Scoring-rule and the choice of a contractor by a financially constrained government

Axel Gautier

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Abstract

Governments and public authorities are increasingly relying on Public-Private Partnerships for the provision of public goods and services. It is often argued that the use of a public private partnership can relieve the government from a strained budget constraint and, in this paper, we analyze the choice of a contractual solution for designing, constructing and managing a project, with observable externalities, by a budget-constrained government. We show that the quality of the project and the designed contractor are affected by the extent of the budget constraint and we show that the use of a PPP may partially overcome the resulting inefficiencies.

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†CREPP, HEC-Université de Liège, Bat B31, Boulevard du Rectorat 7, 4000 Liège, Belgium, and CORE, Université Catholique de Louvain, Belgium. E-mail: agautier@ulg.ac.be, Tel: +32-4-366.30.53.
1 Introduction

Governments and public authorities are increasingly relying on Public-Private Partnerships for the provision of public goods and services. A PPP is a long-term contract between a private firm and a public authority for the provision and the management of an infrastructure project. Its main feature is the delegation of multiple tasks (design, construction and management) to a single private partner (the consortium) and, usually, the contract matches the life span of the assets. This contractual solution should be contrasted with the traditional procurement contract in which each task is delegated separately to a private partner (or provided in-house by the public authority).

It is often argued that bundling several tasks into a single long-term contract can be justified on efficiency grounds. When the quality of the design is difficult to measure and it affects the operating cost the facility, bundling design, construction and management into a single long-term contract allows the private consortium to take these synergies into account (Bennet and Iossa, 2006; Iossa and Martimort, 2008). When a better quality project is less costly to manage and quality cannot be contracted upon, task bundling is likely to be optimal. Despite that, PPP are not necessarily the preferred contractual solution. PPP are, for instance, more vulnerable to (costly) renegotiation (Guash et al., 2006), since there are long-term contracts and partners are locked-in the relationship. Renegotiation originates either in the necessity of adapting the initial contract to unforeseen contingencies or because the partners, the public or the private, behave strategically.

Synergies are probably not the main factors that explain the increasing use of PPP worldwide. As a matter of fact, the use of a public private partnership can relieve the government from a strained budget constraint. Often in a PPP, the private partner finances the infrastructure and recovers its investment by collecting user fees and/or receiving subsidies that are spread over the whole life span of the infrastructure. Thus, a public authority can circumvent its budget constraint by using a PPP. But, a PPP cannot be justified by a public finance argument alone (Engel et al., 2007).

In this paper, we consider a budget-constrained government that lacks of the necessary funds to finance an infrastructure project. Think for example of a local government may that has a limited access to financial markets. As stressed by the World Bank (2000):

"In many developing countries, few of these conditions [for
borrowing] exist. Long histories of macroeconomics instability make long-term financial commitment extremely risky. Information on potential borrowers is unreliable. The legal framework needed to provide investors with recourse in case of default is underdeveloped and often untested. Municipal governments in these countries are viewed -often correctly- as particularly unattractive borrowers because they lack the autonomy to raise revenues or reduce spending, particularly on personnel. Moreover, local governments often have no credible political commitment to long-term financial obligations. Under these conditions, even if long-term private capital is available, local governments generally can borrow only at very high rate of interest, if at all.” (World Bank, World development report page 133)

Thus, even if the future discounted revenues exceed the initial investment cost, the government may lack of the necessary funds to finance the project. This kind of cash-in-advance constraint is the first building block of our analysis. We are interested in the role of such a constraint on the decision to bundle or not tasks. Notice that a limited budget for financing projects is not necessarily justified by borrowing constraints. For Maskin and Tirole (2008), imposing a spending limit may discipline public officials that, otherwise, would have invested in pet projects.

The project we consider is a three-stages project. At date 0, the project must be designed and constructed. Design and construction are delegated to a private contractor. At date 1, the infrastructure is operated. In a PPP, design, construction and management are delegated to a single private firm. In a traditional procurement contract (TPC), design and construction are delegated to a private firm and the government manages the facility itself. The quality of the project designed at date 0 affects the cost of operating the project at date 1. Quality can be for example the energy efficiency of a building: More energy-efficient constructions have lower heating costs. Unlike other papers (for instance Bennet and Iossa, 2006), we consider that the quality of the project can be contracted upon at date zero. And, we consider the case where there are positive synergies between the design/construction and the management i.e. a higher quality project has a lower operating cost.

The second building block of our analysis is to explicitly consider the competition between firms for the contract. There are two firms competing at date 0 for the project. Firms design a project of a given quality $q$ and offers
to construct it for a price $p$. Since quality is observable and verifiable, the government can use a multi-criterion auction to select a contractor. Firms submit a bid that consists of a price-quality package. In this paper, we use a scoring rule (Burgett and Che, 2004) to rank the bids made by the firms. The main advantage of this auction format is its simplicity. Its drawback is that it is vulnerable to corruption. Firms are heterogeneous and, in particular, they can supply different project qualities at different costs. The government selects the best offer and the optimal scoring rule reflects the governmental preferences for price and quality.

We start our analysis by a benchmark case where the government delegates the construction and the design of the project to a private contractor but remains in charge of the operations. In our benchmark case, the weights given to price and quality in the scoring rule reflects the preferences of the buyer and the government trades-off a higher quality with a lower operating cost. Then, we introduce the budget constraint. We assume that the government has a spending limit, and to make the problem interesting, the spending limit is below the cost of the project in the benchmark case. This means that the best offer in the benchmark case is no longer a feasible option. Gautier (2004) and Gautier and Mitra (2006) show that such a budget constraint distorts the optimal regulatory mechanism. Likewise, we show that the optimal scoring rule that applies when the government is financially constrained is distorted and the government puts a higher weight to the price and less to the quality. As a consequence, the firms modify their bidding behavior and, eventually, the government may select another contractor than in the benchmark case. The budget constraint thus modifies both the investment and the operating costs of the project. With a financially constrained buyer, the project has a lower quality (it is less energy efficient) and a higher operating cost (higher heating expenses). Moreover, the selected contractor might be different which is another source of inefficiency.

Finally, we consider the case where construction and operations are bundled into a single long-term contract (a Public-Private Partnership) and we assume that the private contractor is not necessarily more efficient than the government for operating the project i.e. delegation of operations is not justified on efficiency grounds. Despite that, the bundling of tasks might be preferred by the government because the PPP is a way to partially circumvent the budget constraint. We show that the PPP may restore product efficiency and leads to the choice of the most efficient contractor. At the same time, competition at the bidding stage is less intense, leading in par-
ticular to a higher price for the project as a whole. The optimality of a PPP depends on a trade-off between these two dimensions.

In this paper, we construct a very simple model with the following features: firms compete for the contract and the competitive process is explicitly taken into account and the government has a spending limit for building the facility at date zero. We point the following trade-off: with a TPC, the government must reduce the quality of the facility at date zero and thus, operating costs at date 1 are higher. With the PPP, there is no need to match the construction cost with the budget constraint, but competition for a long-term contract is less fierce and the government must leave more profits to the firm at equilibrium. In our model, the choice of a contractual solution depends on a trade-off between these two dimensions.

2 The model

2.1 The project

Let us consider an infrastructure project. The whole project is three stages project: Design, Construction and Management. At date zero, the project is designed and then constructed according to the design. The public authority has no expertise neither in design nor in construction and these tasks must therefore be delegated to a private firm. The design is represented by a quality parameter $q$. Quality is verifiable and can be contracted for ex-ante. The construction of a project of quality $q$ costs $C_i(q)$ to firm $i$.

At date 1, the facility is operated either by the public authority (or another private contractor on its behalf) or by the firm that has designed and constructed the project (the consortium). We assume that the public authority and the consortium are equally efficient for managing the facility. Operating costs at date 1 and given by:

$$d_0 + dq$$

Managing the facility involves two kinds of cost: a fixed cost $d_0$ and a cost $dq$ that depends on the projects design $q$. There are synergies between design and operation; this assumption is quite standard in the literature. In this paper, we will mainly focus on the case of positive synergies: $d < 0$. That is a better design (a higher $q$) reduces the operating cost at date 1.

The government and the firms discount the future with an identical discount factor $\delta$. 

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2.2 The firms

There are two firms that compete for the contract. Competing firms will submit a design $q$ and a price $p$ for the construction of the utility; in the case of a PPP, the contract also includes the management. The firms are heterogeneous. Firm 1 is more efficient to deliver a high quality good while firm 2 is more efficient for a low quality good. The construction of a good of quality $q$ involves two kinds of cost for firm $i$: a fixed cost $k_i$ and a variable cost $\theta_i q^2/2$.

$$C_i(q_i) = k_i + \theta_i \frac{q^2}{2}$$

We assume that, for any given $q$, firm 1 has a higher fixed cost but a lower variable cost than firm 2: $k_1 > k_2$ and $\theta_1 < \theta_2$. Thus, firm 1 is more cost effective than firm 2 for projects of quality above $\tilde{q} = \sqrt{2(k_1 - k_2)/(\theta_2 - \theta_1)} > 0$ and firm 2 is more cost effective for projects of a lower quality. Cost functions for the two firms are represented on figure 1. Firms maximize their profits and conditional on obtaining the contract for a price $p$, with a TPC, firm $i$ has a profit equals to

$$\pi_i = p - C_i(q_i)$$

If a firm does not obtain the contract, its profit is equal to its outside option normalized to zero. Notice that there is no design cost per se. A firm that submitted a design that the authority has not chosen does not incur any cost.
2.3 The government

When a project of quality $q$ is completed, the gross surplus of the government is equal to

$$s_0 + sq$$

from which the payments for the construction and the management must be subtracted.

We assume that the government is financially constrained. We formalize that by assuming that, at date zero, the government has spending limit $T$ to finance the construction of the project. There is no spending limit at date 1, but the government cannot borrow from future income flows.

2.4 The contract

We consider two main contracts: the traditional procurement contract (TPC) where the private firm is in charge of the design and the construction and the public authority in charge of the management and the Public-Private Partnership (PPP) where all the tasks are bundled and delegate to the private consortium. In both cases, the contract specifies a design $q$ that the private partner must deliver and a compensation $p$.

2.5 Attributing contract: the scoring rule

Attributing contracts is a complex task. Contracts can be either negotiated or putted out to tender. There is a large literature comparing the two procedures (see for example Bulow and Klemperer, 1996 and Bajari and Tadelis, 2001).

In this paper, we will consider that the contract is attributed through a multi-dimensional auction. The government being interested in two dimensions of the project, the design $q$ and the price $p$, the multi-dimensional auction proves to be useful to take these two characteristics of the project into account in a transparent way. More specifically, we will use the linear scoring rule of Burguet and Che (2004) to attribute the contract. With the scoring rule, firms submit a quality price offer $(q, p)$ and the scoring rule attributes a bid value to the offer. The bid $b$ associated with $(q, p)$ is

$$b = \alpha q - p,$$
where $\alpha$ is the weight given to the design in the scoring rule. The contract is attributed to the firm who makes the highest bid. The government chooses $\alpha$ according to its preferences for price and quality.

3 The benchmark case

Suppose, for a while, that the government is not financially constrained. Its objective is to design the scoring rule (chose the $\alpha$) to maximize its expected surplus:

$$\max_\alpha (s_0 + sq) - p - \delta(d_0 + dq)$$

Firms submit price-quality package $(p, q)$ and the winning bid is the offers with the associated highest $b$. With the linear scoring, it is possible to determine price and quality independently.

Lemma 1: The firms optimal design is $q_i^* = \alpha/\theta_i$.

Proof: Suppose that firm $i$ submits a bid such that $C_i'(q_i) \neq \alpha$. In such a case, the firm can profitably increase its profit without modifying its probability of winning i.e. without changing the resulting bid $b_i$. Consider the case where $C_i'(q_i) > \alpha$, then if the firm reduces the quality offered by 1 and decreases the price by $\alpha$, its bid $b_i$ (and thus the probability of winning) is left unchanged, $\Delta b_i = 0$ but its profit (conditional on winning) increases by $\Delta \pi_i = C_i'(q_i) - \alpha > 0$. The case where $C_i'(q_i) < \alpha$ is exactly similar.

So, the designs are determined independently of the prices. Firm 1 offers a higher quality $q_1^*$ than firm 2: $q_1^* = \alpha/\theta_1 > \alpha/\theta_2 = q_2^*$. The cost of providing the optimal quality for firm $i$ is given by:

$$C_i(q_i^*) = k_i + \frac{\alpha^2}{2\theta_i}$$

Given the optimal qualities $q_1^*$ and $q_2^*$, firms compete in price to place a winning bid. We have:

Lemma 2: Firm 1 places the winning bid if

$$\alpha \geq \tilde{\alpha} = \sqrt{\frac{2(k_1 - k_2)\theta_1\theta_2}{\theta_2 - \theta_1}}.$$  

Otherwise firm 2 places the winning bid.
Proof: The lowest possible price $p_i$ for firm $i$ is such that it realizes a zero profit: $p_i = C_i(q_i^*)$. Thus, the highest possible bid for firm $i$ is $b_i = \alpha q_i^* - C_i(q_i^*) = \alpha^2/(2\theta_i) - k_i$. Against such a bid, firm $j$ must place a higher bid $b_j$ to win the auction. Given that $q_j^* = \alpha/\theta_j$, firm $j$ wins the auction if

$$p_j < \alpha(q_j^* - q_i^*) + C_i(q_i^*) = \frac{\alpha^2}{\theta_j} - \frac{\alpha^2}{2\theta_i} + k_i$$

Such a price is compatible with a positive profit for firm $j$ if:

$$\alpha(q_j^* - q_i^*) + C_i(q_i^*) - C_j(q_j^*) > 0$$

Rearranging the equation, we immediately see that this condition is satisfied for $j = 1, i = 2$ if $\alpha \geq \tilde{\alpha}$, and for $j = 2, i = 1$ if $\alpha \leq \tilde{\alpha}$.

To summarize, we have:

1. For $\alpha < \tilde{\alpha}$

$$(q_1, p_1) = (\alpha/\theta_1, k_1 + \alpha^2/(2\theta_1))$$

$$(q_2, p_2) = (\alpha/\theta_2, k_1 + \alpha^2/(\theta_2) - \alpha^2/(2\theta_1))$$

$$b_2 > b_1.$$  

2. For $\alpha > \tilde{\alpha}$

$$(q_1, p_1) = (\alpha/\theta_1, k_2 + \alpha^2/\theta_1 - \alpha^2/(2\theta_2))$$

$$(q_2, p_2) = (\alpha/\theta_2, k_2 + \alpha^2/(2\theta_2))$$

$$b_1 > b_2.$$  

3.1 The price functions

Before going further in the analysis, it is interesting to define the price functions, that is the optimal price charged by the firms as a function of the parameter $\alpha$. Price functions are:

$$p_1(\alpha) = \begin{cases} k_1 + \alpha^2/(2\theta_1) & \text{if } \alpha \leq \tilde{\alpha} \\ k_2 + \alpha^2/\theta_1 - \alpha^2/(2\theta_2) & \text{if } \alpha \geq \tilde{\alpha} \end{cases}$$

$$p_2(\alpha) = \begin{cases} k_1 + \alpha^2/(\theta_2) - \alpha^2/(2\theta_1) & \text{if } \alpha \leq \tilde{\alpha} \\ k_2 + \alpha^2/(2\theta_2) & \text{if } \alpha \geq \tilde{\alpha} \end{cases}$$
The function \( p_1(\alpha) \) is increasing and continuous in \( \alpha \). The function \( p_2(\alpha) \) is continuous in \( \alpha \). It is increasing in if \( 2\theta_1 - \theta_2 > 0 \). Otherwise, it is decreasing over the interval \([0, \tilde{\alpha}]\) and increasing for higher values of \( \alpha \). Price functions are represented on figure 2.

Insert fig. 2. here Coming soon!

### 3.2 The objective

The government’s objective thus writes as follows:

\[
\max_{\alpha} S \equiv \begin{cases} 
(\bar{s} + s\frac{\alpha}{\theta_2}) - (k_1 + \frac{\alpha^2}{2\theta_2} - \frac{\alpha^2}{2\theta_1}) - \delta(d_0 + d\frac{\alpha}{\theta_2}) & \text{if } \alpha \leq \tilde{\alpha} \\
(\bar{s} + s\frac{\alpha}{\theta_1}) - (k_2 + \frac{\alpha^2}{2\theta_2} - \frac{\alpha^2}{2\theta_1}) - \delta(d_0 + d\frac{\alpha}{\theta_1}) & \text{if } \alpha \geq \tilde{\alpha}
\end{cases}
\]  

The local maximizer along the second branch of \( S \) is \( \max[\tilde{\alpha}, \alpha^* = \frac{s - \delta d}{\theta_2 - \theta_1}] \) and we will assume that the \( \alpha^* \) is the global maximizer of the \( S \) function.

Without budget constraint, the optimal scoring rule is characterized by \( \alpha^* = \frac{s - \delta d}{\theta_2 - \theta_1} \). Firm 1 is the designated contractor and designs a project of quality \( q_1^* = \frac{s - \delta d}{\theta_2 - \theta_1} \). The total cost decomposes into a construction cost of \( k_2 + \frac{\theta_2}{\theta_1} (s - \delta d)^2 \) and a management cost of \( d_0 + d\frac{\theta_2}{\theta_1} (s - \delta d) \). In this benchmark case, there is no reason to delegate the infrastructure management to the private firm.

### 4 The budget-constrained government

In this section, we introduce the governments spending limit. The maximal budget for investment at date 0 is \( T \) and we assume that this amount is insufficient to finance the construction cost computed in the previous section: \( T < p_1(\alpha^*) \) i.e., the government has not enough resources to finance the high quality project proposed by firm 1. But, to avoid the multiplication of cases and subcases, we assume further that the government is able to finance the low quality project proposed by firm 2. Thus our assumption on the spending limit is:

\[
p_1(\alpha^*) < T < p_1(\alpha^*)
\]

There is no spending limit at date 1. In our view, the budget constraint should be understood as follows: the government has a flow of income at
each period, coming from taxes or user fees. With well-functioning financial markets, the government should be able to borrow at date zero to finance the construction of the facility and pay the money back with its future income flow. A well-constrained government cannot make such financial transfers between periods i.e. it cannot borrow future income.

4.1 The traditional procurement contract

In the traditional procurement contract, the government chooses to manage the facility and delegates the design/construction to a private firm. To choose a contractor, the government still uses the linear scoring auction but the scoring rule must be adapted to avoid bids over $T$.

Adding the budget constraint $p \leq T$ in the government’s objective function, the optimal scoring rule is defined in the following proposition.

Proposition 1 With a spending limit $T$, the optimal $\alpha$ is:

1. If $T > p_1(\hat{\alpha})$, $\alpha = \sqrt{(T - k_2) \frac{2\theta_1\theta_2}{\theta_2 - \theta_1}}$ and firm 1 is the designed contractor.

2. If $p_2(\hat{\alpha}) < T < p_1(\hat{\alpha})$, $\alpha = \hat{\alpha} - \epsilon$ and firm 2 is the designed contractor.

Proof: The optimal $\alpha$ is such that $p_1(\alpha) = T$ and $b_1 > b_2$. If this system of equation has no solution i.e. it is not possible to have a winning firm 1 with the budget constraint, the optimal $\alpha$ is the local maximizer of the first branch of $S$ in (7), which is $\hat{\alpha} - \epsilon$ if $\theta_2 > 2\theta_1$.

Proposition 1 defines the parameter of the scoring rule when the government is budget-constrained. The spending limits implies that the government should put relatively less weight on quality and thus relatively more weight on the price in the bid function. As consequences, prices and qualities offered by firms decline. To match the budget, the project has a lower quality and thus a lower construction cost and a higher operating cost.

And, if the constraint is strong enough $T < p_1(\hat{\alpha})$, the government can no longer buys from firm 1 and the designed contractor will change. In such a case, the quality of the design will be reduced even further (with as a corollary even higher operating costs at date 1) and the construction cost may eventually be below the spending limit $T$. 

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4.2 The Public-Private Partnership

With the PPP, the designed contractor receives at date 0 a payment equals to $Min[p, T]$ and the outstanding balance $Max[p - T, 0]$ at date 1.\(^1\) If construction costs exceed the date-0 payment, the firm must finance partially the construction with its own funds, which is obviously costly for the firm. More precisely, if the $T < C_i(q_i)$, part of the construction cost must be financed by the firm’s own resources. The firm’s profit is

$$\pi_i = T - C_i(q_i) + \delta[(p - T) - d_0 - dq_i] \quad \text{if } T < C_i(q_i)$$

$$\pi_i = P - C_i(q_i) + \delta[-d_0 - dq_i] \quad \text{if } T > C_i(q_i)$$

(8)

With a PPP, firms compete for the three-stages project, design, construction and management. Firms integrate in their design proposal that they will be responsible for managing the project. So, they need less incentives to provide quality, i.e. with an identical $\alpha$, they will supply a higher quality than in the benchmark case. Thus, for a given quality, the government can put relatively less weight on the quality dimension in the scoring rule and relatively more on the price aspect. Price competition between firms is more intense which is a priori good for the buyer. The drawback is that the high quality firm that faces more intense competition from the competitor and that must mobilize costly resources to finance the construction is less likely to win the auction.

Optimal qualities delivered by the firms are given by:

**Lemma 3** $q_i^* = \delta(\alpha - d) \theta_i$ if $T < C_i(q_i)$ and to $q_i^* = \frac{(\alpha - \delta d)}{\theta_i}$ otherwise.

**Proof:** as for lemma 1. \[\square\]

With the results of the lemma, we can identify the winning bid $b_i$ and the price functions. For that, we will assume that $T < C_1(q_1^*)$ and $T > C_2(q_2^*)$. We have:

**Lemma 4** The winning bid and the associated price and qualities are:

1. If $\alpha > \bar{\alpha}$, $b_1 > b_2$ and

   $$q_1 = \frac{\delta(\alpha - d)}{\theta_1}$$

   $$p_1(\alpha) = \frac{\alpha \delta(\alpha - d)}{\theta_1} - \frac{(\alpha - \delta d)^2}{2\theta_2} + k_2 + \delta d_0$$

\(^1\)This is actually, the payment which is the more favorable to the firm, given the budget constraint.
2. If $\alpha < \tilde{\alpha}$, $b_2 > b_1$ and
\[
q_2 = \frac{(\alpha - \delta d)}{\theta_2}
\]
\[
p_2(\alpha) = \frac{\alpha(\alpha - \delta d)^2}{\theta_2} - \frac{\alpha \delta(\alpha - d)^2}{\theta_1} + \frac{k_1}{\delta} + \delta d_0 - \frac{T(1 - \delta)}{\delta}
\]
and $\tilde{\alpha}$ is defined by the following equation:
\[
\frac{\delta(\alpha - d)^2}{2\theta_1} - \frac{(\alpha - \delta d)^2}{2\theta_2} = \frac{(k_1}{\delta} - k_2) + d_0 - \frac{T(1 - \delta)}{\delta}
\]

The budget constraint has two impacts on the bidding of firms. Firstly, the quality gap between the two firms decreases because, firm 1 must partially finance the construction from its own resource while firm 2 should not. Secondly, a lower $T$ extends the parameter range where the winner of the auction is the low quality firm. Moreover, in such a case, the price paid to firm 2 is negatively affected by $T$: $\frac{\partial p_2(\alpha)}{\partial T} < 0$ i.e. a stronger budget constraint implies a higher price paid to firm 2.

The objective of the government is
\[
\max_{\alpha} S \equiv \{ \begin{array}{ll}
(s_0 + s q_2^*) - p_2(\alpha) & \text{if } \alpha \leq \tilde{\alpha} \\
(s_0 + s q_1^*) - p_1(\alpha) & \text{if } \alpha \geq \tilde{\alpha}
\end{array}
\]

In the benchmark case, the local maximizer along the first branch of $S$ is a corner solution at $\tilde{\alpha}$ if $2 \theta_1 < \theta_2$. In such a case, the price function is decreasing in $\alpha$ which means that higher qualities are cheaper up to $q_2^*(\tilde{\alpha})$. This is no longer true in the PPP case and even with $2 \theta_1 < \theta_2$, the price function $p_2(\alpha)$ might be increasing in $\alpha$. Thus there are potentially two interior local maximizers of the objective function, one computed along the first branch of $S$, another computed along the second branch. And the global maximizer of the function depends on the assumptions on the parameters including the strength of the budget constraint.

4.3 Comparisons

Comparisons between the TPC and the PPP are thus complicated. In this section, we first show that the outcome is not equivalent with the two contracting formulas. Second, we show the main trade-off faced by the government and finally, we illustrate the choice of a contracting solution with a numerical example (coming soon).
If the government uses the TPC, its objective function is given by (7) but it applies only for values of $\alpha$ below $\sqrt{(T - k_2) \frac{2\theta_2}{\theta_2 - \theta_1}}$. If the government uses the PPP, its objective function is given by (9). Clearly enough, the two programs are not equivalent and the different contractual solutions lead to different price-quality packages.

There are two main differences between the two programs, that could be summarized as follows:

- the highest qualities are only feasible in (9)
- quality is more costly in (9)

The budget constraint modifies the way firms compete for the attribution of the contract. The PPP is a way to partially circumvent the budget constraint and higher quality projects become feasible with such a contract. But quality is more expensive with this kind of contract. The government thus faces a trade-off between higher quality and more rents left to the firm.

A numerical example: coming soon.

5 Concluding remarks

Our findings can be summarized as follows:

- The budget constraint modifies the way firms compete and, even if externalities can be perfectly contracted for, the traditional contract and the PPP are no longer equivalent.

- The scoring rule used by the government to select a contract depends on the type of contract and the extent of the budget constraint. In general, a budget constrained government puts less weight on quality and more on price.

- The budget constraint leads to lower quality projects with higher operating costs.
References


