_i T}}{\delta_i} > MAX \left\{\frac{1-e^{-\delta_j T}}{\delta_j}\ ; -\ max \left\{\frac{\pi\ (p_0 - \varepsilon, \bar{e}, x_i^s = x^d)}{\pi(p^*, e_M, x^s = x^d)}\ ;\ \frac{\pi(p_0, \bar{e} + \varepsilon\ , x_i^s = x^d)}{\pi(p^*, e_M, x^s = x^d)}\right\}\right\}$$

with $p_0 = c\bar{e}$ and $\varepsilon \rightarrow 0$

If $\int_{s=0}^{T} e^{-\delta_j s}\ \pi(p^*, e_M, x^s = x^d)\, ds > -\pi(o, e_M, x^s = x^d/2)$:

$$\left[1 - e^{-\delta_i T}\right] > \frac{K_p\left[1 - e^{-\delta_i \alpha^*}\right]}{e^{-\delta_i \alpha^*}\ \pi(p^*, e_M, x^s = x^d)}$$

where $K_p = -\pi(o, e_M, x^s = x^d/2)$ and $\alpha^*(K_p) = -\frac{1}{\delta_j} \ln\left[\frac{K_p}{\pi(p^*, e_M, x^s = x^d)\left[1 - e^{-\delta_j T_j}\right] + K_p}\right]$;

Sketch of the proof and discussion

To sketch the proof, assume that firm 'i' is the leader (that is with a higher expected payoff), than it can invest an amount to be shared and shared alike that is exactly equal (for the continuity of the profit function) to the payoff expected by its rival, which will be compelled to exit the market by way of hypothesis (ix). However, as the single period investment cannot exceed its maximum, firm 'i' can face two different situations. If j's expected payoff does not exceed the maximum investment that firms can share, that is

$$\int_{s=0}^{T} e^{-\delta_j s}\ \pi(p^*, e_M, x^s = x^d)\, ds < -\pi(o, e_M, x^s = x^d/2)$$

then firm 'i' will succeed in outperforming 'j' if it simultaneously has a payoff that is higher than both the latter and the minimal instantaneous loss given by (3). If instead firm 'j' has a payoff that is higher than the maximum per period shared investment, firm 'i' will impose the worst possible lost in order to minimize α at any instant, but naturally a single instant will not suffice to force firm 'j' out of the market and predation will therefore last longer. However, firm 'i' discounts such a constant predation cost at a higher discount rate, implying that the extended competition will cost more for it than for firm 'j'. The second case therefore requires not only that firm 'i' have a payoff higher than its rival, but also high enough to continue the predation as long as firm ‘j’ exits the market first because of the irrecoverable losses suffered.

The formal proof, the derivation of the conditions and the discussion on the existence of the solution are reported in Appendix 2.

4.4 Matrix representation of the game

Following the introduction of all of the game’s conditions and assumptions, I now solve it and obtain its equilibria according to which conditions hold for one or both firms. For this purpose it is useful to represent the game in its matrix form and thus show the firm’s payoffs. I use a 2×2 matrix where 2 strategies are allowed for both firms: these are play predation ‘P’ and do not play predation ‘DP’. The equilibria predicted by the game when (4) does not hold for both firms are not included here but instead shown in Appendix 3; the current analysis focuses on the results obtained when (4) always holds and in the cases in which the firms are perfectly symmetric, and when instead a firm has a higher expected income and a dominant strategy to pursue.

I begin the analysis considering the situation in which $\Delta = 0$, that is, $\frac{1-e^{-\delta_i T}}{\delta_i} = \frac{1-e^{-\delta_j T}}{\delta_j} \leftrightarrow \delta_i = \delta_j = \delta$ and the firms are perfectly symmetric. In such a case there is an incentive for both the firms to invest alone onto the market given by the fulfillment of positive profits after predation takes place. Assuming that the monopoly payoff for both firms is less than the limit one-period investment, i.e. that

$\left[e^{-\delta\alpha}\right] \int_{s=0}^{T} e^{-\delta s}\, \pi(p^*, e_M, x^s = x^d)\, ds \leq -\pi(o, e_M, x^s = x^d/2)$ holds, the payoffs matrix is determined as:

Matrix 1

		Firm j: DP	Firm j: P
Firm i	DP	0 , 0	N 0 , z_1
	P	N z_1 , 0	x_1, x_1

where $x_1 = -\pi(p^*, e_M, x^s = x^d)\,\frac{1-e^{-\delta T}}{\delta} = -K$ and $\alpha \to 0$ and

z_1 is the value arising from equation (4).

For the exit-rule of the game and the minimization problem, if firms plunder contemporaneously they will exit the market instantaneously so that $\alpha \to 0$ and will suffer losses exactly equal to their expected monopolistic income. Indeed during predation the total amount that will be invested by firms onto the market will be

$$2K = 2 \cdot \pi(p^*, e_M, x^s = x^d)\,\frac{1-e^{-\delta T}}{\delta} \leq -\pi(o, e_M, x^s = x^d)$$

and it will be shared and shared alike among firms, causing the zeroing of their monopolistic income and their exit from the market. In other words, firms competing with the same 'arms' make consumers indifferent to the traded goods and no shift of customer base will occur so that the total demand will still be equally divided and the single firm losses will amount to K, that is their expected future income. Competition will take place, by the characteristic of the market demand, first for price and then for quality where the instant of such a transition will be determined by the specific cost function employed in the model[18]. If only one firm plays the predation strategy, its payoff is indicated by z_1 in (4). In this case the amount to invest, for a single firm, is the lowest necessary to exclude the rival from the market and the final expected value gained after predation is played will be positive.

As can be seen, no firm has a dominant strategy to play: P is convenient for a firm only if it expects its rival to play DP. In this game we have two easily verifiable pure equilibrium strategies given by (P,DP) and (DP,P), but seeing that there is not a dominant strategy among them, there is a further equilibrium in mixed strategies for the two firms. Formally denoting the probability of firm 'i' to play DP as μ, it will therefore choose a mixed strategy in order to make its rival indifferent between playing 'P' or 'DP'. The utility of playing 'DP' is equal to 0 for 'j', while the utility of playing 'P' is $\mu \cdot z_1 + (1-\mu) \cdot x_1$ in this case. If 'j' is indifferent between the two strategies, it must be valid that:

(8) $\mu \cdot z_1 + (1-\mu) \cdot x_1 = 0 \quad \leftrightarrow \quad \mu = -\frac{x_1}{z_1 - x_1}$

This is the probability that makes firm 'j' indifferent in the face of the two strategies. This probability will be the same for firm 'j' given the game's symmetry. I thus expect firms 'i' and 'j' to play DP with a

[18] The competition is a success only if it resets the rival customer base. This implies that firms will allocate the per-customer budget by dividing the total amount expected by the entire market demand, and after that a particular strategy of price and quality according to the particular cost function will be selected.

probability $\mu = -\frac{x_1}{z_1 - x_1}$ and play P with a probability $(1 - \mu) = \frac{z_1}{z_1 - x_1}$; this will therefore be the mixed equilibrium of the game.

Instead, when the monopolistic profits expected by both firms exceed the instantaneous maximum equally divided cost, i.e. $e^{-\delta\alpha} \int_{s=0}^{T} e^{-\delta s}\ \pi(p^*, e_M, x^s = x^d)\, ds > -\pi(o, e_M, x^s = x^d/2)$, then Δ=0 implies that, for both the firms $[1 - e^{-\delta T}] = \frac{K_p\,[1 - e^{-\delta\alpha*}]}{e^{-\delta\alpha*}\,\pi(p^*, e_M, x^s = x^d)}$ and the condition of Theorem 1 is not verified. The simultaneous predation of both firms will cause their contemporary exit after the same period of time, computed by $\alpha*\,(K_p) = -\frac{1}{\delta}\ln\left[\frac{K_p}{\pi(p^*, e_M, x^s = x^d)\,[1 - e^{-\delta T}] + K_p}\right]$.

The value of $\alpha*$ is minimized because in any instant each firm will suffer the maximal equally divided loss permitted by the market, that is $K_p = -\pi(o, e_M, x^s = x^d/2)$. The equal pay-off expected in the case of common predation is thus given by $x_1 = -\frac{K_p}{\delta}\,[1 - e^{-\delta\alpha^*}]$, whose derivation is reported in App. 2.

The only difference from the previous case is that the reciprocal predation will now last more than a single instant, and the losses are incurred over a longer period of time indicated by $\alpha* > 0$. The firms' choice of investing the maximal instantaneous amount in any period is always the best response to the rival's actions, as no firm can unilaterally increase its payoff by deviating from this strategy and represents the equilibrium of the game. When the expected monopolistic firm income is instead lower than such a level, suffering the highest lost is the shortest way to force the rival out the market for both firms, providing a solution to the predation problem. In this second case so the latter do not vary and the only difference is represented by the x_1's value in Matrix 1.

If the firms are not symmetric and supposing that firm 'i' holds to the conditions in Theorem 1, the predation strategy is a dominant strategy for the latter. The incentive to build a reputation is greater enough to begin competitive dynamics that surely change the market structure. To see how this happens, consider the case in which firm 'j' working as a monopolist expects a future income equal to $[e^{-\delta\alpha}]\int_{s=0}^{T} e^{-\delta_J s}\ \pi(p^*, e_M, x^s = x^d)\,ds$ and that such an income is less than $-\pi(o, e_M,\ x^s = x^d/2)$. If predation is employed by both firms, firm 'i' investing an instantaneous amount exactly equal to this expected income will force firm 'j' to exit the market immediately after predation begins, that is $\alpha \to 0$ because of the hypothesis (ix), thus solving the predation problem. For firm 'j' playing the predation strategy in this way will bring about losses equal to its expected income as in the previous case, but now firm 'i' instead obtains a positive payoff given by the difference between its monopolistic expected income and that of its rival.

The matrix is thus given by:

Matrix 2

		Firm j: DP	Firm j: P
Firm i	DP	0 , 0	0 , z_2
	P	N z_1 , 0	x_1, x_2

Where

$$x_2 = -\int_{s=0}^{T} e^{-\delta_i s}\ \pi(p^*, e_M, x^s = x^d)\, ds = -\pi(p^*, e_M, x^s = x^d)\ \frac{1-e^{-\delta_j T}}{\delta_j} = -\ K < 0$$

and $x_1 = \pi(p^*, e_M, x^s = x^d)\ \frac{1-e^{-\delta_i T}}{\delta_i} - \text{K} > 0$ and $\alpha \rightarrow 0$.

If firms invest alone on the market, given that (4) holds for both, the expected pay-offs are positive and are given again, respectively for firms 'i' and 'j', by:

$$z_1 = \pi(p^*, e_M, x^s = x^d)\frac{1-e^{-\delta_i T}}{\delta_i} - Max\ \{\pi\,(p_0 - \varepsilon\,,\, \bar{e}\,,\, x_i^s = x^d)\,;\, \pi(p_0\,,\, \bar{e} + \varepsilon\ ,\, x_i^s = x^d)\ \}$$

and

$$z_2 = \pi(p^*, e_M, x^s = x^d)\frac{1-e^{-\delta_j T}}{\delta_j} - Max\ \{\pi\,(p_0 - \varepsilon\,,\, \bar{e}\,,\, x_i^s = x^d)\,;\, \pi(p_0\,,\, \bar{e} + \varepsilon\ ,\, x_i^s = x^d)\ \}.$$

In addition, if it is convenient for both of the firms to employ the predation strategy alone, only firm 'i' has a dominant strategy, which is playing 'P' in any case. The best that firm 'j' can do is to exit the market immediately after predation starts; it can otherwise expect losses. The equilibrium of the game is thus represented by the pair of strategies formed (P,DP) respectively for firm 'i' and firm 'j'.

As in the symmetric case, it is possible that the monopoly value for firm 'j' is higher than the maximal instantaneous loss that firm 'i' can bear, i.e. $\int_{s=0}^{T} e^{-\delta_j s}\ \pi(p^*, e_M, x^s = x^d)\, ds > -\pi(o, e_M, x^s = x^d/2)$.

In order to make the predation of firm 'i' a success, it is required that the latter verifies $[1 - e^{-\delta_i T}] > \frac{K_p\, [1 - e^{-\delta_i \alpha^*}]}{e^{-\delta_i \alpha^*}\, \pi(p^*, e_M, x^s = x^d)}$; such a condition guarantees that the expected monopolistic income of firm 'i' is not fully eroded after the finite period of time beyond which firm 'j' quits the market, so that predation is actually a feasible strategy for it.

In this case, the game's payoffs will be given by (see Appendix 2) :

$$x_2 = -\frac{K_p}{\delta_j}\left[1 - e^{-\delta_j\alpha^*}\right] < 0 \qquad \text{with } K_p = -\pi(o, e_M, x^s = x^d/2)$$

and $$\alpha*\left(K_p\right) = -\frac{1}{\delta_j}\ln\left[\frac{K_p}{\pi(p^*,e_M,x^s=x^d)\left[1-e^{-\delta_j T}\right]+K_p}\right].$$

On the other hand, firm 'i', the leader, will earn a profit equal to:

$$x_1 = \frac{\pi(p^*,e_M,x^s=x^d)}{\delta_i}\left[e^{-\delta_i\alpha^*} - e^{-\delta_i(T+\alpha^*)}\right] - \frac{K_p}{\delta_i}\left[1 - e^{-\delta_i\alpha^*}\right] > 0$$

It is clear that also in this case firm 'i' will always employ the predation strategy and the equilibrium will again be given by the pair (P,DP).

Before concluding the equilibrium analysis of the game, it is important to point out the role played by the lack of constraints on the quantities produced by the firms. Indeed, if such constraints existed, there would be no incentives to invest in quality and plunder the market. For the firms, the reason for investing in quality is dictated by the need to reduce the probability of an exit from the market. In the presence of production constraints for high quality producers, such an incentive fails because lower quality goods can be sold to customers who can't buy higher quality ones, but this in turn implies that the market demand of those producers who are unable to introduce high quality never resets, and exit never occurs. In other words, each firm would be a monopolist of a market share removed from any kind of competition and therefore from any incentive to compete in terms of price and quality; in this game, this makes reputation an uninteresting matter.

4.5 Welfare analysis

In this section a brief discussion of what happens from a social point of view when a market transition occurs is reported. In cases 1 and 2 shown in Appendix 3, the game's equilibria implied zero profits for both firms and no reputational process begins in order to increase the customer base. In these situations the utility for customers remains the same throughout the game, i.e. equal to $u^i(p_t = c_{\bar{e}}, q_t)$ and nothing is changed compared to the game's initial equilibrium.

According to the conditions reported above and due to the same technology shared by firms, which implies the same cost function[19], the total profit Π_T in the initial game equilibrium will thus be given for firms 'i' and 'j' by:

$$\Pi_T = \Pi_{it}\ (p_{it}, e_{it}, x_i^s) + \Pi_{jt}\ (p_{jt}, e_{jt}, x_j^s) \leftrightarrow$$

$$p_{it} \cdot x_{it}^s - C(x_{it}^s, e_{it}) + p_{jt} \cdot x_{jt}^s - C(x_{jt}^s, e_{jt}) = \frac{1}{2}(p_{it} - C_{ix}) + \frac{1}{2}(p_{jt} - C_{jx})$$

where the last passage is obtained remembering the market equilibrium condition, i.e. that supply always equals demand[20] and the equality between average and marginal cost. Both firms in this case receive a null pay-off, i.e. their profit equals zero because competition entails prices equivalent to the marginal cost of producing the standard quality $\bar{q}$, i.e. $C_{ix} = C_{jx} = c_{\bar{e}} = p_{it} = p_{jt} = p_t$. It's easy to understand that in such a situation, no incentive is given to firms to increase the quality of their products and consequently no investment will take place. Now consider instead the equilibrium illustrated by Matrix 2. Here we have only one firm, in this case 'i', investing in quality which captures the whole market. If the condition in Appendix 1 holds here, we expect such a firm to provide high effort, offer the maximum product quality and charge a price equal to p^*. Firm 'i' will then earn a profit given by:

$$\Pi_T = \Pi_i = p^* - c_{QM}$$

where c_{QM} represents the average unit cost (or marginal cost) of producing an additional unit of the highest-quality product. At variance with the former situation, the customer surplus is now reduced to zero because of the highest price imposed by firm 'i', but a utility increase occurred for them derived from the augmented quality introduced onto the market. In this situation the customer's price surplus totally vanishes on behalf of the firm's profit. However, in comparison to the previous case, buyers now enjoy the consumption of the qualitative enhanced good. The assumption with regard to the utility function shows however that customers strictly prefer to pay less over receiving higher quality products as long as the quality is not too low, i.e. inferior to $\bar{q}$. Such a hypothesis highlights the higher elasticity of demand for price compared with quality, typical of those markets where the goods traded are almost fungible. This relation can be expressed, in terms of this model, by:

$$u^i(p_t = c_{\bar{e}}\ , \bar{q}) < u^i(p^*\ , \bar{q} + w)\ \ \forall\, w \in [0, q_M - \bar{q}]$$

we observe, in the transition between the initial equilibrium and the new high-quality equilibrium, that buyers experience a net utility decrease even if the quality on the market is appreciated. The price

[19] Remember that the equality between marginal cost and average unit cost was stated in paragraph 3.

[20] That is, $x_i^s + x_j^s = 1$ because of normalization to 1, and in the initial equilibrium $x_i^s = x_j^s$.

customer surplus $(p^* - c_{\bar{e}})$ adds to the firm's profit, breeding a ***reputational quasi rent*** for the latter that is linked to the market structure transition from duopoly to monopoly. It is *quasi* because it is not fully gained by the monopolist, given that introducing high quality increases the production marginal cost $(c_{\bar{e}} \rightarrow c_{QM})$, which in turn implies a reduction of its profit. Such a rent is exactly what is required by the market to provide the firm with incentives to exert high effort and make the promised quality a *self-enforcing* agreement. I can thus obtain a further conclusion from the model: improving on quality is plausible only if customers are ready to renounce their price surplus (and thus receive a lower utility) on behalf of the inflated firms' profits, which furnish the right incentives to investments. The self-enforcement of the agreement is therefore not a costless process but entails a surplus shift from consumers to producers to make it function. With regard to the total welfare generated by the market the model cannot predict worsening or improvement without any further assumption on the cost and utility functions. To conclude the analysis I would like to highlight that during predation, the consumers' welfare improves because the introduced higher quality is sold at a lower price. However, after successful predation occurs I expect the mass of consumers to suffer and the surplus to shift from customers to the monopolist. An increase in a good's quality linked with a structural change is thus not a plausible reason to relax antitrust scrutiny. The basic trouble originates with the lack of competition when monopoly is created; in other words, if competition could always operate on the market[21], predation would not suffice to assure positive profits to the leader. In such a case we should imagine that alternative strategies be carried out along with predation, as, for example, investments to lower production costs. Lowering costs could mean offering a higher quality and charging the same price as before the investment in quality occurred, again moving the reputational rent from monopolist to customers.

Another way to increase customers' wealth relies upon the size of $\bar{q}$. A central planner could decide to pump up the value of $\bar{q}$ after a firm captures the market. A higher value of $\bar{q}$, reducing a firm's single-period profit via a higher effort to be exerted to avoid exit has a positive effect on the customers' surplus. Guaranteeing a higher standard quality to the market therefore implies higher firm costs but as effort can't exceed the maximal value e_M, the probability of exit will rise and competition will become more frequent to a certain extent.

[21] That is if predation doesn't create a monopoly but just a reduction in the number of firms, or alternatively if we permit the plundered firm to pressure the monopolist from outside the market.

5.Comparative Statics

In this section I consider two of the model's parameters (δ and $\bar{q}$) and the consequences deriving from the variation of the values. The arguments exposed here have a broad meaning and are still valid even if condition (4) of Appendix 1 is not satisfied, so the validity of such a condition may also not apply to the results obtained.

We may begin by considering a reduced value of δ for firm 'i'[22]. Consider a shift of δ_i to a new, lower level $\delta_i^{'}$, meaning that firm 'i' will be more patient than before and that this will lead to higher expected future profits. It holds that:

(12) $max_{e_{is}} \int_{s=0}^{T} \mathrm{e}^{-\delta_i^{'} s}\ \pi_{is}(p^*, e_{is}, x^s = x^d)\ ds > max_{e_{is}} \int_{s=0}^{Ti} \mathrm{e}^{-\delta_i s}\ \pi_{is}(p^*, e_{is}, x^s = x^d)\ ds$

This has an important implication for this model: a higher inter-temporal discount factor increasing the future income expected by a firm makes predation more profitable (constraints (4) and (7) are relaxed) and consequently the rival is more likely to exit.

Given that δ_i is the only variable that differs among firms, we also know what happens when the discount factors of the two firms have the same value. If for instance $\delta_i = \delta_j$, the future income expected by both the firms is the same (i.e. Δ = 0) and we consequently see the situations described by Matrix 1.

The implication of this simple analysis leads to an easily verifiable conclusion, i.e. *a firm will employ a predation strategy if and only if it satisfies (4) and has an inter-temporal discount factor that is sufficiently higher than that of its rival.* The essential role played by the discount factor in such a model clearly emerges here: only firms with more patience than their rivals can assume the title of leader on the market. To the limit, for a very high value of the discount factor, the monopolist income for firm 'i' becomes:

$\lim_{\delta_i \to 0} \int_{s=0}^{Ti} \mathrm{e}^{-\delta_i s}\ \pi_{is}(p^*, e_{is}, x^s = x^d)\ ds = \int_{s=0}^{Ti} \pi_{is}(p^*, e_{is}, x^s = x^d)\ ds$

which means that firm 'i' (or 'j') gives the same weight to future pay-offs as to present ones, permitting firm i to invest widely to capture the market and force its rival to quit[23].

[22] The same reasoning obviously holds for firm j.

[23] However, the decision to invest in quality will always depend on the realization of the condition indicated in Appendix 1. The higher the discount factor is, the easier reputation is to build, but it is not enough to guarantee that investment and predation occur.

Now consider a variation in the level of $\bar{q}$. If $\bar{q}$ increases, one expects that the exit will be more difficult to avoid (or alternatively that there will be a shorter monopoly period) and a firm that wants to plunder its rival needs to exert a higher effort than before the new standard was introduced. Indeed it holds that:

(13) $\frac{\partial F(\bar{q} - e_{it})}{\partial \bar{q}} > 0$

i.e. the derivative of F on $\bar{q}$ is strictly positive. Increasing $\bar{q}$ thus reduces the monopoly period for a firm and consequently the future income that it can earn. Intuitively, it is simpler to be a monopolist when the standard requested for quality is relatively low, because the effort necessary to force customers to shift from one firm to another is also lower. However, when $\bar{q}$ increases, it is difficult to satisfy the requests of the market and the leadership of a firm is unlikely to endure. One further consideration can be developed here. When the period of time for a firm as a monopolist is short enough, it could be convenient for such a firm to deviate from the strategy of providing high-effort and offering low-quality at price p*, thus risking exit. In other words, when the standard quality is very high and the difference $\bar{q} - e_M$ is not so negatively large (or at most positive), then exit is highly probable to occur and firms expect to earn incomes as monopolists for few periods. As long as hypothesis *(viii)* holds, the firms know the profits gained from carrying out a low-quality strategy and the profits earned from being a monopolist for a few periods: if the former exceed the latter, low effort strategies cannot be ruled out by the model. To the limit, for a very high value of $\bar{q} - e_M$[24], the exit will occur every period due to $F(\bar{q} - e_M) \cong 1$, and the game is of the one-shot kind so that the classical Bertrand equilibrium will be selected and high effort will never be exerted[25].

[24] One can think that new technologies are developed on the market that immediately involve a boost of $\bar{q}$ with the need to increase, in a dynamic perspective, the efficiency of labor. At the moment the standard is introduced, the quality of labor and consequently the maximum level of effort to be provided are too low relative to the new technologies introduced. In this way we could momentarily have a very high value of $\bar{q} - e_M$.

[25] Here I arrive at a generalization of the conclusion enunciated in *Klein and Leffler [1981]* for a competitive market but with something additional. As in their model, the only incentive for supporting high-effort is given by the price. Instead in a non-cooperative duopoly market, a higher price can exist but high effort could not be exerted in the same way if the time needed to take advantage of it is too short. That means that a short monopoly time can demotivate high quality from being introduced on the market and facilitate a firm's moral hazard.

6. Conclusion

This paper investigated the relation between the cost of reputation, competition strategies, and market structure. In particular I found that the transition from duopoly to monopoly, via reputation, involves a costly process underpinned by the inter-temporal discount factor and other exogenous variables of the firms, such as the standard quality requested by the market and the maximum price at which buyers are available to pay for the traded product. The conclusion presented here is that firms that are more patient with regard to future incomes can more easily support short-term losses in order to increase their market shares over the long-term. The initial losses are due to competition inside the market, which in turn determines the former's extent. When a firm is really patient on the market, its initial investment is broad and more plucky strategies can be executed. This is why, in the initial life cycle of such firms, we often observe strong promotions for traded products, such as freebies or wide discounts[26]. Moving from a duopoly market to the oligopoly, the conclusions can be generalized to take this new market structure into account. As before, firms with more patience can increase their customer base by betting on higher quality and inducing buyers to shift from low-quality producers to high-quality ones[27]. The model also warns of the consequences for society due to a change in the structure of the market. Indeed, if on one hand we expect that firms who take care of their reputation have an interest in increasing and preserving the quality of the good, on the other hand the concentration of market shares in a few firms can reduce the wealth of buyers via a higher price to be paid following such a concentration. Obviously, this matter is more marked in a duopoly than in an oligopoly but in general when customers display a strong elasticity to variation in price, augmented quality is not a sufficient reason to relax antitrust scrutiny. However, customers are often available to pay more and receive a higher quality product; this diminishes the extent of the problem but is not generally true, especially when we consider extremely fungible products or when liquidity constraints can be observed. A potential remedy to this paradox can be found in the capitalization of economies of scale and consequently in the reduction of production costs in order to avoid an excessive price increase. From this point of view, specialization can lead to increasing welfare in a dynamic prospective, not only through a reputation as a high-quality producer, but also via a reputation as an *efficient* producer. This could be an additional argument to be developed in future works.

In conclusion, two further observations must be reported here. The first relies upon the meaning of the exit hypothesis, while the second concerns the interaction between discount factor and credit market. In

[26] The same reasoning is followed in *Hörner [2002]*, with the difference that the author allows price to be negative when new firms enter the market; the conclusion remains the same.

[27] The role of competition in this model thus reinforces the idea exposed in *Dana Jr and Fong [2008]*. It is therein presumed that oligopoly is better than monopoly or perfect competition in preserving reputation and high quality.

the first case, exit can be thought as something that occurs when a firm loses the confidence of all buyers (and not only its loyal clients) after it adopts a morally hazardous behavior. Indeed, the exit hypothesis regards the fact that if a firm fails to produce what the market requests, a shift of its customer base toward some other firms will occur. Abandoning the market can thus be a strong deterrent against morally hazardous behaviors, but this is so only if customers can move easily from one firm to another and beliefs can be promptly updated. Such a concern then emphasizes the role of the social network in informing the market of the firms that are cheating. In the absence of an efficient flow of information between buyers, a central planner should sustain the cost of providing information; however, also in this case troubles arise regarding the ability of such an institution to manage information and its monitoring.

With regard to the role of the interaction between credit market and discount factor, it seems reasonable that the willingness of a firm to wait for future incomes and bear short-term losses depends on the financial resources at its disposal. In the presence of a firm's liquidity constraints, a basic role is held by financial institutions. An efficient and perfectly competitive financial market should assure credit to the firms that wish to profitably and enduringly invest in quality. The role of the credit market in assuring quality is a matter of extreme importance whose implications are noteworthy: a constriction of the credit to firms could cause not only a dearth of the new entrepreneurs necessary to guarantee competition, but also a slowing down in the introduction of higher quality onto the market.

Appendix 1: Condition assuring high effort

In this section I derive the condition under which a firm operating as a monopolist following a rival's exit will always exert high effort. I first write the function of the profits for firm i (the same holds for j) given by:

$$\pi_{it}(p_{it}, e_{it}, x_{it}^{s}) = p_{it} \cdot x_{it}^{s} - \mathrm{C}(x_{it}^{s}, e_{it})$$

The instantaneous profit in a monopoly is maximized when the level of effort exerted is minimal, with the constraint that the standard cannot be lowered to avoid exit when the price is equal to the maximal level payable by consumers. This is the objective function, and is given by:

1) Max e_{it} $\pi_{it}(p*, e_{it}, x_{it}^{s}) = p* \cdot x_{it}^{s} - \mathrm{C}(x_{it}^{s}, e_{it})$

   s.t.

   $e_{it} \in [\, \bar{\mathrm{e}},\ \mathrm{e_M}\,]$

We know that $\frac{\partial \pi_{is}(p, e_{it}, x^{s})}{\partial e_{it}} < 0$ so that the effort level chosen for the objective function will naturally be $\bar{\mathrm{e}}$. With such an effort, firm 'i' will operate on the market as a monopolist for a period that is expected to be equal to $\mathrm{T}_1 = \frac{1}{\mathrm{F}\,(\bar{q} - \bar{\mathrm{e}})}$, and it will gain profits given by:

2) $\int_{s=0}^{T_1} e^{-\delta_i s}\ \ \pi_i(p^{*}, \bar{\mathrm{e}}, x^{s} = x^{d})\, ds$

On the other hand, we know that the period of monopoly for firm 'i' will be maximized when F is minimum. We already know that $\frac{\partial \mathrm{F}(\bar{q} - \mathrm{e_{it}})}{\partial \mathrm{e_{it}}} < 0$ so that T is maximum for an instantaneous constant effort equal to $\mathrm{e_{it}} = e_M$. In this case, the function of T will be given by $\mathrm{T}_2 = \frac{1}{\mathrm{F}\,(\bar{q} - e_M)}$, and it holds that $\mathrm{T}_2 > \mathrm{T}_1$. The monopolistic payoff is instead given by:

3) $\int_{s=0}^{T_2} e^{-\delta_i s}\ \ \pi_i(p^{*}, e_M, x^{s} = x^{d})\, ds$

If the profits obtained by providing a high-effort will be higher than the profits obtained by providing the effort necessary to achieve the standard quality, it means that the best firm i can do is provide high effort over a longer period. Such a condition is expressed by:

$$\int_{s=0}^{T_2} e^{-\delta_i s}\ \ \pi_i(p^{*}, e_M, x^{s} = x^{d})\, ds \ > \ \int_{s=0}^{T_1} e^{-\delta_i s}\ \ \pi_i(p^{*}, \bar{\mathrm{e}}, x^{s} = x^{d})\, ds$$

or alternatively by:

4) $\pi_i(p^*, e_M, x^s = x^d) > \pi_i(p^*, \bar{e}, x^s = x^d)\frac{[1 - e^{-\delta_i T_1}]}{[1 - e^{-\delta_i T_2}]}$

If (4) is satisfied, one will see firm 'i' exerting high effort throughout the entire time that the monopoly exists. It is more likely that (4) will hold when the standard quality is not so elevated and the maximum effort is high compared to the former. That is, by setting $\eta_{it} = \bar{q} - e_{it}$, we know that:

5) $\frac{\partial F(\eta_{it})}{\partial \eta_{it}} = f(\eta_{it}) \sim N(0,1)$

This implies that for very low values of $(\bar{q} - e_{it})$, $F(\bar{q} - e_{it}) \rightarrow 0$ and contemporary $T \rightarrow \infty$. This in turn implies that if the difference $(\bar{q} - e_M)$ is highly negative, we expect $T_2 \rightarrow \infty$ quickly. If this is true, condition (4) becomes:

6) $\pi_i(p^*, e_M, x^s = x^d) > \pi_i(p^*, \bar{e}, x^s = x^d)\left[1 - e^{-\delta_i T_1}\right]$.

Also if in the single instant of time high effort is not profit maximizing for the monopolist, it permits to reduce the probability of quitting the market and consequentially to protract the expected income. If condition 4) holds then the cost of increasing the effort is lower than the expected profits' increasing and the monopolist will introduce high quality also in absence of competition.

Appendix 2: Proof of Theorem 1

Proof.

Assume that (4) and (7) hold for firm 'i'. Remembering that

$$max\ \{\pi\ (p_0 - \varepsilon, \bar{e}, x_i^s = x^d)\ ;\ \pi(p_0\ , \bar{e} + \varepsilon,\ \ x_i^s = x^d)$$

is the same for both firms because of the same technology and that the solution of (1) gives an invariant value of e_{it} [28] throughout the monopoly period, then:

If for firm 'j' it is valid that

$$\int_{s=0}^{Tj} e^{-\delta_j s}\ \pi_j\left(p^*, e_j, x^s = x^d\right) ds\ <\ -\int_{v=0}^{h} e^{-\delta_j v} max\begin{Bmatrix} \pi\left(p_0 - \varepsilon, \bar{e}, x_j^s = x^d\right) \\ \pi\left(p_0, \bar{e} + \varepsilon, x_j^s = x^d\right) \end{Bmatrix} \text{ with } h \rightarrow 0$$

[28] The proof of Theorem 1 is general and doesn't require condition (4) in Appendix 1 be necessarily valid.

if it undertakes a predation strategy (or fights back), it will exit the market immediately upon undertaking the strategy due to hypothesis (ix), as it cannot recover the minimal losses that must be sustained to continue the predation.

If instead for firm ‘j’ it is valid that

$$e^{-\delta_j h} \int_{s=0}^{Tj} e^{-\delta_j s}\ \pi_j\left(p^*, e_j, x^s = x^d\right) ds > -\int_{v=0}^{h} e^{-\delta_j v} \max \begin{Bmatrix} \pi\ (p_0-\varepsilon, \bar{e}, x_j^s = x^d) \\ \pi\ (p_0, \bar{e}+\varepsilon, x_j^s = x^d) \end{Bmatrix} \text{ with h}\rightarrow 0$$

and it combats the predation of ‘i’, two cases must be distinguished.

1. $\int_{s=0}^{Tj} e^{-\delta_j s}\ \pi_j\left(p^*, e_j, x^s = x^d\right) ds \leq -\pi(o, e_M, x^s = x^d/2)$

Hypothesis *(ix)* implies firm ‘j’ can fight the predation for any amount at most equal to its total monopolist income. That means if:

- Firm ‘i’ invests an amount K strictly less than $\int_{s=0}^{Tj} e^{-\delta_j s}\ \pi_j\left(p^*, e_j, x^s = x^d/2\right) ds$ individually in a single instant of time, firm ‘j’ can do better (and consequentially become a monopolist) by investing an amount K+ε with ε→0 and obtaining a positive pay-off given by $\int_{s=0}^{Tj} e^{-\delta_j s}\,\pi_j\left(p^*, e_j, x^s = x^d\right) ds$ – (K+ε);
- Firm ‘i’ invests an amount K equal to $\int_{s=0}^{Tj} e^{-\delta_j s}\ \pi_j\left(p^*, e_j, x^s = x^d/2\right) ds$ individually in a single instant of time, firm ‘j’ cannot do better by investing an amount K+ε with ε→0 and capturing the market because the losses will double immediately and will consume the total monopolistic pay-off[29]. Firm ‘j’, however, will not exit instantaneously because of assumption (ix) and so α is not minimized.
- Firm ‘i’ invests any instantaneous amount K included in the interval

$$\left[\int_{s=0}^{Tj} e^{-\delta_j s}\ \pi_j\left(p^*, e_j, x^s = x^d/2\right) ds\ ,\ \int_{s=0}^{Tj} e^{-\delta_j s}\ \pi_j\left(p^*, e_j, x^s = x^d\right) ds\right]$$

individually, firm ‘j’ will fight the predation and will share the market with ‘i’ because of hypothesis (ix), but will not exit the market so that α is not minimized.

- Firm ‘i’ invests any instantaneous amount K included in the interval

$$\left[\int_{s=0}^{Tj} e^{-\delta_j s}\ \pi_j\left(p^*, e_j, x^s = x^d\right) ds\ ,\ \int_{s=0}^{Ti} e^{-\delta_i s}\ \pi_{\mathrm{i}}\left(p^*, e_{\mathrm{i}}, x^s = x^d\right) ds\right]$$

individually, firm ‘j’ will fight the predation only if the amount chosen by ‘i’ is equal to its expected income. Investing exactly the latter will involve the instantaneous exit of ‘j’ so that α is minimized and ‘i’ obtains a positive pay-off that is the best possible under predation. Indeed, any other amount will not satisfy the h.c.s.p. definition.

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[29] Remember the profit function is such that $\Pi(p_t, e_t, x^s/k) = \Pi(p_t, e_t, x^s)/k$.

Firm 'i' will then sustain an instantaneous loss K to be invested that will satisfy half of the market so that:

a) $K = \int_{s=0}^{Tj} e^{-\delta_j s}\ \pi_j\left(p^*, e_j, x^s = x^d\right) ds\ = \frac{\pi_j\left(p^*,e_j,x^s=x^d\right)}{\delta_j}\left[1 - e^{-\delta_j T_j}\right]$

The total market investment (firm 'i' plus firm 'j') will be given by

$$2\,K = 2 \cdot \int_{s=0}^{Tj} e^{-\delta_j s}\ \pi_j\left(p^*, e_j, x^s = x^d\right) ds$$

and the payoffs will thus be given for firm 'i' and 'j' by the following, respectively:

$x_1 = \int_{s=0}^{Ti} e^{-\delta_i s}\ \pi_i(p^*, e_i, x^s = x^d)\ ds - K = \frac{\pi_i\left(p^*,e_i,x^s=x^d\right)}{\delta_i}\left[1 - e^{-\delta_i\, Ti}\right] - K > 0;$

$x_2 = -K < 0.$

2. $\int_{s=0}^{Tj} e^{-\delta_j s}\ \pi_j\left(p^*, e_j, x^s = x^d\right) ds > -\pi(o, e_M, x^s = x^d/2)$

In such a situation it is still possible for firm 'j' to combat the predation over several periods and avoid exiting the market after one instant of time. Indeed, as π(o, e_M, $x^s = x^d/2$) is the maximum possible instantaneous investment for a firm during a simultaneous predation, i.e. providing the maximum effort, charging a price equal to zero and satisfying half the market, this limit cannot be exceeded by the firms at any instant of time. In order to minimize the predation period thus firm 'i' can impose a maximum total market amount $2K_p$ be invested in the market and to be shared and shared alike so that:

b) $2K_p = -\pi(o, e_M, x^s = x^d) \rightarrow K_p = -\pi(o, e_M, x^s = x^d/2)$

K_p will be invested during predation as long as firm 'j' quits the market and it will be discounted by the latter at the rate δ_j. In order to compute a critical value for firm 'j', beyond which future profits are lost,

$$\int_{s=0}^{\alpha} e^{-\delta_j s}\ K_p ds\ \geq e^{-\delta_j \alpha}\int_{s=0}^{Tj} e^{-\delta_j s}\ \pi_j\left(p^*, e_j, x^s = x^d\right) ds$$

must be valid for hypothesis (ix), with α being the period after which firm 'j' exits the market.

The value of the minimal α is determined by the exit-rule with the equation:

c) $e^{-\delta_j \alpha}\int_{s=0}^{Tj} e^{-\delta_j s}\ \pi_j\left(p^*, e_j, x^s = x^d\right) ds - \int_{s=0}^{\alpha} e^{-\delta_j s}\ K_p\ ds = 0$

Which, for the stationarity of the profit maximization function, in turn implies:

d) $\alpha^*\left(K_p\right) = -\frac{1}{\delta_j}\ \ln\left[\frac{K_p}{\pi_j\left(p^*,e_j,x^s=x^d\right)\left[1-e^{-\delta_j T_j}\right]\ +K_p}\right] > 0$

After $\alpha^*(K_p)$ is computed for firm 'j' we cannot exclude that firm 'i' can carry out a successful predation strategy with an instantaneous constant investment of $K_p = -\pi(o, e_M, x^s = x^d/2)$. This is because firm 'i' discounts future losses at a rate higher than its rival and this implies that firm 'i' can lose its monopolistic income after a period $\alpha_i < \alpha^*$. In order to exclude such a case, it must be valid that:

$$e)\quad e^{-\delta_i\alpha^*}\int_{s=0}^{T_i} e^{-\delta_i s}\,\pi_i(p^*, e_i, x^s = x^d)ds - \int_{s=0}^{\alpha^*} e^{-\delta_i s}\,K_p\,ds > 0$$

which in turn implies:

$$f)\ \int_{s=0}^{T_i} e^{-\delta_i s}\ \pi_i(p^*, e_i, x^s = x^d)\ ds > \frac{K_p}{\delta_i}\left[\frac{1}{e^{-\delta_i\alpha^*}} - 1\right]$$

As $\pi_i(p^*, e_i, x^s = x^d)$ is constant due to the problem's stationarity, the condition for predation to be successful thus becomes:

$$g)\ \pi_i(p^*, e_i, x^s = x^d) > \frac{K_p\left[1 - e^{-\delta_i\alpha^*}\right]}{\left[e^{-\delta_i\alpha^*} - e^{-\delta_i[\,T_i+\alpha^*]}\ \right]} > 0$$

or alternatively :

$$\left[1 - e^{-\delta_i T_i}\ \right] > \frac{K_p\left[1 - e^{-\delta_i\alpha^*}\right]}{e^{-\delta_i\alpha^*}\,\pi_i(p^*, e_i, x^s = x^d)} > 0$$

If g) holds this means that firm 'i' quits the market after a period $\alpha_i > \alpha^*$ and that predation can be successfully employed. Condition (g) thus indicates a necessary and sufficient condition for predation to be successful that must hold when $\int_{s=0}^{Tj} e^{-\delta_j s}\ \pi_j\left(p^*, e_j, x^s = x^d\right) ds > \pi(o, e_M, x^s = x^d/2)$. After α^* periods, 'j' exits the market with losses equal to

$$x_2 = -\,e^{-\delta_j\alpha}\int_{s=0}^{Tj} e^{-\delta_j s}\ \pi_j\left(p^*, e_j, x^s = x^d\right) ds = -\ K_p\int_{s=0}^{\alpha^*} e^{-\delta_j s}\ ds = -\frac{K_p}{\delta_j}\left[1 - e^{-\delta_j\alpha^*}\right],$$

while instead firm 'i' will earn a payoff given by

$$x_1 = e^{-\delta_i\alpha^*}\int_{s=0}^{Ti} e^{-\delta_i s}\,\pi_i(p^*, e_i, x^s = x^d)ds - K_p\int_{s=0}^{\alpha^*} e^{-\delta_j s}\,ds > 0\,,\ \text{that is:}$$

$$x_1 = \frac{\pi(p^*, e_i, x^s = x^d)}{\delta_i}\left[\,e^{-\delta_i\alpha^*} - e^{-\delta_i(T_i+\alpha^*)}\ \right] - \frac{K_p}{\delta_i}\left[1 - e^{-\delta_i\alpha^*}\right] > 0$$

Existence of the solution.

More generally, the condition allows the firm 'i' undertaking the predation to receive all of the discount factors for firm 'i' and firm 'j' such that:

$$h)\quad \alpha_i > \alpha_j$$

Assume now that condition (4) in Appendix 1 is verified for both firms[30] . If this is so, then both firms prefer to provide high effort. Given that the profit function and the exit probability with high effort is the same for both of them, we have that:

$$T_i = T_j = T$$

and

$$\pi_{is}(p^*, e_M, x^s = x^d) = \pi_{js}(p^*, e_{Ms}, x^s = x^d) = \pi(p^*, e_M, x^s = x^d)$$

The last equality derives from the stationarity of the objective function.

Condition (h) implies that:

$$-\frac{1}{\delta_i}\ln\left[\frac{K_p}{e^{-\delta_i t}\int_{s=0}^{Ti} e^{-\delta_i t}\,\pi_i(p^*,e_M,x^s=x^d)\,ds + K_p}\right] > -\frac{1}{\delta_j}\ln\left[\frac{K_p}{e^{-\delta_j t}\int_{s=0}^{Tj} e^{-\delta_j t}\,\pi_j(p^*,e_M,x^s=x^d)\,ds + K_p}\right] \quad \leftrightarrow$$

$$\ln\left[\frac{K_p}{\pi_{is}(p^*,e_M,x^s=x^d)\left[1-e^{-\delta_i T}\right] + K_p}\right] < \ln\left[\frac{K_p}{\pi_{js}(p^*,e_M,x^s=x^d)\left[1-e^{-\delta_j T}\right] + K_p}\right]^{\frac{\delta_i}{\delta_j}} \quad \leftrightarrow$$

$$\frac{K_p}{\pi(p^*,e_M,x^s=x^d)\left[1-e^{-\delta_i T}\right] + K_p} < \left[\frac{K_p}{\pi(p^*,e_M,x^s=x^d)\left[1-e^{-\delta_j T}\right] + K_p}\right]^{\frac{\delta_i}{\delta_j}} \quad \leftrightarrow$$

$$\frac{\pi(p^*,e_M,x^s=x^d)\left[1-e^{-\delta_i T}\right] + K_p}{K_p} > \left[\frac{\pi(p^*,e_M,x^s=x^d)\left[1-e^{-\delta_j T}\right] + K_p}{K_p}\right]^{\frac{\delta_i}{\delta_j}} \quad \leftrightarrow$$

$$i)\quad 1 + \frac{\pi(p^*,e_M,x^s=x^d)}{K_p}\left[1-e^{-\delta_i T}\right] > \left[1 + \frac{\pi(p^*,e_M,x^s=x^d)}{K_p}\left[1-e^{-\delta_j T}\right]\right]^{\frac{\delta_i}{\delta_j}} \quad \leftrightarrow$$

Note that if ‘i’ has a payoff higher than its rival, $e^{-\delta_i} > e^{-\delta_j} \leftrightarrow \delta_i < \delta_j$ and so $0 < \frac{\delta_i}{\delta_j} < 1$.

[30] The same conclusion also holds when no firm satisfies condition (4) in Appendix 1. When only one satisfies (4), the equations are slightly different but it is still possible to find α_i and α_j so that $\alpha_i > \alpha_j$.

Since $\left[1 - e^{-\delta_i T}\right] < \left[1 - e^{-\delta_j T}\right]$, we know that the left-hand side of i) is always lower than the expression within brackets on the right-hand side. Furthermore, both the right-hand and left-hand side are greater than 1. For the continuity of the exponential function, there will thus be at least a couple of δ_i and δ_j , with $\frac{\delta_i}{\delta_j} < 1$for which i) is satisfied and predation will occur with success. It is possible to show that condition i) is more likely to hold when $\frac{\pi(p^*, e_M, x^s = x^d)}{K_p}$ is higher.

The reason for this is simple: the higher the instantaneous monopolistic profit compared to the maximal instantaneous loss, the more profitable it will be to capture the market and extend the monopoly's permanence compared to the cost of predation.

Q.E.D.

Appendix 3: Equilibrium solution without predation and no investment in quality.

It is necessary to distinguish between two cases, the first where both conditions (4) and (7) are not verified for any firm, and the second where both do not satisfy (4) but they are not symmetric. In the latter case, there is still a possibility for the stronger firm to reach a positive payoff but such a chance is denied by the presence of a dominating strategy for the weak firm. When condition (4) does not hold for the firms, this means that they cannot recover the minimal loss to pursue the predation strategy and consequently the second hypothesis of Theorem 1 is never considered.

Case 1: both (4) and (7) do not hold for the two firms.

In such a case we know that any firm will not invest individually in quality (or in reducing price) because it cannot recover the losses by operating as a monopolist. Because of the invalidity of (7), no firm has a higher payoff than its rival and so the game is perfectly symmetric. The payoffs will be as follows:

a) $z_1 < 0$ *if only a firm invests*

and

b.i) $x_1\ (<\ z_1 < 0\ )$ *if both of them invest and*

$$\int_{s=0}^{T} e^{-\delta_i(=\delta_j)s}\ \pi(p^*, e_M, x^s = x^d)\, ds > Max \left\{ \begin{array}{c} \pi\left(p_0 - \varepsilon, \bar{e}, x_i^s = \frac{x^d}{2}\right) \\ \pi\left(p_0, \bar{e} + \varepsilon, x_i^s = \frac{x^d}{2}\right) \end{array} \right\}$$

with $x_1 = - K = - \int_{s=0}^{T} e^{-\delta_i(=\delta_j)s} \ \pi(p^*, e_M, x^s = x^d) \ ds = - \frac{\pi_j(p^*, e_M, x^s = x^d)}{\delta_i}\left[1 - e^{-\delta_i T_j}\right]$

or

b.ii) $Max \begin{Bmatrix} \pi\left(p_0 - \varepsilon, \bar{e}, x_i^s = \frac{x^d}{2}\right) \\ \pi\left(p_0, \bar{e} + \varepsilon, x_i^s = \frac{x^d}{2}\right) \end{Bmatrix}$ $(> x_1$ but $< z_1)$ *if both of them invest and*

$$\int_{s=0}^{T} e^{-\delta_i(=\delta_j)s} \ \pi(p^*, e_M, x^s = x^d) \ ds \leq Max \begin{Bmatrix} \pi\left(p_0 - \varepsilon, \bar{e}, x_i^s = \frac{x^d}{2}\right) \\ \pi\left(p_0, \bar{e} + \varepsilon, x_i^s = \frac{x^d}{2}\right) \end{Bmatrix}$$ [31]

I now only consider case (a) with case (b.i) while case (a) with (b.ii) can easily be obtained by changing the values of (b), with the result remaining unaltered. In this case if a firm invests alone in the market, then it will suffer losses equal to the minimal investment to force its rival to quit and it will operate as a monopolist, but given that (4) is not valid, it receives negative profits at the end of predation. When the two firms employ predation at the same time, hypothesis *(ix)* indicates that they will continue to employ it as long as their monopolist income is higher than the cost to sustain their reputation. However, in such a situation hypothesis *(ix)* and the firms' symmetry implies that $\alpha \rightarrow 0$, i.e. the firms employ predation and will exit the market together after one instant of time in which they suffer losses equal to their monopolist income[32]. The equilibrium of the game can be shown by employing the following matrix:

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[31] If the monopolistic income is less than the minimal individual investment under predation, i.e. the minimal investment to satisfy half market demand, then if firms decide to begin the predation strategy they will exactly meet this loss. Lower losses are not admitted because no investment (in price or quality) can be carried out with lower amounts.

[32] Note that hypothesis (ix) is silent with regard to the decision to begin a predation strategy. Indeed, an exit rule is stated therein that indicates when to top off such a strategy, and not a rule that indicates when the firm should start it. This means that no constraint is imposed on firms whose monopolistic profit is already less than the cost of predation in the beginning, except for the fact that in these cases the predation strategy must be abandoned the instant following its implementation, causing the realization of the losses already sustained. In summary, hypothesis (ix) only indicates a rule that corresponds to a temporal debt limit so that firms cannot continue to borrow after a specific instant of time.

		Firm j DP	Firm j P
Firm i	DP	N 0 , 0	0 , z_1
	P	z_1 , 0	x_1, x_1

It is easy to see that the equilibrium of the game in this case is given by the pair of strategies (DP,DP) where no investment in quality and predation occurs. In this case the cost of capturing the market via reputation is thus too high and the game will remain in the initial duopoly situation.

Case 2: (4) is not satisfied by the firms and (7) is satisfied by only one of them.

In this case, for the same reason as before, it is not convenient for a firm to invest alone in the market, but now if both invest and we assume that 'i' has a higher payoff than its rival and higher than the minimal individual investment under predation, that is

$$Max\ \pi\{ (p_0 - \varepsilon, \bar{e}, x_i^s = x^d/2)\ ;\ \pi(p_0, \bar{e} + \varepsilon, x_i^s = x^d/2)\}$$

then 'i' can force 'j' to quit the market, investing a total amount K = $\int_{s=0}^{T} e^{-\delta_j s}\ \pi(p^*, e_{\mathrm{M}}, x^s = x^d)$

(or exactly $Max\ \pi\{ (p_0 - \varepsilon, \bar{e}, x_i^s = x^d/2)\ ;\ \pi(p_0, \bar{e} + \varepsilon, x_i^s = x^d/2)\})$

to be shared and shared alike and obtaining positive profits as a monopolist[33]. The matrix of the game can thus be expressed as follows:

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[33] In the contrary case that:

$\int_{s=0}^{T} e^{-\delta_i s}\ \pi(p^*, e_{\mathrm{M}}, x^s = x^d)\, ds\ \leq Max\ \pi\{ (p_0 - \varepsilon, \bar{e}, x_i^s = x^d/2)\ ;\ \pi(p_0, \bar{e} + \varepsilon, x_i^s = x^d/2)\}$

then the matrix is equal to the previous case except for the presence of z_2 for firm 'j'. This is because firm 'i', also satisfying only half market during predation, can't recover the minimal individual loss and will instantaneously exit the market because of hypothesis (ix).

		Firm j	
		DP	P
Firm i	DP	N 0 , 0	0 , z_2
	P	z_1 , 0	x_1, x_2

With $x_2 = -\int_{s=0}^{T} e^{-\delta_i s}\ \pi(p^*, e_{\mathrm{M}}, x^s = x^d)\, ds = \pi(p^*, e_{\mathrm{M}}, x^s = x^d)\ \frac{1-e^{-\delta_{\mathrm{j}}\mathrm{T}}}{\delta_j} = -$ K < 0

(or alternatively x_2 = $Max\ \pi\{ (p_0 - \varepsilon, \bar{e}, x_i^s = x^d/2)\ ;\ \pi(p_0, \bar{e} + \varepsilon, x_i^s = x^d/2) \}$ = K)

and $x_1 = \pi(p^*, e_{\mathrm{M}}, x^s = x^d)\ \frac{1-e^{-\delta_i \mathrm{T}}}{\delta_i}$ – K > 0 and α→0 and

z_1 , z_2 < 0 arising from equation (4).

The equilibrium of the game, as in the previous case, is (DP,DP). For firm 'j', 'P' is a dominant strategy of the game and with assumption *(viii)* of perfect information, firm 'i' will never play 'P' obtaining negative profits. The consistent equilibrium of the game will thus be DP for both firms and no investment in quality will occur.

At variance with Case 1, there is now a chance for firm 'i' to invest and obtain positive profits as a monopolist, but such a possibility is denied by a dominant strategy for firm 'j'[34].

[34] Also if we assume that *(viii)* doesn't hold and allow randomization, there is no dominant strategy for firm i to employ predation. In that case, investment will occur, constricting firms with appropriate beliefs computed via Bayesian rule.

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