Writing Legally Unenforceable Contracts to Facilitate Relationships∗

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First Version: August 16, 2009
This Version: May 13, 2010

∗Preliminary and incomplete. Comments are welcome. I am grateful to Kohei Kawamura for helpful conversation at an earlier stage of this research, Junichiro Ishida, Shingo Ishiguro, Shinsuke Kambe, Michael Riordan, and the seminar participants at Osaka University, University of Stavanger, Norwegian School of Economics and Business Administration (NHH), University of New South Wales, CTW, and Hitotsubashi University for helpful comments, and especially Andrew Daughety for detailed comments and encouragement.

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Abstract

Transacting parties sometimes write contracts that are unenforceable in courts. Why do they write such contracts despite of “ink costs”? To answer this question, I analyze contractual relationships in context in the sense that there is a large population of principals and agents, a principal and an agent are randomly matched and engage in transaction, and at the end of each period, they can choose to continue or terminate the current partnership. I adopt an extreme assumption that written contracts are never legally enforced. I then show that writing a contract can help relational contracting between principals and agents more enforceable than relying on tacit understanding of their agreement for three reasons: (i) ink costs of writing a contract make a new match more costly and hence continuing the current match more valuable; (ii) the existence of a written document, with signatures of a principal and an agent, helps parties in the matching pool to identify (some of) those who reneged in the previous transaction; and (iii) the existence of a written document can raise motivation to engage in prosocial behavior (e.g., go to court to punish reneging parties), and hence increasing the probability that the matching pool learns about past transaction.

JEL Classification Numbers: D86 (Economics of Contract: Theory), K12 (Contract Law), L14 (Transactional Relationships; Contracts and Reputation; Networks).

Keywords: relational contracting, community enforcement, random matching, legally unenforceable contract, prosocial behavior
There are contracts in societies that have no formal law enforcement machinery...Someone known not to perform his side of bargains will find it difficult to find people willing to make exchanges with him in the future.

(Posner, 2007, p.94)

1 Introduction

Why do trading parties write a contract? The main “rationale for contracting is to lock in a commitment *ex ante* that one or both parties would otherwise not wish to honor *ex post*... The use of a contract to establish such commitment is undermined,..., if the contract will not be enforced in the way the parties anticipate (Hermalin et al., 2007, p.99).”

However, as Djankov et al. (2003) argue and convincingly show, economists “have been generally most optimistic about courts as the institution securing property and enforcing contracts (p.454).” Standard economic theories of contracts assume that (i) contractual terms contingent on verifiable states and/or actions are perfectly enforced by courts; (ii) legal enforcement is all-or-nothing; and (iii) verifiability of actions or states are exogenously given. Although research based on these assumptions has contributed to our understanding of optimal contracts, they are also extreme. Judicial enforcement depends on contract law, courts’ discretion, the parties’ ex post costly action (e.g., submit evidence), and parties’ ex ante costly contracting (e.g., costs of thinking of future contingencies and writing documents). Recent (law and) economics literature attempts to relax the standard extreme assumptions and incorporates some of these features into formal analysis.\(^1\) However, they still take substantial (although imperfect) degree of legal enforcement for granted.

More attention has recently been paid to private/informal enforcement mechanisms alternative to courts, such as relational contracting under which contract enforcement is carried out within a bilateral relationship, and community enforcement that third parties in the mar-

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ket play disciplinary roles. While these enforcement mechanisms are obviously important in developing/transition economies where legal protections are limited and unreliable (Dixit, 2004; Johnson et al., 2002; McMillan and Woodruff, 1999), the importance and prevalence of informal enforcement are also true even in economies with well-developed legal systems (Djankov et al., 2003; Macaulay, 1963). “The upshot is that private ordering is central to the performance of an economy whatever the conditions of lawfulness (Williamson, 2005, p.2).”

In this paper I also focus on informal enforcement and its interaction with formal contracts. In contrast to existing literature studying such interaction (Baker et al., 1994; Schmidt and Schnitzer, 1995; Pearce and Stacchetti, 1998; Itoh and Morita, 2009; Battigalli and Maggi, 2008; Kvaløy and Olsen, 2009), where a bilateral relationship is isolated from markets or communities, I analyze contractual relationships in context, by adopting the framework of matching games where a large population of buyers and sellers interact in bilateral relationships and they are neither tied permanently to one another, nor are they forced to dissolve their current partnerships at the end of every period.

There is relevant work in game theory that analyzes (mostly) prisoners’ dilemma in a community setting (see Mailath and Samuelson (2006, Chapter 5) for an overview). Kandori (1992) and Ellison (1994) establish a folk theorem in a random matching game where the population is finite, each pair must break up exogenously at the end of each period, and each player can only observe the outcomes of the games he played previously. Closer to my analysis are Ghosh and Ray (1996), Kranton (1996), Sobel (2006), Rob and Yang (2010), and Fujiwara-Greve and Okuno-Fujiwara (2009) where there is a continuum of players in the population and matched players can choose whether to continue or terminate the current partnership. However, their focus is on strategies in prisoner’s dilemma, and hence no contract or transfer is considered.

By developing a “matching game theory meets relational contracting” framework, I examine roles of written contracts that are not legally enforceable. While the literature on

\[^2\text{See Deb (2008) and Takahashi (2010) for recent extension.}\]
relational contracting illuminates the logic of informal enforcement, it does not answer the following question: Do the parties need to write agreements in documents? It appears that tacit understanding of informal promises is enough and saves “ink costs.” An answer to this question, suggested by Kvaløy and Olsen (2009) and Sobel (2006), is that writing more costly contracts is more likely to be legally enforced, and can complement relational contracting under some conditions.

What if costly contracts are not legally enforceable? In this paper I adopt this extreme assumption in order to highlight roles of written contracts other than those related to legal enforcement. An influential article by a legal scholar Llewellyn (1931) in fact argues that “official aid on the contract side consists most commonly not in what we know as enforcement but rather in an official declaration—or merely official recognition...—that an obligation is owed and forfeit (p.711).” According to him, the main role of legal contract is to provide an adjustable framework that “almost never accurately indicates real working relations, but which affords a rough indication around which such relations vary, an occasional guide in cases of doubt, and a norm of ultimate appeal when the relations cease in fact to work (p.737).”

Writing a legally unenforceable contract is not entirely unrealistic. A classic paper Macaulay (1963) provides several examples of parties writing legally unenforceable contracts:

... it is likely that businessmen are least concerned about planning their transactions so that they are legally enforceable contracts. For example, in Wisconsin requirements contracts—contracts to supply a firm’s requirements of an item rather than a definite quantity—probably are not legally enforceable. Seven people interviewed reported that their firms regularly used requirements contracts...None thought that the lack of legal sanction made any difference...Three of these people were house counsel who know the Wisconsin law before being interviewed.

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Another example of a lack of desire for legal sanctions is found in the relationship between automobile manufacturers and their suppliers of parts. The manufacturers draft a carefully planned agreement, but one which is so designed that the supplier will have only minimal, if any, legal rights against the manufacturers. The standard contract used by manufacturers of paper to sell to magazine publishers has a pricing clause which is probably sufficiently vague to make the contract legally unenforceable (p.60).

Goldberg (2008) argues that the manufacturing agreement in the 1919 contract between Fisher Body and General Motors was legally unenforceable: “Nothing precluded Fisher from selling some, or all, of its body production to Ford...if Fisher is free not to supply auto bodies if it so decides, then GM, despite the specific promises made in the contract, has no obligations; it is free to buy auto bodies from other suppliers.” Goldberg (2008) further points out that even if the executives were unaware, or acted as if they were unaware of this, their counsel should have known when drafting the agreement that it would not be enforceable. He conclude by arguing that “the unenforceable agreements can be effective...If, as I suspect, such agreements are fairly common, any serious theory of the organization of economic activity will have to take this mechanism into account.”

Ryall and Sampson (2009), based on their analysis of a sample of joint technology development contracts in telecommunications and microelectronics industries, argue that “partners who deal with each other repeatedly may find it worthwhile to write a detailed agreement, including performance terms and related penalty clauses, not due to their usefulness in court, but instead, their usefulness in maintaining a smoothly functioning relational contract (p.923).”

So how can costly written unenforceable contracts be useful in maintaining good relationships? An obvious answer is that writing down terms and obligations enable the parties to remember them (as well as communicate them with relevant members who belong to the same organization), and to minimize misunderstandings that might jeopardize repeated
transactions. I do not pursue these roles of contracts, and assume instead that there is no
problem in communication between the trading parties. I also exclude the possibility that
contracts serve as a signaling device (Bénabou and Tirole, 2003; Maskin and Tirole, 1990).

I however show that when transactions are repeated, writing a particular form of con-
tracts, although they are unenforceable, may help the market identify whose reputation must
suffer should dissolution occur, and hence may facilitate self-enforcement of relational con-
tracting. The relevant work cited above hints at a communication role of written contracts.
I attempt to make this role precise.

The structure of the rest of the paper is as follows. In section 2, a matching game
theoretic model of principal-agent relationships is introduced. The stage game is a simple
principal-agent model with symmetric information, where the agent chooses an action from
a binary set and the principal offers a price. Since I assume no payment is contractible, in
the one-shot transaction the principal “holds up” the agent by expropriating all the benefit,
and hence the agent does not choose the costly action. This incentive problem is not at all
mitigated in repeated interaction, when there is a continuum of principals and agents and
a principal and an agent are matched to play the game, given that no information about a
new matched player is available in the matching pool: Each principal always chooses to pay
nothing, terminate the current relationship, and return to the matching pool to start a new
life.

Following existing literature on matching games, I introduce heterogeneity in the princi-
pal’s population so as to create a cost of starting a new match: Principals are either of “bad”
type who never make positive payments or of “opportunistic” type, and look for equilibria
where principals of the opportunistic type compensate their agents for choice of costly action.
This creates a cost of starting a new match because the principal must guarantee higher pay
to provide the agent, who does not know the principal’s type, with incentives to choose an
appropriate action. However, in section 3, I assume that no contract is written, and show
that there exists no equilibrium in which opportunistic principals make positive payments:
The cost of a new match is not large enough to prevent the principal from reneging every period.

In section 4, I assume a contract is written. Although writing a contract is costly, it is not at all enforced in courts. However, I show that writing a costly contract can help relational contracting between principals and agents more enforceable for three reasons. First, writing a contract makes a new match more costly via “ink costs.” The role of ink costs is a familiar one, similar to that of “burning money” in literature on matching games.

Second, the existence of a written document, with signatures of a principal and an agent, serves as a communication device, by helping agents in the matching pool to distinguish old principals who did not pay, from new principals who do not have a bad record. I obtain conditions under which there exists an equilibrium where opportunistic principals make positive payments.

The written contract can serve as a communication device only if agents in the matching pool can observe the document. For example, the Securities and Exchange Commission (SEC) requires that publish firms submit contracts in some categories as part of their filings. However, for those transaction parties who are not subject to such requirements, voluntary disclosure is necessary. The needed disclosure is achieved if agents whose principals did not make the promised payments file lawsuits. However, while going to court is costly, no monetary benefit is expected since the agents win with probability zero.

Now the third reason why writing a contract could help comes in. Suppose that agents are heterogenous in terms of their intrinsic preferences for prosocial activity, which is to file costly lawsuits in order to punish reneging principals. There is ample evidence showing that people engage in costly punishment.\(^3\) Prosocial behavior may bring direct payoffs as well as reputational benefits from social or self signaling (Bénabou and Tirole, 2006). However, without a written document, intrinsic motivation may not be high enough for them to going

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\(^3\) See, for example, Fehr and Gächter (2000) and Gintis (2009, Chapter 3) for an overview. According to the latter (p.51), “Recent neuroscientific evidence supports the notion that subjects punish those who are unfair to them simply because this gives them pleasure.”
to court. The existence of a written document can raise motivation to go to court in order to
punish reneging principals, by making such behavior look “legitimate” personally and hence
can increase the probability that the matching pool learns from the document.

In section 5, I discuss several extensions and conclude in section 6.

2 Model

Principal-agent setting  There is a continuum of players called principals and agents,
each in the unit interval. In every period, a principal and an agent are matched and play the
following simple principal-agent stage game. The principal makes an offer to the agent, and
the agent decides whether or not to accept it. If the agent rejects the offer, the stage game
ends and the payoffs are zero for both players. After accepting the offer the agent chooses
action \( a \in \{0, 1\} \) with personal cost \( d(a) = da \), where \( d > 0 \) is a constant representing the
cost of action \( a = 1 \). The benefit to the principal is \( y(a) = ya \), where \( y > 0 \) is a constant.
The total surplus is written as \( s(a) = sa \) where a constant \( s = y - d > 0 \) is the total surplus
under \( a = 1 \). The principal then chooses a payment \( w \in [0, y] \) to the agent. The payoffs in
the period are \( ya - w \) for the principal and \( w - da \) for the agent.

All the relevant variables are observable but none of them is contractible. The principal
hence expropriates all the benefit by offering \( w = 0 \), and the agent, anticipating the principal’s
response, chooses \( a = 0 \) in the one-shot game. There is no initial contract that can bind the
principal and the agent ex post.

Matching process  At the beginning of each period, a principal and an agent are randomly
matched and play the stage game. At the end of the period, after the outcome of the game
realizes, the matched players simultaneously decide whether to continue or terminate the
current relationship. If both choose to continue, they move to the next period to play the
same stage game. If at least one of them chooses to terminate, they both go back to the
matching pool where each of them is matched with a new player. Furthermore, at the end
of each period (either before or after the separation decision), with exogenous probability 
$1 - \rho \in (0, 1)$ both players must leave the population and are replaced by new players who 
join the pool of unmatched players so as to keep the total population constant.\footnote{The assumption that \textit{both} players must leave the population is adopted to simplify calculation, the results do not essentially change if we instead allow the possibility that one player leaves while the other remains.} The players 
discount their payoffs with common discount factor $\delta_0 \in (0, 1)$, and the effective discount 
factor is denoted by $\delta = \delta_0 \rho$.

Note that there is no possibility that players in the pool are matched with someone they 
have previously met. I further assume that while players observe (and remember) the history 
of play in their own relationships, each player in the pool obtains no information about the 
new partner’s past history of play with others, including the number of periods that the new 
partner has been in the population.

In this setting, it is immediate to observe that there is no stationary symmetric equilib- 
rium where the agent chooses $a = 1$ every period. Here stationarity means that each player 
chooses the same strategy every period. Symmetry means that the equilibrium strategy of all 
the principals is identical, and the same is true for all the agents. To see this claim, suppose 
instead that there is a symmetric stationary equilibrium in which every period each principal 
offers $w \geq d$ if $a = 1$ and zero otherwise, and each agent chooses $a = 1$. It is optimal for 
the agent to choose $a = 1$ if he expects the principal to pay $w$. However, given the strategies 
of agents and other principals, the principal’s optimal response is to renege by paying zero 
instead, and to terminate the relationship. When she goes back to the matching pool, she can 
make the same offer to a new agent who will choose $a = 1$ following the equilibrium strategy. 
Her per period payoff hence increases from $y - w$ in equilibrium to $y$. A contradiction.

The problem is that no one can punish the reneging principal: The matched agent cannot 
do so since the principal terminates the relationship, and new partners cannot, either, because 
you are unable to distinguish her from other principals. The principal hence incurs no cost 
for reneging and returning to the matching pool.
Following Ghosh and Ray (1996), Kranton (1996), and Rob and Yang (2010), I introduce the following heterogeneity in the population as one source of a lower value of starting a new match than that of continuing the relationship. There are two types of principals in the population: type B ("bad") in proportion $\beta \in [0, 1)$, and type O ("opportunistic") in proportion $\gamma = 1 - \beta$. The bad type behaves mechanically and never makes positive payments, for example, due to a large opportunity cost from paying for the partner, or being myopic with a discount factor close to zero. The focus of the analysis is hence the behavior of opportunistic principals whose payoffs are as specified above. Each principal’s type is her private information. On the other hand, I assume all agents are opportunistic.

**Equilibria** I restrict my attention to pure-strategy, symmetric equilibria with the following features. First, all types of indistinguishable principals make an identical offer because I want to explore roles of contracting different from the well-studied signaling role (Bénabou and Tirole, 2003; Maskin and Tirole, 1990). Since I have not specified the preferences of type B principals, I can exclude “separating” equilibria by appropriately defining their payoff functions. Second, all the agents choose the efficient action $a = 1$ every period (unless they meet principals who are identified as type B).

Third, type O principals choose to pay a promised positive amount contingent on the agent’s action choice $a = 1$. This implies that the agent, observing the principal’s payment behavior in their first stage game, can know whether she is of type B or type O. Fourth, the equilibrium payment schemes depend only on whether or not the relationship between

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5In subsection 5.1, I extend the model by introducing the third type, “good” principals. The main message of the paper is not affected by the existence of such a type.

6The analysis can be extended to the case in which there exist agents of bad type (who are myopic or never choose $a = 1$). The principal’s reputation is at stake in the current model, and introducing heterogeneity of agents does not change the results substantially. See subsection 5.2 for more on this.

7Symmetry here means that the equilibrium strategy of all the principals of the same type is identical, and so is that of all the agents.

8Inducing the agent to choose $a = 1$ in later periods is possible in non-stationary equilibria. Suppose that in their first interaction a principal offers zero payment irrespective of action, a matched agent chooses $a = 0$, and both choose to continue the relationship. From the second period on, the principal offers to pay $w = d$ if $a = 1$, the agent chooses $a = 1$, and both continue. Any deviation leads to termination of the relationship. This strategy profile generates costs of starting a new match endogenously. See the analysis of repeated prisoners’ dilemma in Mailath and Samuelson (2006, Chapter 5).
a principal and an agent is new. In their continuing match, in particular, payment schemes
do not depend on how many times they play stage games. When the relationship is new,
the agent does not know the principal’s type initially, while he finds out her true type at the
end of the period and terminates the relationship with type B. The agent does not learn any
more information from the second game on in which he knows the principal is of type O.
Following Ghosh and Ray (1996), I call a principal and an agent are in stranger phase (phase
\(S\)) when they first interact and hence the agent does not know the principal’s type, and they
are in friendly phase (phase \(F\)) when they have already interacted and hence he knows the
principal is of type O.

Fifth and finally, the equilibrium is in a steady state in the sense that the distribution of
the principals’ types in each of phases \(S\) and \(F\) does not change over time. From hereafter I
call the equilibrium with all the features given above simply the “good” equilibrium.

3 Analysis: When No Contract Is Written

In this section I assume no contract is written, and examine the existence of the good equi-
librium where type O principals make promised payments, and hence in phase \(S\) type B
principals who do not pay are screened. The agent terminates the relationship if and only if
the principal does not pay. If the principal makes payments (so that she is of type O), then
they move to phase \(F\) (if they survive) by continuing the relationship. Type B principals go
back to the matching pool and repeat phase \(S\) with probability \(\rho\).

Steady state I first obtain the steady-state distribution of the principals’ types under the
equilibrium. Figures 1 and 2 summarize the transition of type B and type O Principals, re-
spectively. In the equilibrium, type B principals never move to phase \(F\). Suppose proportion
\(x \in [0, \gamma]\) of type O principals is in phase \(S\), of which \(\rho x\) moves to phase \(F\) and \((1 - \rho) x\) exits
from the population. Proportion \(\rho \beta\) of type B principals stays in and repeats phase \(S\), and
\((1 - \rho) \beta\) dies. To keep the population constant, \(1 - \rho\) of newborn principals enter phase \(S\)
who consist of $(1 - \rho)\beta$ type B and $(1 - \rho)\gamma$ type O principals. Note that for type B, the rate of inflow and that of outflow are equal to $(1 - \rho)\beta$. For type O, the rate of inflow $(1 - \rho)\gamma$ must be equal to that of outflow $(1 - \rho)x + \rho x$, or $x = (1 - \rho)\gamma$.

Figure 1: Transition of Type B Principals without Written Contracts

![Figure 1](image)

Figure 2: Transition of Type O Principals

![Figure 2](image)

In phase $F$, there is proportion $\gamma - x$ of type O principals, of which $\rho(\gamma - x)$ repeats phase $F$ and $(1 - \rho)(\gamma - x)$ exits. Note that if $x = (1 - \rho)\gamma$, then the rate of outflow $(1 - \rho)(\gamma - x)$ is equal to that of inflow $\rho x$. Given this steady-state distribution of type O principals in phase

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denote the probability of an agent’s meeting a type B principal in phase $S$ by

$$q = \frac{\beta}{x + \beta}.$$  

(1)

Note that $q$ is increasing in $\beta$, $q = 0$ if $\beta = 0$, and $q \uparrow 1$ if $\beta \uparrow 1$.

**Agents** Suppose a principal and an agent reach the following informal agreements: In phase $i$, the agent chooses $a = 1$, and the principal pays $w_i$ if $a = 1$, and pays nothing if $a = 0$. They terminate their relationship if and only if the principal fails to abide by the payment scheme. If the agent chooses $a = 0$, then the principal does not pay $w_i$ and the relationship continues. In other words, the agent is, without loss of generality, provided with the incentive to choose $a = 1$ via the current payment only.\(^9\)

Let $U_F$ and $U_S$ be the agent’s present values in phases $F$ and $S$, respectively. They are obtained as follows:

$$U_F = w_F - d + \delta U_F$$

$$U_S = (1 - q)(w_S - d + \delta U_F) + q(-d + \delta U_S)$$

In phase $F$, all the principals are of type O, and hence the agent is paid $w_F$ and repeats phase $F$ with probability $\rho$. In phase $S$, each agent meets a type O principal with probability $1 - q$, and is paid $w_S$ upon his action choice $a = 1$. Then they move to phase $F$ with probability $\rho$. On the other hand, the agent meets a type B principal with probability $q$, in which case he is paid nothing and hence terminates the relationship and moves back to phase $S$ with probability $\rho$.

The agent’s incentive compatibility constraints, implying he chooses $a = 1$ in both phases,

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\(^9\)This is without loss of generality. I could justify the assumption by interpreting the current model as a limit of an extended model where $a$ is unobservable to the principal and benefit 0 can result from $a = 1$ with a positive probability (Levin, 2003).
are given as follows:

\[ w_F - d \geq 0 \]
\[ (1 - q)w_S - d \geq 0 \]

Since the principal makes a take-it-or-leave-it offer, these constraints bind. The optimal payments are thus \( w_F^0 = d \) and \( w_S^0 = d/(1 - q) \), and the equilibrium values are \( U_F^* = U_S^* = 0 \).

**Type O principals** I now turn to the principal’s payment decisions. Let \( V_F \) and \( V_S \) be the type O principal’s present values in phases \( F \) and \( S \), respectively, which are given as follows:

\[ V_F = y - w_F + \delta V_F \]
\[ V_S = y - w_S + \delta V_F \]

Note that the difference is

\[ V_F - V_S = w_S - w_F. \] (2)

Upon the agent’s action choice \( a = 1 \), the principal pays \( w_i \) in phase \( i \) if the following conditions hold:

\[ V_F \geq y + \delta V_S \]
\[ V_S \geq y + \delta V_S \]

In either phase, the principal can deviate by paying nothing and terminating the relationship, which yields the same right-hand side of the conditions. Since \( V_F \geq V_S \), only the second constraint binds. Using (2) yields the following self-enforcing condition:

\[ w_S \leq \delta(w_S - w_F). \] (3)
This condition, along with $w_i^0 > 0$ from the agent’s incentive compatibility constraints, leads to the following result.

**Proposition 1** The good equilibrium does not exist.

The proof is obvious from condition (3): No contract $(w_S, w_F)$ with $w_i > 0$ for $i = S, F$ satisfies the condition. It is actually costly for the type O principal to go back to the matching pool because then she starts at phase $S$ where she has to make a higher payment than in phase $F$. However, this pay difference in payment is not high enough to make the contract self-enforcing. The principal prefers enjoying the reneging temptation $w_S$ every period than paying as promised and moving to phase $F$, that only increases her future payoff by $\delta(w_S - w_F)$.

**Comparison with Bilateral Repeated Relationship Setting** It is instructive to contrast this result to the one in the standard setting where a principal and an agent engaged in repeated transaction infinitely, with zero reservation payoff for both players.\(^{10}\) Each agent in phase $S$ meets a type B principal with probability $q$ and a type O principal with probability $1-q$. The agent meeting type B earns zero payoff forever. The agent meeting type O moves to phase $F$ and stays there forever. The agent’s incentive compatibility constraints are the same as above, and hence $w_F^0 = d$ and $w_S^0 = d/(1-q)$.

In phase $F$, the type O principal’s reneging temptation is $w_F^0 = d$ (not pay $w_F^0$) and the future loss is $[\delta/(1-\delta)](y-d)$ (earns zero instead of $y-d$ every future period). Hence the principal’s promise in phase $F$ is self-enforcing if and only if $d \leq [\delta/(1-\delta)](y-d)$ or

$$\delta \geq \frac{d}{y}$$

holds.

\(^{10}\)See MacLeod and Malcomson (1989) and in particular Levin (2003). Note that different from them where fixed transfers are contractible, the model here assumes that no contractible transfer is feasible.
Similarly, in phase $S$, the principal’s reneging temptation is $w^0_S = d/(1 - q)$, while the future loss is $[\delta/(1 - \delta)](y - d)$ (earns zero instead of $y - d$ every future period in phase $F$). The self-enforcing condition is then given by $d/(1 - q) \leq [\delta/(1 - \delta)](y - d)$ or

$$q \leq q_0(\delta) \equiv \frac{\delta y - d}{\delta(y - d)}. \quad (5)$$

Note that $q_0(\delta)$ is increasing in $\delta$ and satisfies $q_0(d/y) = 0$ and $\lim_{\delta \to 1} q_0(\delta) = 1$. Since (5) is sufficient for (4), condition (5) is necessary and sufficient for $a = 1$ to be implemented every period. In Figure 3, $(\delta, q)$ pairs in the shaded region satisfy the condition.

Figure 3: Existence of Good Equilibria in the Bilateral Repeated Relationship Setting

4 Analysis: When Contracts Are Written

When a principal and an agent write a contract, two changes occur. First, writing a contract is costly. I assume that a principal, who writes a contract and makes an offer in the take-it-or-leave-it fashion, must incur “ink cost.” The second change is that, as pointed out by Llewellyn
(1931), Macaulay (1963), and Ryall and Sampson (2009) among others, a written contract serves as a communication device. I assume that the existence of written documents, each with signatures of a principal and an agent, enables each agent in the matching pool to identify with some probability whether or not his next partner has been in the population in previous periods, and to infer the partner’s type from the content of the contract. For example, publish firms must submit M&A contracts, employment contracts, joint technology development contracts, and other material contracts as part of their filings, under the Securities and Exchange Commission (SEC)’s disclosure requirements. Even for those transaction parties who are not subject to such requirements, a public lawsuit between a principal and an agent can serve a similar role.

We thus assume that a written document in phase $i = S, F$ is revealed to the matching pool with probability $\lambda_i$, while with probability $1 - \lambda_i$, the matching pool remains ignorant. Let $c_S$ be the ink cost in phase $S$, and $c_F$ the ink cost in the first game in phase $F$. From the second period on in phase $F$, there is no need to modify the contract and hence the ink cost is assumed to be zero.

An interpretation is that when a principal wrote a contract with ink cost $c_i$ but did not pay the specified wage $w_i$ upon the agent’s choice of $a = 1$, the agent goes to court with probability $\lambda_i$. By going to court he incurs the (expected) monetary cost of filing the lawsuit $\ell > 0$ while no monetary benefit is expected whether or not a contract is written, because it is not enforceable anyway and hence no remedy is paid. However, he enjoys private benefit from punishing the reneging principal, which may include direct intrinsic “joy” of engaging altruistic punishment, as well as reputational concern such as a desire to appear prosocial by others or by his later self (see the literature cited in Introduction). Let $v$ be this private benefit from going to court, which is drawn from some probability distribution. Only those agents with $v \geq \ell$ file lawsuits when their principals renege on payments.

I assume that the very existence of a written document can increase the probability of going to court, by making such behavior look “legitimate” and hence increasing $v$. Suppose
is drawn from a probability distribution with density $f_0(v)$ under no contract, while the density is $f_i(v)$ if a contract is written in phase $i$, and the latter dominates the former in the sense of first-order stochastic dominance. Furthermore, suppose for simplicity that no agent goes to court under no contract. Then the probability of going to court $\lambda_i$ when a contract is written in phase $i = S, F$, is defined by $\lambda_i = \int_{v > \ell} f_i(v) dv > 0$.\(^{11}\)

Before analyzing the effects of these two changes mentioned above, I first examine in subsection 4.1, how the ink cost of writing a contract alone restores the possibility of an equilibrium in which all the agents choose $a = 1$. There I assume $\lambda_i = 0$ for $i = S, F$: no contract is disclosed and hence written contracts do not serve as a communication device. The general case is analyzed in subsection 4.2.

### 4.1 No Communication

Throughout this subsection I assume $\lambda_i = 0$ for $i = S, F$, and hence the transition of types B and O is as in Figures 1 and 2, respectively. The only difference from the no contract case is that principals can incur ink costs.

Suppose a principal and an agent reach the following informal agreements: In phase $S$, each principal writes a contract $(c_S, w_S)$ where $c_S$ is the ink cost and $w_S$ is the payment contingent on $a = 1$, and the agent chooses $a = 1$. In phase $F$, each principal (who is of type O) writes a contract $(c_F, w_F)$, and the agent chooses $a = 1$. In either phase, they terminate their relationship if the principal fails to offer the contract or to pay $w_i$ contingent on $a = 1$. Contracts are said to implement the good equilibrium if there exist contracts under which the good equilibrium exists.

The present values and the incentive compatibility constraints of the agent does not change from the previous no contract case: The optimal payments are $w^0_F = d$ and $w^0_S = d/(1 - q)$, and the present values are $U^*_F = U^*_S = 0$.

---

\(^{11}\)On the other hand, I exclude the possibility that the agent uses the action to file a grievance to hold up the principal (such as “pay me for $a = 0$ or I file”), because such a threat is not credible: I assume that the agent does not enjoy $v$ from going to court to the purpose of extracting rent from the principal.
Let $V_S$ be the type O principal’s present value in phase phase $S$, and $V_F$ be the present value at the first period of phase $F$. These are given as follows:

$$V_F = y - w_F - c_F + \delta(V_F + c_F)$$
$$V_S = y - w_S - c_S + \delta V_F$$

Note that the difference is

$$V_F - V_S = w_S - w_F + c_S - (1 - \delta)c_F$$  \hspace{1cm} (6)$$

The principal makes promised payments if the following incentive compatibility constraints are satisfied:

$$V_F \geq y - c_F + \delta V_S \iff w_F \leq \delta(V_F - V_S + c_F) = \delta(w_S - w_F + c_S + \delta c_F)$$
$$V_S \geq y - c_S + \delta V_S \iff w_S \leq \delta(V_F - V_S) = \delta(w_S - w_F + c_S - (1 - \delta)c_F)$$

Since $w^0_S \geq w^0_F$, the first condition is slack and can be ignored. Then the ink cost $c_F$ in phase $F$ only reduces the present values without relaxing the second constraint. Hence $c_F = 0$ must hold: no contract is written in phase $F$.

On the other hand, writing a contract in phase $S$ relaxes the second incentive compatibility constraint:

$$\frac{1}{1 - q}d \leq \delta(V_F - V_S) = \delta\left(\frac{q}{1 - q}d + c_S\right).$$  \hspace{1cm} (7)$$

Since this constraint does not hold for $c_S = 0$, writing a contract ($c_S > 0$) in phase $S$ is necessary.

If the principal deviated by not writing the contract in phase $S$, then both the principal and the agent would choose to terminate the relationship. Given this, the type O principal would optimally choose not to pay $w_S$, and hence the agent also would choose $a = 0$. The
principal’s payoff in the current period would be thus zero and the continuation payoff $\delta V_S$. The principal does not deviate from writing a contract if $V_S \geq \delta V_S$ or $V_S \geq 0$. The ink cost hence cannot be too high:

$$V_S = y - w_S^0 + \delta V_F - c_S \geq 0.$$  \hspace{1cm} (8)

**Proposition 2** There exists a contract with ink cost $c_S > 0$ in phase $S$ that, along with no contract in phase $F$, implements the good equilibrium, if and only if condition (5) holds.

**Proof** Conditions (7) and (8) are rewritten as follows:

\begin{align*}
(1 - \delta) \frac{q}{1-q} d & \leq \delta c_S - d \quad \hspace{1cm} (9) \\
(1 - \delta) \frac{q}{1-q} d & \leq \delta c_S - d + y - c_S \quad \hspace{1cm} (10)
\end{align*}

Hence (9) binds if $c_S \leq y$, while (10) binds otherwise. For a given $c_S$, both conditions hold if and only if

$$q \leq q_{ic}(\delta, c_S) = \frac{\delta c_S - d - \max\{c_S - y, 0\}}{\delta(c_S - d) - \max\{c_S - y, 0\}}$$  \hspace{1cm} (11)

is satisfied. The right-hand side is increasing in $c_S$ up to $c_S = y$ and then decreasing. When $c_S = y$, the right-hand side is equal to the right-hand side of (5), that is, $q_{ic}(\delta, y) = q_0(\delta) = \frac{(\delta y - d)}{\delta(y - d)}$. \hspace{1cm} \text{Q.E.D.}

Intuitively, as the proportion of type B principals is higher, the incentive compatible payment $w_S^0$ in phase $S$ must be higher, which fact raises the principal’s reneging temptation. Increasing the ink cost mitigates this incentive problem by making termination more costly. If the ink cost is very high ($c_S > y$), then the self-enforcing condition (9) is no longer an issue while the non-negative condition (10) must be satisfied.

By Proposition 2, the good equilibrium exists in the same shaded region as the bilateral repeated relationships sustain, as figure 3 shows. Note however that different from the bilateral case, there is a welfare loss due to the ink cost in phase $S$, which must be positive and sufficiently large to generate costs of starting a new relationship.
4.2 Communication

I now return to the general case in which written documents serve as a communication device, by assuming $\lambda_i > 0$ for $i = S, F$. Consider an equilibrium where each type $O$ principal writes an identical contract that looks like this: “If the agent chooses $a = 1$ in phase $F$, the principal pays $w_F$. If the agent chooses $a = 1$ in phase $S$, the principal pays $w_S$. If the principal fails to pay these, the relationship is terminated.” Suppose that this contract, with signature of the principal and the agent, is revealed to the matching pool. A new agent matched with the principal whose name is specified in the contract can infer that the old principal did not make a specified payment previously, and hence she be of type $B$. On the other hand, a new principal matched with the agent who appear in the contract can infer that the old agent is in the pool because his previous principal reneged, and hence he can start the new relationship without any disadvantage.

**Steady state** The steady-state distribution of type $O$ principals is the same as in Figure 2: Proportion $x = (1 - \rho)\gamma$ is in Phase $S$ and $\rho\gamma$ is in Phase $F$. Type $B$ principals are always in Phase $F$, as in the equilibrium without written contracts. However, now there are two kinds of type $B$ principals, those who did not pay and faced lawsuits and hence is identified as old, and those indistinguishable from type $O$ principals, that consist of new type $B$ and old type $B$ who were not sued by the agents. I say that the former type $B$ principals are in state $SI$ and the latter in state $SN$. The steady-state distribution of type $B$ principals are summarized in Figure 4.

Suppose proportion $z \in [0, \beta]$ of type $B$ principals is in state $SN$, of which $(1 - \lambda_S)\rho z$ stays in state $SN$, $\lambda_S\rho z$ moves to state $SI$, and $(1 - \rho)z$ exits from the population. To keep the population constant, $(1 - \rho)\beta$ of newborn type $B$ principals enter state $SN$. From the conditions that the rate of inflow and that of outflow are equal in each of the states, the
steady-state distribution of type B principals in state $SN$ is obtained as

$$z = z(\lambda_S) = \frac{1 - \rho}{1 - \rho + \lambda_S \rho} \beta,$$

which is decreasing in $\lambda_S$. As $\lambda_S$ goes to 1 (all the agents go to court), $z$ approaches to $(1 - \rho)\beta$, the proportion of new type B principals. That is, all the old type B principals go to and stays in state $SI$.

The probability distribution of principals in phase $S$ is summarized in Table 1, where $q = \beta/(x + \beta)$ is the probability that an agent meets a type B principal in phase $S$, as defined in (1). And $r$ is defined by

$$r = r(\lambda_S) = \frac{\lambda_S \rho}{1 - \rho + \lambda_S \rho},$$

and then $z = (1 - r)\beta$. Note that $r(\lambda_S)$ is increasing in $\lambda_S$, with $r(0) = 0$ and $r(1) = \rho$.

<table>
<thead>
<tr>
<th>Information</th>
<th>Type</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>state $SI$</td>
<td>type B</td>
<td>$rq$</td>
</tr>
<tr>
<td>state $SN$</td>
<td>type B</td>
<td>$(1 - r)q$</td>
</tr>
<tr>
<td></td>
<td>type O</td>
<td>$1 - q$</td>
</tr>
</tbody>
</table>
If an agent meets a principal in state $SI$, she must be of type B who never compensates for $a = 1$. The agent hence chooses $a = 0$ even if he accepts a contract offered by the principal. The identified principal hence has no incentive to pay $c_S$ to write a contract, and the payoffs are zero for both the principal and the agent for that period, and they repeat phase $S$ again with probability $\rho$. If an agent meets a principal in state $SN$, she is of either type B or type O, with probabilities as summarized in Table 1.

Suppose a principal and an agent reach the following informal agreements: In phase $S$, each type O principal and type B principal in state $SN$ writes a contract $(c_S, \lambda_S, w_S)$, and the agent chooses $a = 1$. In phase $F$, each principal (who is of type O) writes a contract $(c_F, \lambda_F, w_F)$, and the agent chooses $a = 1$. They terminate their relationship if the principal fails to offer the contract or to abide by the payment rule. When a principal in state $SI$ and an agent meet, they earn zero payoff in the current period and terminate the relationship to go back to the matching pool.

**Agents** The agent’s present value in phase $S$ changes as follows:

$$U_S = (1 - q)(w_S - d + \delta U_F) + (1 - r)q(-d + \delta U_S) + rq(\delta U_S)$$

In phase $S$, each agent meets a type O new principal with probability $1 - q$, and is paid $w_S$ upon his action choice $a = 1$. Then they move to phase $F$. The agent meets a type B indistinguishable principal with probability $(1 - r)q$, in which case he is paid nothing though he chooses $a = 1$, and terminates the relationship and moves back to phase $S$. With probability $rq$, the agent meets an identified old principal who is of type B. He earns zero payoffs and repeats phase $S$.

The agent’s incentive compatibility constraint in phase $S$ is as follows:

$$(1 - q)(w_S - d) + (1 - r)q(-d) \geq 0,$$
which is binding. The optimal payment scheme is hence \( w^*_F = w^0_F = d \) and

\[
w^*_S(\lambda_S) \equiv (1 + \sigma(\lambda_S))d \equiv \left(1 + \frac{q}{1 - q}(1 - r(\lambda_S))\right)d,
\]

and the agent’s equilibrium values are \( U^*_S = U^*_F = 0 \). Note that \( \sigma(\lambda_S) = (1 - r(\lambda_S))q/(1 - q) \) is decreasing in \( \lambda_S \) and increasing in \( q \), with \( \sigma(0) = q/(1 - q) \). Hence \( w^*_S(\lambda) \) is lower than the optimal payment in phase \( S \) without written contracts, which is \( w^0_S = d/(1 - q) = (1 + \sigma(0))d \), because the agent need not be motivated to choose \( a = 1 \) when he meets an identified old principal. If old principals are more likely to be identified (\( \lambda_S \) higher), then the lower payment is needed to induce the agent to choose \( a = 1 \) in phase \( S \). This payment, however, increases with the proportion of type B principals.

**Type O principals** The type O principal’s present values in phase \( S \), and at the first period of phase \( F \) are the same as before:

\[
V_F = y - w_F - c_F + \delta(V_F + c_F)
\]
\[
V_S = y - w_S - c_S + \delta V_F
\]

And hence condition (6) continues to hold. The type O principal makes promised payments if the following conditions hold:

\[
V_F \geq y - c_F + \delta(1 - \lambda_F)V_S
\]
\[
V_S \geq y - c_S + \delta(1 - \lambda_S)V_S.
\]

To explain the right-hand sides, suppose the principal reneges on payments. In phase \( i \), the agent goes to court with probability \( \lambda_i \), and then the principal’s present value becomes zero from the next period on. Otherwise, she goes back to the matching pool and restarts phase
Using (6) yields the following incentive compatibility constraints:

\[ w_F \leq \delta(V_F - V_S + c_F + \lambda_F V_S) = \delta(w_S - w_F + c_S + \delta c_F + \lambda_F V_S) \quad \text{(ICF)} \]

\[ w_S \leq \delta(V_F - V_S + \lambda_S V_S) = \delta(w_S - w_F + c_S - (1 - \delta)c_F + \lambda_S V_S). \quad \text{(ICS)} \]

The present values also have to be nonnegative:

\[ V_F \geq 0 \quad \text{(NF)} \]

\[ V_S \geq 0 \quad \text{(NS)} \]

I examine under what conditions all of these constraints hold and hence the good equilibrium exists. The main result is summarized as follows.

**Proposition 3** If

\[ q \leq q_1(\delta) \equiv \frac{\delta y - d}{\delta y - d + (1 - \delta)(1 - \rho)d} \quad \text{(13)} \]

holds, then there exists a contract \((c_S, \lambda_S)\) in phase \(S\) implementing the good equilibrium. No contract is written in phase \(F\). If (13) does not hold, no written contract can implement the good equilibrium.

The proposition is formally proved in the appendix. Here I offer an intuitive explanation of the result.

It is easy to see that if the principal does not write a contract in phase \(S\) (that corresponds to \(c_S = 0\) and \(\lambda_S = 0\)), then (ICS) never holds. A contract thus has to be written at least in phase \(S\). Must a contract be written also in phase \(F\)? If a contract \((c_F, \lambda_F)\) is written, the right-hand side of (ICF) increases, and hence the principal’s incentive compatibility constraint in phase \(F\) is easier to be satisfied. However, it makes the other constraints more stringent.

Based on these observations, in the appendix I first show that when no contract is written in phase \(F\), there is a contract in phase \(S\) under which both (ICS) and (NS) hold if and only
if $(\delta, q)$ satisfy condition (13). And then it is shown that (ICF) is slack under the combination of no contract in phase $F$ and a contract in phase $S$: Writing a contract in phase $F$ hence does not expand the set of $(\delta, q)$ in which the good equilibrium exists.

Figure 5: Existence of Good Equilibria under Written Contracts with Communication

![Diagram showing the existence of good equilibria with written contracts.](image)

The good equilibrium exists in the shaded region in Figure 5. It is more likely to exist as the discount factor is higher and/or the probability of meeting a type B principal is lower. The figure also shows that the region expands from that under bilateral repeated interaction/written contracts without communication (ink cost only). The positive effect of communication is in particular strong in the north-west region where the discount factor is low and the proportion of type B principals is high. This is exactly the situation where the principal’s self-enforcing condition is hard to satisfy.
5 Extensions

5.1 “Good” Principals

The formulation of preferences in reputation models typically includes the third, “good” type. Suppose in addition to types B and O, there is the third good (G) type in proportion $\alpha$, where $\alpha + \beta + \gamma = 1$. The type G principals always pay what they promised, because, for example, they enjoy private benefit from making promised payments, they suffer from large negative disutility from reneging, their discount factor is close to one (or larger), and so on.\footnote{However, I exclude the possibility that principals make payment offers exceeding benefit $y$.}

The analysis of good equilibria where type O principals make promised payments is not affected by the addition of type G, because type O behave exactly like type G principals. However, the existence of type G opens the possibility of another equilibrium under which type O principals do not pay in phase $S$. Only type G principals make payments and hence are screened to phase $F$. Types B and O principals go back to the matching pool and repeat phase $S$ (with probability $\rho$).

In this “low-friendly-phase” equilibrium, type O principals behave exactly like type B principals, and hence I call them type NG who stay in phase $S$. The steady-state distribution of types can be obtained in a way similar to the distribution under the previous good equilibrium. For type NG principals, the rate of inflow is $(1 - \rho)(1 - \alpha)$ which is equal to that of outflow. The remaining $\rho(1 - \alpha)$ repeats phase $S$. For type G principals, the rate of inflow $(1 - \rho)\alpha$ is equal to the sum of the exit rate $(1 - \rho)g$ and the rate of moving to phase $F$ equal to $\rho g$, where $g \in [0, \alpha]$ is the proportion of type G in phase $S$. Hence $g = (1 - \rho)\alpha$. In phase $F$, the rate of inflow $\rho g = \rho(1 - \rho)\alpha$ is equal to that of outflow $(1 - \rho)(\alpha - g) = \rho(1 - \rho)\alpha$. Denote the probability of an agent’s meeting a type G principal in phase $S$ by $p = g/(g + 1 - \alpha)$.\footnote{However, I exclude the possibility that principals make payment offers exceeding benefit $y$.}
The agent’s present values are given as follows:

\[ U_F = w_F + \delta U_F - d \]
\[ U_S = p(w_S + \delta U_F) + (1 - p)\delta U_S - d \]

The incentive compatibility constraints are \( w_F \geq d \) and \( pw_S \geq d \), and hence the optimal payments are \( \hat{w}_F = d \) and \( \hat{w}_S = d/p \), and the agent’s equilibrium values are \( U^*_F = U^*_F = 0 \).

The type O principal’s present value in phase S is \( V_S = y + \delta V_S = y/(1 - \delta) \) since she does not compensate for \( a = 1 \). This is the maximum attainable value, and hence what positive level \( w_S \) is, she has no incentive to pay, and instead chooses not to pay and terminates the relationship. The only condition to be satisfied for the existence of the low-friendly-phase equilibrium is that the payment in phase S, \( \hat{w}_S = d/p \), cannot be higher than \( y \), or

\[ p = \frac{g}{g + 1 - \alpha} \geq \frac{d}{y}. \]

**Proposition 4** There exists \( \alpha \in (0, 1) \) such that if \( \alpha > \alpha \), a low-friendly-phase equilibrium exists.

**Proof** By simple calculation the left-hand side of (14) is equal to \( \alpha(1 - \rho)/(1 - \alpha \rho) \), which is increasing in \( \alpha \), goes to 0 as \( \alpha \downarrow 0 \), and goes to 1 as \( \alpha \uparrow 1 \).

Q.E.D.

Note that the low-friendly-phase equilibrium is efficient (all the agents choose \( a = 1 \)). There is hence no efficiency loss if the proportion of type G principals is sufficiently high. And no party has an incentive to write a costly contract in this equilibrium if it exists. My previous analysis thus corresponds to the case in which the proportion of type G principals is sufficiently low (\( \alpha < \alpha \) holds).
5.2 Heterogeneity in the Agents’ Population

In the main model I have assume that all the agents are opportunistic. Introducing agents of myopic type with zero discount factor does not change the analysis because such a type can also be induced to choose $a = 1$ by compensation in the same period.

If the definition of type B agents is such that they never choose $a = 0$, the main messages of the paper still go through with some modification. First, phase $S$ becomes more costly for principals and hence an equilibrium in which type O principals make positive payments can exist even without written contracts. Second, even type O principals with contracts may return to phase $S$ if they meet type B agents. Then while the information value of written contracts is reduced, it does not disappear because type B principals always return to phase $S$ (if they survive).

5.3 Contractible Fixed Payments

Throughout the paper I have assume that no payment is contractible, in contrast to literature in relational contracting such as Levin (2003) in which fixed transfers are contractible. The payment schedule in my analysis corresponds to a “bonus” contract where the principal compensates the agent by a bonus pay in the current period.

Suppose instead that fixed payments are contractible, or equivalently, principals can pay some fixed amounts in advance before agents choose their action. Then without written contracts, it is in the interest of the agents to choose $a = 0$ and terminate their relationships.

Similar to the previous analysis, writing a “termination” contract in which an agent is paid a fixed amount along with the termination clause after shirking can restore his incentive to choose $a = 1$, if the agent must incur ink costs. However, there is no communication role in writing such a contract, because there is heterogeneity only in the principals’ population, and hence it is the principals’ reputation that is at stake. The parties should hence write a bonus contract to facilitate informal enforcement via communication. What kind of contract to write thus matters.
This conclusion is reversed if only the agents’ population is heterogeneous. In this case, the agents’ reputation is at stake, and hence the parties should write a termination contract in which the principal has no chance to deviate while the agent’s incentive to choose $a = 1$ is provided with future rent.

If both players’ population is heterogeneous, what should be written in a contract is an issue. Writing either a bonus contract or a termination contract only serves as a device of communicating which side is at stake, while writing a contract where both sides obtain future rents reduces the information value of the contract. This problem is to be explored in future research.

6 Concluding Remarks

To make a communication role of writing a contract suggested by empirical literature more precise, I have developed a simple but novel framework where there is a large population of principals and agents and they are matched and decide whether or not to continue their partnerships. And I have then shown that writing a costly contract, although it is unenforceable, contributes to its informal enforcement and facilitates relationships.

The current paper is just a start and does not offer an explanation of particular empirical evidence nor testable implications. Offering them is an obvious next step, and to this purpose it is important to extend the current model to heterogeneity in both principals’ and agents’ population. Another possible extension is to moral hazard, in which the agent’s action is unobservable to the matched principal while her benefit is observable but stochastic.
References


Appendix

In this appendix I prove Proposition 3 by proving the following three lemmas. By Lemma 1 I can confine attention to sufficiently high discount factor $\delta \geq d/y$. In Lemma 2 I obtain the condition on $(\delta, q)$ under which there exists a contract in phase $S$ such that both (ICS) and (NS) hold when no contract is written in phase $F$. In Lemma 3 I show that (ICF) holds without a written contract in phase $F$. Since writing a contract in phase $F$ only makes the other constraints (ICS) and (NS) more stringent, no contract in phase $F$ turns out to be without loss of generality.

**Lemma 1** There is no good equilibrium if $\delta < d/y$.

**Proof** Suppose $\delta < d/y$. By (NS),

$$c_S \leq \Pi(\delta) - \sigma(\lambda_S) - \delta c_F \quad (A1)$$

holds, where $\Pi(\delta) \equiv V_F + c_F = (y - d)/(1 - \delta)$. (ICS) is rewritten as

$$(1 + \sigma(\lambda_S))d \leq \delta(1 - \lambda_S)\sigma(\lambda_S)d + \delta(1 - \lambda_S)c_S - \delta(1 - \delta(1 - \lambda_S))c_F + \delta\lambda_S\Pi(\delta). \quad (A2)$$

Using (A1) and summarizing yield

$$(1 + \sigma(\lambda_S))d \leq \delta\Pi(\delta) - \delta c_F \leq \delta\Pi(\delta),$$

which in turn leads to

$$\sigma(\lambda_S)d \leq \delta\Pi(\delta) - d = \frac{\delta y - d}{1 - \delta} < 0$$

A contradiction. \[Q.E.D.\]

Hereafter assume $\delta \geq d/y$.  

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Lemma 2 Suppose no contract is written in phase $F$. There exists a contract $(c_S, \lambda_S)$ in phase $S$ satisfying both (ICS) and (NS) if and only if

$$q \leq q_1(\delta) = \frac{\delta y - d}{\delta y - d + (1 - \delta)(1 - \rho)d}$$  \hspace{1cm} (A3)$$

Proof (ICS) and (NS) under no contract in phase $F$ are, respectively, summarized as follows:

$$\sigma(\lambda_S) d \leq \frac{\delta \lambda_S \Pi(\delta) - d + \delta(1 - \lambda_S)c_S}{1 - \delta(1 - \lambda_S)} \hspace{1cm} (A4)$$

$$\sigma(\lambda_S) d \leq \Pi(\delta) - c_S \hspace{1cm} (A5)$$

It is easy to find that (A4) binds if $c_S \leq y$, while (A5) binds otherwise. Since the right-hand side of (A4) is increasing in $c_S$ and $\lambda_S$, and that of (A5) is decreasing in $c_S$, the right-hand sides of (A4) and (A5) are maximized at $c_S = y$. The left-hand side is, on the other hand, minimized at $\lambda_S = 1$ since $\sigma(\lambda_S) = (1 - r(\lambda_S)q/(1 - q)$ is decreasing in $\lambda_S$. Substituting $c_S = y$ and $\lambda_S = 1$ into (A4) yields

$$\frac{q}{1 - q(1 - \rho)} \leq \frac{\delta y - d}{(1 - \delta)d},$$

or

$$q \leq q_1(\delta) = \frac{\delta y - d}{\delta y - d + (1 - \delta)(1 - \rho)d}$$  \hspace{1cm} (A6)$$

which is the condition in Proposition 3. If $q > q_1(\delta)$, no contract can satisfy (ICS) and (NS). If $q \leq q_1(\delta)$, which is not empty for $\delta \geq d/y$, is attainable by $c_S = y$ and $\lambda_S = 1$.  \hspace{1cm} \textbf{Q.E.D.}

Lemma 3 Contracts with $c_S = y$ satisfy (ICF) when no contract is written in phase $F$.

Proof Substituting $c_F = \lambda_F = 0$ into (ICF) and summarizing yield

$$\sigma(\lambda_S) \geq \frac{1}{\delta} - \frac{c_S}{d}.$$  \hspace{1cm} (A7)
When $c_S = y$, the right-hand side becomes

\[ \frac{1}{\delta} - \frac{y}{d} = \frac{1}{\delta d}(d - \delta y) \]

which is nonpositive by $\delta \geq d/y$. Condition (A7) hence holds for $c_S = y$. Q.E.D.