Commitment in utility regulation:

A model of reputation and policy applications

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Abstract

This paper builds a dynamic model of utility regulation where a government cannot commit to a time-inconsistent policy of not expropriating investment. By allowing the government’s type to change over time, I explore how reputation concerns may generate partial commitment. Restricting attention to equilibria that are strongly renegotiation proof, I show that there is a unique perfect Bayesian equilibrium. This contains episodes of investment and good behaviour followed by periods of expropriation and non-investment. I then apply the model to consider how the power of the incentive scheme and decentralization may influence the properties of this equilibrium. In the case of the power of incentives, the model suggests that price-caps may worsen commitment in developing countries, but not in developed ones. Similarly, the model suggests that decentralisation is likely to have a significant effect on commitment, but that this effect will depend on the general ability of the government to commit. Overall, we conclude that the effect of such policies on commitment will be different across countries, depending on the institutional environment.

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1 Introduction and previous literature

In many contexts, governments enter into a relationship with the private sector where the government’s optimal policy is time-inconsistent. This is to say that the government would like to commit itself to carrying out certain actions in the future, but when it actually comes to that point in time it would prefer to carry out different actions. Examples include the taxation of investment, sovereign debt, inflation policy and the regulation of utilities, which is the focus of this paper. In some situations the government can use a third party to tie its hands, but where institutions outside the executive are relatively weak such constraints may not be possible. When the government cannot constrain itself in this way, it may still be able to commit through the use of reputation.

In general, reputation encourages commitment since the government fears a period of ‘punishment’ by the private sector should it lose its reputation. In some instances, government reputation is always maintained and hence such a punishment is never carried out. However, in other cases, particularly in developing countries, a loss of government reputation is a very real prospect. In order to understand how commitment can be aided in developing countries, it is therefore important to use a model of government reputation that includes periods where reputation is lost. This article therefore builds a simple model which aims to capture such a situation. In particular, we study a situation where the government promises to a single firm not to expropriate its gains from investment.

Central to the model is the fact that the government’s preferences vary over time. This could be due to external factors such as a change in the need for government revenue. These environmental changes then translate into how much the government has to gain from expropriating. In good times, the gains may be fairly small, whilst in bad times the payoff from expropriating will be larger. Crucially, the firm is not aware of the government’s preferences, and hence its investment decision may be dependent on its current beliefs as to the government’s type.

One novelty of the paper is that we restrict our attention to studying equilibria that are renegotiation proof. This aims to rule out equilibria where at some point both parties would like to forget what has passed and instead play as if some other history had occurred. We
believe that the renegotiation proofness criterion is an important one in a model between the government and one other player because in this situation renegotiation is a very real possibility. For example, Guasch et al. (2007, 2008) show that in infrastructure regulation in Latin America, renegotiation is a very common event.\footnote{Of course, renegotiation can occur for a variety of different reasons. Indeed, Guasch et al. (2007, 2008) argue that many of these renegotiations are in fact beneficial only for the firm or the government. ‘Expropriation’ in our model can therefore be viewed as renegotiation that disadvantages the firm and favours the government. For a study of renegotiation that benefits the firm and not the government, see the model of Guasch et al. (2006). Stern (2009) provides a discussion of the positive side of many renegotiations.} We show that by restricting ourselves to renegotiation proof equilibria, we rule out traditional ‘trigger’ strategies where the firm proposes an arbitrarily long punishment period should the government expropriate.\footnote{For examples of models where trigger strategies are used to generate time-inconsistent behaviour, see Gilbert and Newbery (1994) and Salant and Woroch (1992) in the case of regulation; Green and Porter (1984) and Shapiro (1989) in the case of inter-firm collusion; Baker et al. (2002) in the case of inter-firm relational contracts; Barro (1986) and al Nowaihi and Levine (1994) in the case of monetary policy; Thomas and Worrall (1994) and Aguiar et al. (2007) in the case of FDI; Chari and Kehoe (1990) in the case of taxation.} These are not credible equilibria since, at some point during these proposed punishments, both the firm and the government would like to pretend expropriation had never happened and return to the firm investing.

Having rejected these equilibria, we then show that there is a unique perfect Bayesian strongly renegotiation proof equilibrium that may contain investment. In this equilibrium, the government’s needs are sufficiently low in good times that it will not expropriate. However, when needs are particularly great, i.e. times are bad, the government finds expropriating the preferable option. Given this strategy, the firm’s beliefs become the key factor in determining its investment. If the firm believes that the government is likely to be facing ‘good’ times, it will invest, since it believes the probability of expropriation to be low. On the other hand, when a firm believes the governments needs are high, it will fear expropriation and not invest. Since expropriation is equated with a government in ‘bad’ times, a period of non-investment will follow government expropriation. It is fear of this non-investment that keeps the government from expropriating when the state of the world is good. The non-investment period is not set by the firm as an amount of punishment but instead it is the time it takes for the firm to believe that the government’s type has probably changed and it is safe to invest again.

Since the firm does not observe the government’s needs directly, the equilibrium relies
on a system of signalling and built up reputation. At any given time, the government will wish to expropriate, but doing so sends the firm a signal that the government’s needs are high, and hence the government is likely to expropriate next time. Not expropriating, on the other hand, sends the signal to the firm that needs are low (i.e. times are good), and hence expropriation next time is unlikely. The government therefore builds a reputation for having low needs by not expropriating, and the desire to keep this reputation is the only reason it does not always expropriate.

Building the model in this way allows us to apply it to the case of monopoly regulation in developing countries, where reputation (rather than the rule of law) is a crucial factor in making sure governments provide the promised return on their investment. In particular, we can use the model to analyse how policy decisions affect the governments ability to commit. We study the difference between two incentive schemes - a cost-plus contract and a price cap - and the effect of decentralising regulation.

Looking first at the effect of the incentive scheme, we find that in situations where the firm’s profits are valued in a similar way to government revenues or consumer surpluses, a price-cap may enhance commitment. This is because, in this situation, expropriating is most tempting when the private firm is found to be inefficient, and the price-cap scheme makes sure that it is in this case that the firm’s profits are smallest. On the other hand, in developing countries where the firm’s profits are likely to be significantly less valued, price-caps weaken a government’s commitment ability. This is due to the fact that here the most tempting moment to expropriate is when profits are highest, and it is a price-cap system that creates the highest levels of potential profit.

In terms of decentralisation, we find that regulating at a more centralised level can average out the incentives to deviate in the two regions. The implication for policy thus depends on the context within a particular country. Centralisation may be a good idea if there is one region where private sector participation is very valuable, as this can incentivise a regulator not to expropriate in a less valuable region. On the other hand, if commitment under centralisation can not be acheived, then decentralising may at least allow those regions with the most valuable projects to receive investment. Overall therefore, we find that in both the case of the incentive scheme and decentralisation there can be a significant effect of the
policy on the equilibrium. Moreover, the implications are likely to be different for developing countries.

A few other papers have looked at the issue of government commitment in utility regulation. Gilbert and Newbery (1994) consider a case where the regulator and utility are restricted to a framework set by the legislator. Declared strategies are maintained by the utility threatening not to invest and the regulator threatening to minimise future profits. These threats are trigger strategies in which, once defection occurs, the two parties play the punishment strategies forever. They also compare the sustainability of a rate-of-return system to one with state-contingent linear price regulation when future demand is uncertain, and find a parameter range for which a rate-of-return regime is credible but not price regulation. Salant and Woroch (1992) offer a similar model where investment is maintained through trigger strategies and provide a description of many alternative equilibria, including ones where punishments are finite. Since in their model the two players effectively act simultaneously, the first-best solution cannot be achieved, but is tended to in the limit. In both models, expropriation does not take place in equilibrium.

Dassiou and Stern (2009) consider a continuous level of trust that the private sector has in the government honouring its commitment, which is updated over time dependent on the governments actions. They show that a misalignment in trust levels leads to suboptimal investment and explore how alternative contract types interact with this effect. Faure-Grimaud and Martimort (2003) build a model of limited commitment whereby a government may choose to constrain future governments by setting up an independent regulator. In their model it is certain that the identity of the political principle changes, and they suggest that making elections contestable and endogenous would be a valuable extension. Both these latter two models assume that the probability of a government reneging is an exogenous parameter. Guasch et al. (2006) also build a model whereby contracts are renegotiated, but here it is due to imperfect enforcement of the regulatory contract rather than the temptation of the government to expropriate. They explore the effects of a range of parameters, including the degree of state capture and the cost of public funds, and find that the implications are broadly consistent with the results of their empirical work.

On a more theoretical level, several other papers have also explored the role of reputation
where players types changed in an unobserved way. For instance, Cole et al. (1995) and Phelan (2006) study government commitment in the cases of sovereign debt and taxation respectively. Cole et al. (1995) consider an equilibrium where unstable governments renege on their debt commitments, and foreign lenders as a result do not lend to governments they believe to be unstable. Once the government becomes stable, they resume paying off their debts and hence signal to foreign lenders that they will not renege. In Phelan (2006) a low-tax government is committed to taxing reasonably, but an opportunistic type may decide to expropriate all capital. Phelan shows that the unique Markov equilibrium involves an opportunistic government playing a mixed strategy for some time after expropriation, since they benefit from households gradually increasing their production. In a different setting, Mailath and Samuelson (2001) model firms as either being able to only produce low quality goods or having the option of producing higher quality goods. Since firms names can change hands without consumers being aware, firms that can have an incentive to produce high quality products in order to indicate to consumers that their products are worth spending more on.

The model presented in this article differs in two significant ways from this previous literature. First, since we are studying a situation where there are only two players, we use renegotiation proofness as an equilibrium selection criterion. In the papers mentioned above, the authors have instead generally restricted themselves to looking at equilibria where strategies are dependent only on beliefs and types, rather than histories in general, i.e. Markov strategies. Such an assumption may be reasonable in the case where there are many players, as the coordination required for more general strategies would not be feasible. However, we cannot restrict our strategies in such a way here because when there are only two players, no such coordination is required and it is quite conceivable that contracts between the firm and the government include conditions on previous behaviour. In our setting renegotiation proofness lends itself as the natural criteria to use, whereas in the case with many players it is likely to be unnecessary since renegotiation would be infeasible.

Second, unlike the majority of the literature on reputation, we do not use a ‘Stackelberg type’ in our model. Stackelberg types are players who do not optimise their payoff function,

3These are also called ‘commitment types’ or ‘crazy types’.
but instead are forced to play the strategy they would like to play where they able to commit (they are called Stackelberg types as we can imagine them as a type that can publicly set their entire strategy before any other play takes place). The use of such types is common in the reputation literature since, as shown by Kreps et al. (1982), they allow cooperation when the horizon is finite. In a finite game, if one player knows for sure that another player is not such a type, then they know that this player will play the time-consistent strategy in the last period, and hence through backward induction reputation mechanisms will be ineffective. Whilst in many situations the existence of Stackelberg types may be a reasonable assumption, we believe that when modelling some cases of government behaviour it may be more realistic to assume that the horizon is infinite and that such types do not exist.

The article proceeds as follows. Section 2 presents the framework of the game, introducing the players and their payoffs. Section 3 then considers the equilibria of the game, focusing first on Perfect Bayesian equilibria before introducing the concept of renegotiation proofness and showing this produces a unique equilibrium. Section 4 then uses the model to study the implications of a couple of policy decisions, before Section 5 concludes. Proofs of all propositions are given in the Appendix.

2 The game

In the game there are two players, F and G, and nature. We can think of player F as representing the private sector and player G as the government. For analytical ease, we construct our model in continuous time. However, the following results will also hold in a discrete time model, and indeed our continuous time framework can be shown to be the limit of a discrete time one as the time between periods tends to 0. Both players seek to maximise their discounted expected payoff, with each discounting at a constant rate \( r \).

At a particular time \( t \), nature decides whether the state of the world is ‘high’ or ‘low’, which we can regard as G’s type. Player G’s type is G’s private information - player F is not aware of G’s type. Player F chooses whether to invest or not. The choice is discrete - they can either invest (at a cost of 1) or not invest (at a cost of 0). Player G sets \( R \) (subject to some restrictions), a return received by player F. If player F has invested at time \( t - \tau \),
then player G’s payoff is $R - \theta R$, where $\theta = \theta_L$ if the state of the world is low and $\theta = \theta_H$ if the state of the world is high. We assume $0 < \theta_L \leq \theta_H < 1$. If player F did not invest at time $t - \tau$, then player G’s payoff is 0, and they are forced to set $R = 0$. In this way, we can view investment as creating a total return of $\bar{R}$ at a time $\tau$ after the investment is made.\(^4\) The government then decides how much of this return is received by the firm, and the firm’s payoff is $R - 1$ if they invested and $R$ if they did not.

We can view the government’s payoff in the following way. Investment generates a total social high of $\bar{R}$. This return is then divided between the firm, which receives $R$, and the rest of society, which receives $\bar{R} - R$. The government’s payoff is then a weighted sum of these two returns, with the firm’s welfare weighted by $1 - \theta$ and the rest of society’s welfare weighted by 1. $\theta = 0$ therefore represents a government who is equally concerned with a firm’s profit as tax revenue or consumer welfare, whilst $\theta = 1$ represents a government who does not value the firm’s profit directly. Since the part $\bar{R} - R$ can be viewed as the returns that accrue directly to the government (through taxation say), we can view the variation in $\theta$ as the variation in the government’s need for public funds.

In order to incentivise player F to invest, player G may make a non-binding promise as to what $R$ will be in the future. In order to impose some structure on the contracts, we restrict the government to only being able to offer the firm a constant level of $R$ indefinitely. We assume that the firm does not take this offer as a signal of the government’s type in any way. In other words, if the government sets a level of $R$ at time $t$, the only non-zero rate or return it may pay between time $t$ and $t + \tau$ is this same level of $R$.\(^5\) The government can however at any time set $R = 0$, essentially expropriating the firm’s promised return on its investment. If the government does so, it is restricted to setting $R = 0$ for a length of time $\tau$.\(^6\)

We consider the game to be played over the infinite horizon. The use of an infinite

\(^4\)We are thus assuming that both investment, and the returns to investment, are constant over time. Extending the model to consider a situation where investment is increasing or decreasing in value would complicate the model whilst probably adding little insight - if investment is increasing in value, expropriation would be less tempting, whilst it would be more tempting if the value of investment was decreasing.

\(^5\)This restriction simplifies our analysis without changing any fundamental results, since the firm is only concerned with the expected level of $R$ over the infinite horizon, and this restriction does not decrease the range of potential expected returns that the government can offer.

\(^6\)This restriction is necessary to prevent any type of signalling on the side of the government, which would substantially change the model.
horizon is crucial in our analysis, since many of the following equilibria would not occur in an equivalent finitely repeated game.\footnote{More precisely, our equilibria cannot be created by considering games repeated a finite number $T$ times, and then taking the limit as $T \to \infty$.} Our analysis is therefore not relevant to situations where there is a known limit to the number of times that the interaction will be repeated. However, we suggest that, in the majority of cases where the government interacts with the private sector, an infinite horizon is the most appropriate way to model the situation. This is not to say that the interaction has no finite limit, but rather that the government tends to approach the game without consideration of a finite horizon. For more details on this argument, see Osborne and Rubinstein (1994, p.135-136).

Players are restricted to playing pure strategies. Though this assumption is fairly restrictive, it significantly simplifies parts of the analysis.

The state of the world $\theta$ evolves according to a time-homogeneous Markov process. We define the transition matrix $Q$ to be

$$Q = \begin{pmatrix} -\nu \beta & \nu \beta \\ \nu (1 - \beta) & -\nu (1 - \beta) \end{pmatrix} \quad (1)$$

Here $0 < \beta < 1/2$ is the long-run probability that the state of the world is high (or ‘bad’) and $\nu > 0$ is the rate at which the government’s type changes. Thus for a small time period $\epsilon$, the probability that the state of the world switches from high to low is approximately $\nu \beta \epsilon$ and the probability that it switches from low to high is approximately $\nu (1 - \beta) \epsilon$. The length of time that the government’s type stays constant is then exponentially distributed.

Let us define the matrix $P(s)$ to be made up of the elements $p_{ij}(s)$ where

$$p_{ij}(s) = \mathbb{P}(\theta(t+s) = j | \theta(t) = i) \quad (2)$$
where \(i, j \in \{\theta_H, \theta_L\}\). Then, as proven in the Appendix, we have

\[
P(s) = \begin{pmatrix}
p_{\theta_L \theta_L} & p_{\theta_H \theta_L} \\
p_{\theta_L \theta_H} & p_{\theta_H \theta_H}
\end{pmatrix}
= \begin{pmatrix}
1 - \beta (1 - e^{-\nu s}) & \beta (1 - e^{-\nu s}) \\
(1 - \beta) (1 - e^{-\nu s}) & 1 - (1 - \beta) (1 - e^{-\nu s})
\end{pmatrix}
\]

(3)

This matrix represents the probabilities that the government is of a particular type at time \(t+s\) given its type at time \(t\). We can see, for example, that at \(s = 0\), this matrix is simply the identity matrix, since the probability of changing states in 0 time is 0. At the other extreme, if we consider the limit as \(s \to \infty\), then the probability of being in the high state is \(\beta\) and the probability of being in the low state is \(\beta\), regardless of the state at time \(t\). This reflects the fact that the state of the world at time \(t\) becomes gradually less important as we move away from \(t\). Since the probability that the state of the world is high at time \(t\) may not be the same as at time \(t+s\), we are therefore considering a dynamic game rather than a simply repeated one.

In this dynamic game, we assume that both player F (the firm) and player G (the government) are long-lived. At all times they are therefore maximising their expected discounted payoff over the infinite horizon. We must also specify the commitment powers of each player. Since we are investigating a situation of weak government commitment, we assume that it can not commit itself - that is, its strategy must always be time-consistent.

### 2.1 Perfect Bayesian Equilibria

Let us first consider some of the perfect Bayesian equilibria of the game, without restricting ourselves to those that are renegotiation proof. In order to do so, it is helpful first to define two broad categories of equilibria: Full-investment equilibria and episodic-investment equilibria.

**Definition 1.** A **full-investment equilibrium with non-investment length** \(T\) **is an** equilibrium that meets the following description:

- Player F invests if and only if player G has not expropriated within a length of time \(T\).
where \( T \) is a constant in \([0, \infty]\).

- Along the equilibrium path, player \( G \) always shares.
- Hence, in equilibrium, there is constant investment.

Full investment equilibria are essentially those that achieve the first best - constant investment - and do so with the threat of a non-investment period should expropriation occur. Such a non-investment period never occurs however on the equilibrium path since governments share investment in both states of the world.

It is useful for us now to define some functions that summarise players’ payoffs. In particular, define \( W_H(T, R) \) as:

\[
W_H(T, R) = \frac{1 - e^{-r(T + \tau)}}{r} (\overline{R} - (\beta \theta_H + (1 - \beta) \theta_L)R) \\
- \frac{1 - e^{-(r+\nu)(T + \tau)}}{r + \nu} (1 - \beta)(\theta_H - \theta_L)R - \frac{1 - e^{-r\tau}}{r} \overline{R} \tag{4}
\]

Here \( W_H(T, \overline{R}) \) is the relative payoff to \( G \) of sharing rather than expropriating in a full-investment equilibrium if \( \theta = \theta_H \) (this is shown in the proof of Proposition 2 in the Appendix). The first term is the payoff the government would receive between \( t \) and \( T + t + \tau \) from not expropriating were \( \theta \) independently drawn at each time \( t \). \( \beta \theta_H + (1 - \beta) \theta_L \) is the unconditional expectation of \( \theta \), and hence \( \overline{R} - (\beta \theta_H + (1 - \beta) \theta_L)R \) is the expected payoff at each time in between time \( t \) and \( t + T + \tau \) (the term \( \frac{1 - e^{-r(T + \tau)}}{r} \) reflects the fact this payoff is discounted).

The second term then corrects for the fact that \( \theta(t) = \theta_H \), since this implies that the payoff from investment over the period \([t, t + T + \tau]\) is lower than if we did not know \( \theta(t) \). We can see that this is the case by taking the limit as \( \nu \to \infty \), i.e. by reducing the persistence of states to 0. Finally, the last term is the payoff that the government would receive from expropriating investment.

The following proposition then gives conditions under which there exists a perfect Bayesian full-investment equilibrium, where a perfect Bayesian equilibrium is one as defined by Fudenberg and Tirole (1991, pp.325-326). Perfect Bayesian equilibria are essentially equilibria where at each information set (a set of histories that the player cannot distinguish between) a player’s strategy is optimal given their beliefs. Furthermore, their beliefs are based on
players’ equilibrium strategies and updated using Bayes rule.

**Proposition 1.** There exists a perfect Bayesian full-investment equilibrium with non-investment length $T$ if and only if $W_H(T, R) \geq 0$. In such an equilibrium, $R$ will be set according to the equation

$$e^{-r\tau} R = 1$$  \hspace{1cm} (5)

The proof of the proposition is given in the appendix. Since the strategy of player $G$ does not depend on her type, we do not need to consider the beliefs of player $F$. Hence we are essentially looking for a subgame perfect equilibrium. In order to prove these strategies form a subgame perfect equilibrium, we check that neither player wishes to deviate using a version of the one-stage deviation principle suitable for perfect Bayesian equilibria.\(^8\) In our case the principle effectively states:

A player’s strategy is optimal from all his information sets if and only if there is no information set from which the player can gain by changing his strategy there, keeping it fixed at all his other information sets.

The principle is useful as it means we only need to check that each player can do no better by deviating just once from their equilibrium strategy - we do not have to check all possible deviations. In our case, we only need to check that they do not wish to deviate from their equilibrium strategy for a time $\epsilon$ as $\epsilon \to 0$.

The equilibrium can be sustained by $F$ threatening not to invest for a length of time $T$ were $G$ to expropriate. The threat is credible if $G$’s strategy is to expropriate all investment during this punishment phase, and hence $F$ has no incentive to invest. The condition $W_H(T, R) \geq 0$ signifies the requirement that this punishment period needs to be sufficiently long to incentivise the government not to expropriate even when $\theta = \theta_H$. Clearly it is when needs are high that the temptation to expropriate is greatest, and therefore if this expression holds the government will not deviate when $\theta = \theta_L$ either. $R$ is set according to equation (5).

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since this is the value of $R$ at which the firm is indifferent between investing and not, and the government has no incentive to offer anything higher.

Hence we can see that an efficient outcome can be generated as a perfect Bayesian equilibrium through player G fearing potential punishment for expropriating. This proposition would be the same in a game of complete information, since player G’s type plays no role in equilibrium behaviour.

Let us now define a different type of equilibria which, rather than having investment all of the time, experiences episodes of investment and episodes of non-investment.

**Definition 2.** An episodic-investment equilibrium with non-investment length $T$ is an equilibrium that meets the following description:

- Player F invests if and only if player G has not expropriated within a length of time $T$, where $T$ is a constant in $(0, \infty]$.
- Along the equilibrium path, player G shares if the state of the world is high and expropriates if the state of the world is low.
- Hence, in equilibrium, there are some periods of investment and some periods of non-investment.

Hence, episodic-investment equilibria are those where the first-best is not achieved, but there are periods of investment. Periods of non-investment are caused by the government entering the bad state of the world and expropriating. Investment resumes after a period of time $T$, independent of the state of the world. If needs happen to be high when investment resumes however, it will be expropriated and we immediately enter a new phase on non-investment.

Let us now consider when an episodic-investment equilibrium is perfect Bayesian. Before coming to the proposition, it is again helpful to define some notation. In particular, let us first define define $R(T)$ as:

$$R(T) = \frac{1 - e^{-rT}}{r + \nu \beta}$$

Here $R(T)$ is the level of $R$ at which F is indifferent between investing and not investing.
in episodic-investment equilibrium. It is more complicated than the equivalent level of $R$ in the full-investment equilibrium, since the firm must now take into account the probability that the government will not keep its promise. $(1 - \beta) \left(1 - e^{-r(T+\tau)}\right)$ is the probability that the state of the world is low at time $T + \tau$ after it was high, and $\frac{R(T) - 1 + r}{\nu + r}$ is the expected discounted payoff if the state of the world is low. The term on the right hand side, $\frac{1 - e^{-r\tau}}{r}$, is the cost of investment before the firm can expect any return.

Now let us define $W_L(T, \overline{R})$ as:

$$W_L(T, \overline{R}) = \frac{1 - e^{-(r+\nu)(T+\tau)}}{(1 - \beta)(1 - e^{-(r+\nu)(T+\tau)})r + \beta(1 - e^{-r(T+\tau)})(\nu + r)} \times \left( (1 - e^{-r(T+\tau)})(\overline{R} - \theta_L R) - (1 - e^{-r\tau})\overline{R} \right)$$

(7)

Similar to before, $W_L(T, \overline{R})$ is the payoff to $G$ of sharing rather than expropriating in an episodic-investment equilibrium if $\theta = \theta_L$. The expression is simpler than the previous one because the only potential payoff that accrues to player $G$ under shared investment is $\theta_L$, since expropriation will occur anyway when $\theta(t) = \theta_H$. We can see that this is clearly dependent on $R(T)$ as defined above, since a greater share going to the firm decreases the government’s payoff.

We can now give the proposition of when an episodic-investment equilibrium is perfect Bayesian.

**Proposition 2.** There exists a perfect Bayesian episodic-investment equilibrium with non-investment length $T$ if and only if:

$$W_L(T, \overline{R}) \geq 0$$

(8)

$$W_H(T, \overline{R}) \leq 0$$

In such an equilibrium, $R(T)$ is given by equation (6)

The proof of the proposition is in the appendix, and is very similar to the proof of Proposition 1 above. Here the punishment period is sufficiently long such that $G$ does not expropriate when needs are low (Condition (8)) but is at the same time sufficiently short that $G$ will expropriate when needs are high (Condition (9)). From examining the expressions for
$W_L(T, \bar{R})$ and $W_H(T, \bar{R})$, we can see that $W_L(T, \bar{R}) > W_H(T, \bar{R})$, and hence there is a range of $T$ for which both conditions can hold.

These two propositions show that trigger strategies can sustain equilibria that include investment. A given period of non-investment $T$ that results from expropriation may be enough to prevent the government expropriating in both states of the world (if condition (4) holds) or at least when the state of the world is high (if condition (4) doesn’t hold but conditions (8) and (9) do). Indeed, since $T$ can take any value in $(0, \infty)$, both types of equilibrium may well exist for a given set of parameters, and a range of values of $T$ will be able to support both. For the full-investment equilibrium this might not be a great concern, since the non-investment period is only hypothetical. However, for the episodic-investment equilibria, the value of $T$ effects the discounted payoff of both players, and hence it would be useful to analyse whether one particular value of $T$ is more likely to arise than another. The next section therefore considers how we may use renegotiation proofness as an equilibrium selection device.

### 2.2 A unique renegotiation-proof equilibrium

We have thus shown how there are multiple equilibria that include at least some investment, each using a trigger-strategy style form of reputation. However, these equilibria in general suffer from not being robust to renegotiation. In particular, during the period of non-investment (the ‘punishment phase’) both players receive lower continuation payoffs than they do in the non-punishment phases. Hence they may both like to renegotiate and start over as if the government had never expropriated.

Formally, we use the criteria of an equilibrium being ‘weakly’ or ‘strongly’ ‘renegotiation-proof’ formulated by Farrell and Maskin (1989). To understand this criterion, we model players’ strategies as being dependent on a ‘state’, in addition to their payoffs and beliefs. A ‘state’ is an equivalence class of histories such that players’ strategies are functions of this state, their beliefs and their type only. In our case, our state is the time since the government last reneged, $s$.

An equilibrium is then weakly renegotiation proof (WRP) if the payoffs at any pair of
states cannot be Pareto ranked.\textsuperscript{9} This is to say that there is no point in the equilibrium strategies where both players would prefer to play as if the state were different. In the case of the equilibria above, they would be WRP if there was no point in the players’ equilibrium strategies where they would both like to pretend that the government had expropriated more or less recently than it had.

Before we come to applying these conditions, it is again useful to define another expression. Let $T_B$ be defined by the following differential equation:

$$
\frac{\theta_L}{\nu \beta + r} \frac{dR}{dT}(T_B) + R - \theta_L R(T_B) = 0 \tag{9}
$$

The value $T_B$ represents the value of $T$ at which the government would most like the firm to recommence investment. This optimal point represents the solution of a trade-off between two competing desires. On the one side, the government would like investment to recommence sooner (smaller $T$) since this brings closer in time the prospective gains, and hence they are discounted less. On the other hand, to incite investment at this earlier point, the government needs to promise the firm a higher value of $R$ (as seen in the definition of $R(T)$), and hence it would like to delay investment in order to reduce this required incentive.

Given this value of $T$, we can now move on to describe the set of WRP perfect Bayesian equilibria.

**Proposition 3.** There exists a WRP perfect Bayesian episodic-investment equilibrium with non-investment length $T$ if and only if $W_L(T, R) \geq 0$, $T \geq T_B$ and $W_H(T, \bar{R}) \leq 0$. In such an equilibrium, $R = R(T)$. Furthermore, any WRP perfect Bayesian equilibrium is such an episodic-investment equilibrium with non-investment length $T$.

We can see therefore that the range of values of $T$ is a subset of the range for which there are perfect Bayesian equilibria set out in Proposition 2. If $T^A < T^B$, where $T^A$ is the solution to the equation $W_L(T^A, \bar{R}) = 0$, we can see that this will be a strict subset, and hence the perfect Bayesian equilibria with $T^A \leq T < T^B$ are not weakly renegotiation proof.

Prior to this time, the government would prefer not to offer the firm a reasonable contract and instead wait until it could offer a rate of return that was lower.

\textsuperscript{9}The definition given by Farrell and Maskin (1989) in fact uses strict Pareto ranking, but for our purposes the definition with weak Pareto ranking is more appropriate.
This proposition thus shows that a subset of the equilibria discussed above are not weakly renegotiation proof. This includes any full-investment equilibria, since both parties would be willing to renegotiate out of the non-investment phase as they do better pretending expropriation hadn’t happened. In the episodic-investment equilibrium, any equilibrium where the firm agrees to accept a contract now but threatens not to accept a contract at a later stage is ruled out, since such a threat is not credible to renegotiation. Hence, if it wishes, the government can postpone investment to a point where the required rate of return is lower.

The proposition however does not lead us to a unique equilibrium since we can not rule out equilibria where the non-investment period is longer than the government would desire, i.e. where \( T > T^* \). This is because within the equilibria with non-investment length \( T > T^* \) there is no state where the government offers \( R(T^*) \), and hence the parties cannot renegotiate here. However, we can arrive at a unique equilibrium by requiring ‘strong renegotiation proofness’. Again formulated by Farrell and Maskin (1989), an equilibrium that is strongly renegotiation proof (SRP) is essentially one where at all points the players would not both wish to renegotiate to some state of an alternative WRP equilibrium. This then gives us the following proposition

**Proposition 4.** Suppose that there exists a value \( T^A > 0 \) that solves equation \( W_L(T^A, \bar{R}) = 0 \). Then, letting \( T^* = \max\{T^A, T^B\} \),

- If \( W_H(T^*, \bar{R}) \leq 0 \), then the unique strongly renegotiation proof perfect Bayesian equilibrium is an episodic investment equilibrium with non-investment length \( T^* \).

- If \( W_H(T^*, \bar{R}) \geq 0 \), then the unique strongly renegotiation proof perfect Bayesian equilibrium is that with no investment.

If there is no value \( T^A > 0 \) that solves the equation \( W_L(T^A, \bar{R}) = 0 \), then the unique strongly renegotiation proof perfect Bayesian equilibrium is that with no investment.

We have thus identified a unique perfect Bayesian equilibrium by using strong-renegotiation proofness as an equilibrium selection criterion.
3 Applications

The previous section built a model of government reputation and commitment where non-investment resulted if the government reneged upon her promises for future rates of return. We then found a unique equilibrium by restricting our attention to those that were perfect Bayesian and strongly renegotiation proof. This section proceeds to apply this model to two policy issues. The model is first expanded to include uncertainty over future costs, and this is used to investigate the differences between a cost-plus regime and a price-cap regime. The chapter then considers the regulation of two non-competing regional enterprises and analyses how decentralising regulation to the regions may alter investment. In each case, the previous model and results are used to consider how the government’s ability to commit interacts with the regulatory framework.

3.1 Price-Cap vs. Cost-Plus

One of the key decisions when forming regulatory policy is whether to operate a price-cap or a cost-plus/rate-of-return style mechanism. In essence, under a cost-plus or rate-of-return regime the enterprise is guaranteed a fixed amount of profits. By being allowed to set prices sufficiently above costs, the enterprise will make the amount of profit agreed to. On the other hand, a price-cap system sets future prices significantly in advance, and hence the amount of profit the enterprise makes is variable. The debate over which style is more appropriate has many facets, but fundamentally, under a rate-of-return framework, the investor is relatively insured against risk, and this may encourage investment. On the other hand, since profits are guaranteed, the incentives to lower costs are fewer than under price-cap regulation, where the enterprise will keep any efficiency gains. This trade-off may therefore take the form of weighing up the need for observable infrastructure investment against the need for unobservable cost-reducing investment.

In developing countries the decision between the two regimes is present, although some of the factors to consider may be different. Kirkpatrick et al. (2005) show in their cross-country survey that regulators in developing countries are varied in which approach they choose. Laffont (2005, p.23) argues that we need to build a general understanding of how the type of
regime affects the commitment problem. Armstrong and Sappington (2006) describe that, since price-caps allow for greater volatility of profits, they may lead to either particularly large profits or financial distress, both possibly prompting renegotiation. They thus argue that a rate-of-return regime may increase the credibility of commitment. We seek to develop this argument by extending the model of the previous section to consider the two regimes.

To compare a price-cap to a cost-plus regime we need to add to our model some uncertainty over future costs. Let us move to a situation where $R$ may either be $R_L$ or $R_H$, and define $\Delta R = R_H - R_L$. In other words, the total return to society may be high or low, implicitly depending on the costs involved. For analytical simplicity, we assume that $R$ follows a process whereby it is constant for a period of time $\tau + S$, and then is independent drawn anew at this time, with probability $\mu$ that $R = R_L$ and $1 - \mu$ that $R = R_H$. $S$ is a period of time greater than 0 but less than $T^*$, were $T^*$ is the non-investment period in equilibrium.

We can then represent the two different types of regulatory regime in the following way:

- **Cost-Plus** - As before, the contract offered to the firm consists of a single value $R$ which the firm will receive no matter the value of $R$ (so long as the firm invests and the government does not expropriate).

- **Price-Cap** - The regulator essentially fixes a price that the enterprise will be allowed to charge that is independent of $R$, and in doing so transfers all the variation in costs to the firm. In this case the firm will receive a return of $R_L$ when $R = R_L$ and $R_H = R_L + \Delta R$ when $R = R_H$. Hence $E(R)(T) = R_L(T) + \mu \Delta R$

Now, let us consider the cost-plus case. In place of $W_L(T, R)$ we have the following expression

$$W^{CP}_L(T, R_L) = \frac{1 - e^{-(r + \nu)(T + \tau)}}{(1 - \beta)(1 - e^{-(r + \nu)(T + \tau)})r + \beta(1 - e^{-(r + \tau)})(\nu + \tau)} \times \left[ (1 - e^{-r(S + \tau)})(R_L - \theta_L R(T)) + (e^{-r(S + \tau)} - e^{-r(T + \tau)})(E[R] - \theta_L R(T)) - (1 - e^{-r\tau}R_L) \right]$$

As shown in the proof of Proposition 5 this is the relative payoff of sharing rather than
expropriating for player G in equilibrium when \( \theta = \theta_L \) and \( \overline{R} = \overline{R}_L \). Similarly,

\[
W_{H}^{CP}(T, \overline{R}_H) = -1 - e^{-r(T+\tau)}(\beta \theta_H + (1 - \beta) \theta_L)R(T) + \frac{e^{-r(T+\tau)} - e^{-r(S+\tau)}}{r} \overline{R}_H
\]

\[
+ \frac{e^{-r(S+\tau)} - e^{-r(T+\tau)}}{r} \mathbb{E}[\overline{R}]
\]

\[
- \frac{1 - e^{-(r+\nu)(T+\tau)}}{r + \nu} (1 - \beta)(\theta_H - \theta_L)R(T)
\]

We also need to slightly tweak our definition of \( T^B \), since this will now be a function of \( \mathbb{E}[\overline{R}] \) rather than \( \overline{R} \).

\[
\frac{\theta_L}{\nu \beta + r} \frac{dR}{dT} (T^B) + \mathbb{E}[\overline{R}] - \theta_L R(T^B) = 0
\]

We can now give the equivalent of Proposition 4 for the cost-plus case:

**Proposition 5.** Suppose that there exists a value \( T^A_{CP} \) that solves the equation \( W_L^{CP}(T^A_{CP}, \overline{R}_L) = 0 \). Then, letting \( T^*_{CP} = \max\{T^A_{CP}, T^B_{CP}\} \),

- If \( W_H(T^*, \overline{R}_H) \leq 0 \), then the unique strongly renegotiation proof perfect Bayesian equilibrium is an episodic investment equilibrium with non-investment length \( T^* \).

- If \( W_H(T^*, \overline{R}_H) \geq 0 \), then the unique strongly renegotiation proof perfect Bayesian equilibrium is that with no investment.

If there is no value of \( T^A_{CP} \) that solves the equation \( W_L(T^A_{CP}, \overline{R}_L) = 0 \), then the unique strongly renegotiation proof perfect Bayesian equilibrium is that with no investment.

In order to compare, let us now calculate the equivalent values under price-cap regulation.

In place of \( W_L(T, \overline{R}) \) we have the following two expressions:

\[
W_{L}^{PC}(T, \overline{R}_H) = (1 - e^{-r(S+\tau)})(\overline{R}_H - \theta_L R_H(T)) + (e^{-r(S+\tau)} - e^{-r(T+\tau)})\mathbb{E}[\overline{R}] - \theta_L \mathbb{E}[R(T)]
\]

\[
- (1 - e^{-r\tau})\overline{R}_H
\]
and

\[ W^PC_L(T, R_L) = (1 - e^{-r(S+\tau)})(R_L - \theta_L R_L(T)) + (e^{-r(S+\tau)} - e^{-r(T+\tau)})(E[R] - \theta_L E[R(T)]) \\
- (1 - e^{-r\tau})R_L \]

Again, we show in Proposition 6 that this is the relative payoff of sharing rather than expropriating for player G in equilibrium when \( \theta = \theta_L \). We require the two functions here, since it is ambiguous under which value of \( R \) it is more tempting for player G to expropriate. This is also the case for when \( \theta = \theta_H \), and hence we also require the following two expressions:

\[
W^PC_H(T, R_H) = -\frac{1 - e^{-r(S+\tau)}}{r} (\beta \theta_H + (1 - \beta) \theta_L) R_H(T) \\
- \frac{e^{-r(S+\tau)} - e^{-r(T+\tau)}}{r} (\beta \theta_H + (1 - \beta) \theta_L) E[R(T)] \\
+ \frac{e^{-r\tau} - e^{-r(S+\tau)}}{r} R_H + \frac{e^{-r(S+\tau)} - e^{-r(T+\tau)}}{r} E[R] \\
- \frac{1 - e^{-(r+\nu)(S+\tau)}}{r + \nu} (1 - \beta)(\theta_H - \theta_L) R_H(T) \\
- \frac{e^{-(r+\nu)(S+\tau)} - e^{-(r+\nu)(T+\tau)}}{r + \nu} (1 - \beta)(\theta_H - \theta_L) E[R(T)]
\]

and

\[
W^PC_H(T, R_L) = -\frac{1 - e^{-r(S+\tau)}}{r} (\beta \theta_L + (1 - \beta) \theta_L) R_L(T) \\
- \frac{e^{-r(S+\tau)} - e^{-r(T+\tau)}}{r} (\beta \theta_H + (1 - \beta) \theta_L) E[R(T)] \\
+ \frac{e^{-r\tau} - e^{-r(S+\tau)}}{r} R_L + \frac{e^{-r(S+\tau)} - e^{-r(T+\tau)}}{r} E[R] \\
- \frac{1 - e^{-(r+\nu)(S+\tau)}}{r + \nu} (1 - \beta)(\theta_H - \theta_L) R_L(T) \\
- \frac{e^{-(r+\nu)(S+\tau)} - e^{-(r+\nu)(T+\tau)}}{r + \nu} (1 - \beta)(\theta_H - \theta_L) E[R(T)]
\]

Since the return given to the firm is now variable, we use an expression for \( E(R(T)) \) rather than \( R(T) \), i.e.

\[
(1 - \beta) \left( 1 - e^{-\nu(T+\tau)} \right) e^{-r\tau} E[R(T)] - \frac{1}{r + \nu \beta} = \frac{1 - e^{-r\tau}}{r}
\]

(11)
We also need to slightly tweak our definition of $T^B$, since this will now be a function of $E[R]$ rather than $R$.

\[
\frac{\theta_L}{\nu^\beta + r} \frac{dE[R]}{dT} (T^B) + E[R] - \theta_L E[R(T^B)] = 0 \tag{12}
\]

We can now state the equivalent proposition for the price-cap case:

**Proposition 6.** Suppose that there exists values $T^A_{PC,L}, T^A_{PC,H} > 0$ that solve the equations $W^P_L(T^A_{PC,L}, \overline{R}_L) = 0$ and $W^P_L(T^A_{PC,H}, \overline{R}_H) = 0$. Then, letting $T^*_{PC} = \max\{T^A_{PC,H}, T^A_{PC,L}, T^B_{PC}\}$,

- If $W_H(T^*_{PC}, \overline{R}_L) \leq 0$ and $W_H(T^*_{PC}, \overline{R}_H) \leq 0$, then the unique strongly renegotiation proof perfect Bayesian equilibrium is an episodic investment equilibrium with non-investment length $T^*$.

- If $W_H(T^*_{PC}, \overline{R}_L) \geq 0$ or $W_H(T^*_{PC}, \overline{R}_H) \geq 0$, then the unique strongly renegotiation proof perfect Bayesian equilibrium is that with no investment.

If there are no solutions to the equations $W^P_L(T^A_{PC,L}, \overline{R}_L) = 0$ and $W^P_L(T^A_{PC,H}, \overline{R}_H) = 0$, then the unique strongly renegotiation proof perfect Bayesian equilibrium is that with no investment.

In order to compare the price cap case with that of cost-plus, it is useful to define $\theta^*_L$ as follows:

\[
\theta^*_L = \frac{\overline{R}_H - \overline{R}_L}{R^P_H(T^*_{PC}) - R^P(T^*_{CP})} \frac{e^{-r(S + \tau)} - e^{-r(S + \tau)}}{1 - e^{-r(S + \tau)}} \tag{13}
\]

We can now formally compare the two in the following corollary.

Having characterised the unique strongly renegotiation proof equilibrium in both the case of a cost-plus regime and a price cap, we can now compare the two situations. This is done in the following corollary:

**Corollary 1.** Let $\theta^*_L$ be defined as in equation (13). Then,

- If $\theta_L \geq \theta^*_L$, the non-investment length $T^*$ in the unique SRP PBE under the cost-plus regime is greater than or equal to the non-investment length $T^*$ in the corresponding equilibrium under the price-cap regime.
If $\theta_L \leq \theta^*_L$, then the non-investment length $T^*$ under the cost-plus regime is less than or equal to the non-investment length $T^*$ under the price-cap regime.

This corollary essentially says that the choice of regulatory regime may make a difference to the length of non-investment period in the model. If the non-investment period is determined by the government waiting until it is optimal to offer a contract to the firm (i.e. $T^B > T^A$), then the choice of regime does not affect the length of non-investment. However, if the non-investment period is determined by the first time at which the firm feels comfortable that the government will not expropriate (i.e. $T^B < T^A$), then the choice of regime matters.

The corollary shows that which regime is best can go either way, depending on the value of $\theta_L$, the amount by which the government under-values profits compared to consumer surplus. If $\theta_L$ is low, and the government values profits fairly highly, then a price-cap regime is best for preventing expropriation. This is because a low $\theta_L$ implies the government is mainly concerned with the joint surplus produced by investment. The most tempting time for the government to expropriate therefore is when $R$ is low - i.e. costs are high. When the investment of the firm is bringing relatively less to the economy (i.e. the firm is inefficient), the cost of non-investment is lower, and so the temptation to expropriate is greater. A price-cap is therefore helpful, because it is precisely when the firm is inefficient that the return they make is lowest. Hence this dampens the temptation of the government to expropriate compared to a cost-plus regime when the amount going to the firm is constant.

However, if $\theta_L$ is high, and the government cares relatively little about profits, then it is in fact the profits of the firm that determine which cost-level is most tempting to renego under. Since under a price-cap profits may reach higher levels than at any time under a cost-plus regime (when $R$ is high), it is more difficult to prevent expropriation under a price-cap regime. Hence a high $\theta_L$ favours a cost-plus regime.$^{10}$

One final point to note is that we might expect $\theta_L$ to be higher in developing countries. This stems from two reasons. First, the population of the country is less likely to benefit

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$^{10}$One potential extension that could be made would be to make the choice of regime endogenous to the model, such that it was chosen by the government. In this case, the government would simply pick that regime which was optimal (i.e. that with the smallest non-investment period). There would be no possibility of signalling, since a government with high needs is indifferent between the two regimes, since they would not pay the firm anything under either.

23
from the firm’s profits than in a developed country since share ownership is lower and utility firms are often foreign owned. Second, if the government is concerned with equity, the gap between the majority of the population and those that receive the firm’s profits is likely to be greater, and hence less weight will be placed on a firm’s profits. This higher value of $\theta_L$ would then translate into the fact that expropriation is most tempting when profits are high, and therefore a cost-plus regime would result in greater investment.

3.2 Decentralisation

Decentralisation of regulation has been a lively debated issue in many developing countries, particularly in federal ones like Brazil (see, for example, Laffont and Pouyet (2004)). Devolving power to more local levels has been advocated more generally by institutions such as the World Bank (Bardhan, 2002) and some of the characteristics that have made it popular are very appropriate to improving regulation. Two key arguments given are that it makes government more responsive and that it means that better information is available to them. The first of these arguments is exemplified by the study of Faguet (2004), which finds that decentralisation in Bolivia did make local government more responsive to local needs, including extra investment in infrastructure (particularly water). Meanwhile, it is commonly assumed in the theoretical literature that local governments know more about the local environment than can be easily transmitted to the centre (Gilbert and Picard, 1996).

One issue that there has been very little consideration of is how decentralisation interacts with the commitment problem. This is despite the fact that the two issues just outlined, responsiveness and information, have strong implications for government opportunism. In particular, if a local government is responsive to the local population, then it is likely to be tempted to renege on a regulatory contract if it benefits that region. Meanwhile, if they are well informed about the profits the enterprise is making, it may be less risky to attempt to steal them. We now consider the implication local responsiveness may have for decentralisation by extending our model of commitment.

We look at the setting where a country has two regions (Region 1 and Region 2), with each region having an independent enterprise running its utility. We consider two setups, one where regulation takes place at the regional level, and one where a central government
regulates both enterprises. In each case, the enterprises’ objective functions are as before, and we assume the firms are identical. Similarly, in the case where there is a local regulator, we assume that it is under the control of the local government which operates exactly as the government did in the basic model. Let \( \theta \) be the needs of government \( i \) at time \( t \), where governments 1 and 2 represent regions 1 and 2 respectively and the central government is labelled as government 0. We assume that for each government \( \theta \) follows an independent Markov process as described above. The only parameter that varies between the two regions is the social return of the investment, \( R_i \).

It is clear that the case of local regulation is the same as in the basic model above. Hence \( W_H(T, R) \) and \( W_L(T, R) \) are defined as above in equations (4) and (7) and \( T_i^B \) is defined according to the equation

\[
\frac{\theta_L}{\nu \beta + r} \frac{dR(T_i^B)}{dT} + \mathcal{R}_i - \theta_L R_i^B = 0
\]

where \( R(T) \) is as defined above in equation (6). We then have the following proposition:

**Proposition 7.** Suppose that there exists a value \( T_i^A > 0 \) that solves the equation \( W_L(T_i^A, R_i) = 0 \). Then, letting \( T_i^* = \max\{T_i^A, T_i^B\} \),

- If \( W_H(T_i^*, R_i) \leq 0 \), then the unique strongly renegotiation proof perfect Bayesian equilibrium in region \( i \) under local regulation is an episodic investment equilibrium with non-investment length \( T_i^* \).
- If \( W_H(T_i^*, R_i) \geq 0 \), then the unique strongly renegotiation proof perfect Bayesian equilibrium in region \( i \) under local regulation is that with no investment.

If there is no solution to the equation \( W_L(T_i^A, R_i) = 0 \), then the unique strongly renegotiation proof perfect Bayesian equilibrium in region \( i \) under local regulation is that with no investment.

Now let us consider the case where regulation is centralised. Since both enterprises will act in the same way, we can treat them as just one enterprise. Hence the only difference will be that the social gain is now \( \mathcal{R}_1 + \mathcal{R}_2 \) and the transfer given to the enterprise(s) is \( 2R(T) \).
This time, $T_B^0$ will be defined by the equation

$$2\frac{\theta_L}{\nu\beta + r}\frac{dR}{dT}(T_B^0) + \overline{R_1} + \overline{R_2} - 2\theta_L R(T_B^0) = 0$$  \hspace{1cm} (15)$$

Hence we have the following proposition:

**Proposition 8.** Suppose that there exists a value $T_A^0 > 0$ that solves the equation $W_L(T_A^0, \frac{\overline{R_1} + \overline{R_2}}{2}) = 0$. Then, letting $T_0^* = \max\{T_A^0, T_B^0\}$,

- If $W_H(T_0^*, \frac{\overline{R_1} + \overline{R_2}}{2}) \leq 0$, then the unique strongly renegotiation proof perfect Bayesian equilibrium in region $i$ under local regulation is an episodic investment equilibrium with non-investment length $T_i^*$.

- If $W_H(T_0^*, \frac{\overline{R_1} + \overline{R_2}}{2}) \geq 0$, then the unique strongly renegotiation proof perfect Bayesian equilibrium in region $i$ under local regulation is that with no investment.

If there is no solution to the equation $W_L(T_A^0, \frac{\overline{R_1} + \overline{R_2}}{2}) = 0$, then the unique strongly renegotiation proof perfect Bayesian equilibrium in region $i$ under local regulation is that with no investment.

We see that in the proposition, $R$ has been replaced by $\overline{R_1} + \overline{R_2}$. This is because the governments actions now essentially depend on the average of the two social returns. We can now compare the local regulation case to the central regulation case in the following corollary

**Corollary 2.** When $\overline{R_1} > \overline{R_2}$, centralising regulation produces the following results:

1. The range of parameters for which the WRP PBE with investment exists is smaller in Region 1 but larger in Region 2.

2. If a WRP PBE with investment exists in all cases, then $T_2^* \geq T_0^* \geq T_1^*$.

If $R_1 < R_2$, the opposite of each of these statements apply.

Intuitively, centralising regulation is sharing the risk between the two regions. The temptation to renege in the region where future investment has a lower return is mitigated by the regulator not wishing to give up investment in the more profitable region. This result is similar to that of Bernheim and Whinston (1990) in their model of multi-market collusion.
They find that firms competing in multiple markets are just as able to sustain collusion as when they operating in a single market so long as the markets are symmetric, but that with certain asymmetries collusion may be more easily achieved. In our case, this is equivalent to the fact that constant investment in both regions is easier to achieve in the centralised case when returns are different. Collusion in Bernheim and Whinston’s model is equivalent to the agreements between the regulator and the enterprise that investment will take place and profits will be allowed.

The implications for (de)centralisation as a policy option are therefore mixed. Depending on the asymmetries between the regions, the risk pooling mechanism has ambiguous effects on investment. In some cases, centralising regulation can create investment in an area where previously the threat of expropriation was too high. This may explain why Gomez Ibanez (2003, p.132) finds that in North and South America “nationalization was generally slower or less likely where the responsibility for regulation was at the national or provincial level”. On the other hand, when it is difficult producing credible commitment anywhere, we have seen that the best option is to decentralise, at least to those regions where some commitment is possible. These are the regions where the social return on investment is greatest, and as such is consistent with the recommendation of Walker et al. (1999, p.77) that “the priority in the decentralisation program should be given to the cities where the problems are greatest and the potential for service improvement is highest”.

We can therefore see that the impact of decentralisation will depend on the context, and indeed if regions are fairly symmetric there may be no change in the commitment credibility at all. Spiller and Savedoff (1999, p.19) argue on the whole that decentralising itself is not a particularly effective tool in improving commitment, and instead one should focus on using it as a tool to fragment regulation. In this way, it would be interesting to extend the model to consider a hierarchy of regulation, where both central and local government play a role. By making regulation less dependent on the interests of a single executive, renegotiation may be harder. In Argentina, for example, the central government credibly committed to Aguas Argentina by devolving day-to-day regulation to the local level, but reserving the right to intervene in the case of dispute. Thus when the local regulator was taken over by a local government hostile to the privatised enterprise, they were unable to completely
renegotiate the contract since the central government overruled with a more profitable deal (Alcazar et al., 2002). On the other hand, such a scheme has not worked so well in the Indian electricity sector, where the federal government has had less success in exerting its authority on state regulators, perhaps due to the large information asymmetry (Rufin, 2003). Modeling this process explicitly would enable us to better understand how a hierarchical system may work and what can be done to improve its effectiveness.

Another extension to this model of decentralisation that would also be interesting to pursue would be to introduce yardstick competition as a means to lessen the central government’s informational disadvantage. Such an inclusion is likely to add another dimension to the role regional asymmetry places in decentralisation. A further aspect not considered here is that, when limited investment is available, localities may compete for capital by offering high powered incentives. This is modeled by Laffont and Pouyet (2004), who show that decentralisation can lead to more stability since competition between the regions leaves less leeway for expropriation, and it would be worthwhile to mix this effect with that of risk pooling. Finally, it is sometimes argued that local governments are more vulnerable to capture by a local elite (Bardhan, 2002), and incorporating such an idea may be necessary if the general model were extended to allow for capture by the enterprise.

4 Conclusion

Overall, the model offers a step forward in thinking about ways in which reputation can sustain time-inconsistent government policy. We have shown how the government may play an efficient strategy through fear of future non-investment in a way that is robust to two criticisms that often apply to typical ‘trigger-strategy’ reputation mechanisms: The punishment length being arbitrary and the possibility of renegotiation. Such an equilibrium is sustained by considering the uncertainty that a firm may have about the government’s precise payoffs.

Unlike previous models with reputation created through changing types, the article has focused on a situation with two long-lived players. This has the advantage of being the appropriate context to model renegotiations in utility regulation, an issue that is particularly significant in developing countries. This has allowed us to consider how different regula-
tory policy may influence a government’s ability to commit. In particular, we have shown that the choice of regime and amount of centralisation may well have an impact on commitment. Furthermore, we have seen how the impact on commitment is likely to be different in developing countries where governments’ commitment abilities are much weaker. We can therefore conclude that it is necessary to model explicitly the way a government commits to a time-inconsistent policy in order to understand the ways in which policy is likely to affect this commitment.

5 Appendix

Proof of equation 3. Let $P(s)$ be a $2 \times 2$ matrix whose elements $p_{ij}(s)$ are defined by the equation

$$p_{ij}(s) = \mathbb{P}(x_{t+s} = j|x_t = i)$$

where $i, j \in \{\theta_H, \theta_L\}$. For a finite state space, the Chapman-Kolmogorov equations then give:

$$P(s) = e^{sQ}$$

(16)

where $Q$ is the transmission matrix defined in equation 1. Furthermore, from the definition of exponential matrices, we have that if $Q$ can be diagonalised such that $D = M^{-1}QM$ then

$$P(t) = Me^{tD}M^{-1}$$

(17)

In particular, we have

$$D = \begin{pmatrix} 0 & 0 \\ 0 & -\nu \end{pmatrix}$$
and 

\[ M = \begin{pmatrix} 1 & -\nu \beta \\ 1 & -\nu (1 - \beta) \end{pmatrix} \]

Hence

\[ P(s) = \begin{pmatrix} 1 - \beta (1 - e^{\nu s}) & \beta (1 - e^{\nu s}) \\ (1 - \beta) (1 - e^{\nu s}) & 1 - (1 - \beta) (1 - e^{\nu s}) \end{pmatrix} \]

which is as required by equation 3.

Proof of Proposition 1. We first show that the condition \( W_H(T, R) \geq \) is sufficient by positing strategies for the two players and then showing that these indeed form a perfect Bayesian equilibrium.

Players’ strategies are dependent on the time \( s \) since the government last expropriated. Player F plays a strategy of investing if and only if \( s \geq T \) and \( R \geq e^{rT} \). We then posit that player G plays the strategy of sharing when \( s \geq T \) and expropriating when \( s < T \). As stated above, we check the equilibrium is subgame perfect by considering the payoff of deviating for a time \( \epsilon \) as \( \epsilon \to 0 \).

First let us consider player F’s strategy. Deviating when \( s \geq T \) and \( R \geq e^{rT} \) means not investing for a time \( \epsilon \), giving an increase in the expected discounted payoff of

\[ \int_0^\epsilon e^{-rt} dt - \int_T^{T+\epsilon} e^{-rt} R dt = \frac{1 - e^{-r\epsilon}}{r} - e^{-rT} R \frac{1 - e^{-r\epsilon}}{r} \]

Dividing by \( \epsilon \) and taking the limit as \( \epsilon \to 0 \) then gives that this expression is equal to \( 1 - e^{-rT} R \). Hence if \( R \geq e^{rT} \), the payoff of deviating is sub-optimal. Similarly, if \( R < e^{rT} \), then it is optimal to not invest and hence the firm has no incentive to deviate when \( s \geq T \).

If \( s < T \), then any investment will be expropriated and hence clearly the firm does not wish to deviate by investing.

Let us now consider player G’s promise of \( R \) to the firm. Clearly, if \( R < e^{rT} \), then player F will never invest. Hence it is a dominant strategy for the government to choose \( R = e^{rT} \),
since investment always generates a positive return for the government.

Now let us consider player G’s decision of whether or not to expropriate. First, we consider the case when \( \theta(s) = \theta_H \). Let us first show that equation (4) is indeed the relative payoff to player G of sharing rather than expropriating in equilibrium. Since at a time \( \tau + T \) in the future the payoff will be independent of the government’s action now, we need only consider the differences in payoffs over this time. Hence:

\[
W_H(T, \overline{R}) = \mathbb{E} \left[ \int_0^{T+\tau} e^{-rt} (\overline{R} - \theta(t)R) dt - \int_0^{\tau} e^{-rt}\overline{R}dt \right] \bigg| \theta(0) = \theta_H
\]

\[
= \int_0^{T+\tau} e^{-rt} \left( \mathbb{E} [\theta(t) | \theta(0) = \theta_H] R \right) dt - \int_0^{\tau} e^{-rt}\overline{R}dt
\]

\[
= \int_0^{T+\tau} e^{-rt} \left( \mathbb{E} [\theta(t) | \theta(0) = \theta_H] R \right) dt - \int_0^{\tau} e^{-rt}\overline{R}dt
\]

\[
= \int_0^{T+\tau} e^{-rt} \left( (1 - \beta (1 - e^{-rt}))\theta_L + e^{-rt}[\beta (1 - e^{-rt})]\theta_H \right) R dt
\]

\[
- \int_0^{\tau} e^{-rt}\overline{R}dt
\]

\[
= \frac{1 - e^{-r(T+\tau)}}{r} \left( \overline{R} - (\beta\theta_H + (1 - \beta)\theta_L)R \right)
\]

\[
- \frac{1 - e^{-(r+\nu)(T+\tau)}}{r + \nu} (1 - \beta)(\theta_H - \theta_L)R - \frac{1 - e^{-r\tau}}{r} \overline{R}
\]

Given this expression is greater than 0 (a condition of the proposition) it is then trivial that the government has no incentive to deviate when \( \theta(t) = \theta_H \). When \( \theta(t) = \theta_L \), we can similarly obtain the following expression for the payoff of sharing rather than expropriating:

\[
\frac{1 - e^{-r(T+\tau)}}{r} \left( \overline{R} - (\beta\theta_H + (1 - \beta)\theta_L)R \right) + \frac{1 - e^{-(r+\nu)(T+\tau)}}{r + \nu} \beta(\theta_H - \theta_L)R - \frac{1 - e^{-r\tau}}{r} \overline{R}
\]

Since this expression is clearly greater than \( W_H(T, \overline{R}) \), the government has no incentive to deviate when \( \theta(t) = \theta_H \). Therefore, overall, the government has no incentive to deviate, and the equilibrium is subgame perfect and thus (trivially) a perfect Bayesian equilibrium.

It only remains to show that the condition \( W_H(T, \overline{R}) \geq \) is necessary. If this were not to hold, then, as shown above, player G would rather expropriate when \( \theta(t) = \theta_H \) and \( s > T \). Hence they cannot be best-responding, and therefore an equilibrium where player F plays
such a strategy is not perfect Bayesian.

This therefore concludes our proof that condition $W_H(T, \overline{R}) \geq$ is both necessary and sufficient.

Proof of Proposition 2. We again first show that these conditions are sufficient by positing strategies for the two players. This time we also need to specify beliefs for player F.

Players’ strategies are as before dependent on the time $s$ since the government last expropriated. Furthermore, player G’s strategy is dependent on her type.

Player F plays a strategy of investing if and only if $s \geq T$ and $R = R(T)$, where $R(T)$ is the rate of return given by equation (6).

If player G gets the opportunity, and the state of the world is high, she will share when $s \geq T$ and $R = R(T)$, and expropriate otherwise. If player G gets the opportunity, and she is the low type, she will expropriate.

Since player G’s strategy is dependent on her type, we need to posit beliefs for player F. Since player F must update her beliefs according to Bayes’ rule when she sees actions that are in the equilibrium strategies, she will believe that the state of the world is high if player G shares and believe that it is low if player G expropriates. Furthermore, we assume she similarly updates her belief according to Bayes rule for any off-equilibrium investment. If player F does not invest, then player G will not have the opportunity to act. In this case, they will update their beliefs as is consistent with the Markov process - i.e. according to matrix (3).

To show that these strategies and beliefs constitute a perfect Bayesian equilibrium, we need to check that neither player can do better by deviating from their posited strategies given their beliefs. Again, we use the one-stage deviation principle stated in the previous proof.

First let us consider player F’s strategy. Since $R(T)$ is set by the government, it will set the firm’s participation constraint to 0. Deviating is therefore uninteresting since all actions produce a 0 payoff. We need only check therefore that the definition of $R(T)$ given by equation (6) is indeed that which satisfies the firm’s participation constraint exactly. The
payoff to the firm of investing until the next expropriation is

\[-\frac{1 - e^{-r\tau}}{r} + e^{-r\tau} p_{\theta_L \theta_H}(s + \tau) \mathbb{E} \left[ \int_0^{S^B} e^{-r(t)} (R - 1) dt \right] \]

where $p_{\theta_L \theta_H}(s)$ is defined by equation (2) and $S^B$ is the next time the state of the world switches from good to bad, as before. Substituting in the the definition of $p_{\theta_L \theta_H}(s)$ and the density function of $S^B$ then gives that her expected payoff is

\[-\frac{1 - e^{-r\tau}}{r} + (1 - \beta) \left( 1 - e^{-\nu(s+\tau)} \right) e^{-r\tau} \int_0^\infty \int_0^{S^H} (R - 1) e^{-rt} \nu \beta e^{-\nu S^B} dtdS^B \]

\[= -\frac{1 - e^{-r\tau}}{r} + (1 - \beta) \left( 1 - e^{-\nu(s+\tau)} \right) e^{-r\tau} \int_0^\infty \frac{1}{r} (R - 1) \left( 1 - e^{-\nu S^H} \right) \nu \beta e^{-\nu S^B} \]

This is, as we can see, set to zero if $R(T)$ is as defined by equation (6).

Now let us consider player G. In order to do this, it is helpful to define $W(\theta)$ to be the expected discounted payoff of player G at time $s$ when $\theta(s) = \theta$ and players follow their equilibrium strategies. Then we have:

\[W(\theta_L) = \int_0^\infty \left[ \int_0^{S^H} e^{-rt} (R - \theta_L R) dt + e^{-rS^H} W(\theta_H) \right] \nu \beta e^{-\nu S^H} dS^H \]

\[= \int_0^\infty \left[ e^{-\nu S^H} - e^{-(r+\nu\beta)S^H} (R - \theta_L R) + e^{-(r+\nu\beta)S^H} W(\theta_H) \right] \nu \beta dS^H \]

\[= \frac{1}{\nu \beta + r} \left[ (R - \theta_L R) + \nu \beta W(\theta_H) \right] \]

(18)

Where $S^H$ is the next time at which the state of the world becomes high. For $W(\theta_H)$, we have

\[W(\theta_H) = \frac{1 - e^{-r\tau}}{r} \]

\[+ e^{-r(T+\tau)} \left[ W(\theta_H) + (1 - \beta)(1 - e^{-\nu(T+\tau)})(W(\theta_L) - W(\theta_H)) \right] \]

(19)
Rearranging gives

\[
W(\theta_H)(1 - e^{-r(T+\tau)}) = \frac{1 - e^{-r\tau}}{r} \mathcal{R} + (1 - \beta)(e^{-r(T+\tau)} - e^{-(\nu+r)(T+\tau)})(W(\theta_L) - W(\theta_H))
\]

From equation (18) we then have

\[
rW(\theta_H) = \mathcal{R} - \theta_L R - (r + \nu \beta)(W(\theta_L) - W(\theta_H))
\]  

(21)

substituting this into (19) and rearranging gives

\[
W(\theta_L) - W(\theta_H) = \frac{(1 - e^{-r(T+\tau)})(\mathcal{R} - \theta_L R) - (1 - e^{-r\tau})\mathcal{R}}{(1 - \beta)(1 - e^{-(\nu+r)(T+\tau)})r + \beta(1 - e^{-r(T+\tau)})(\nu + r)}
\]  

(22)

If \(\theta(t) = \theta_H\), deviating when \(s \geq T\) would mean delaying expropriation for time \(\epsilon\). The relative payoff of doing so is

\[
E \left[ \int_0^\epsilon e^{-rt} (\mathcal{R} - \theta(t) R) \, dt + e^{-r\epsilon} W(\theta(\epsilon)) \bigg| \theta(0) = \theta_H \right] - W(\theta_H)
\]

Expanding this out and taking the limit as \(\epsilon \to 0\) gives

\[
\lim_{\epsilon \to 0} \frac{1}{\epsilon} \left[ \int_0^\epsilon e^{-rt} (\mathcal{R} - \theta_H R) \, dt + e^{-r\epsilon} (1 - \beta)(1 - e^{-(\nu+r)})(W(\theta_L) - W(\theta_H)) - (1 - e^{-r\epsilon})W(\theta_H) \right]
\]

\[
= \mathcal{R} - \theta_H R + \nu(1 - \beta)W(\theta_L) - W(\theta_H) - rW(\theta_H)
\]

Substituting in equation (21) then tells us this is equal to

\[
-(\theta_H - \theta_L)R + (r + \nu)(W(\theta_L) - W(\theta_H))
\]

Now, substituting in our expression for \(W(\theta_L) - W(\theta_H)\) from equation (22) gives us that the condition for the government to not wish to deviate when \(\theta = \theta_H\) is

\[
\frac{(1 - e^{-r(T+\tau)})(\mathcal{R} - \theta_L R) - (1 - e^{-r\tau})\mathcal{R}}{(1 - \beta)(1 - e^{-(\nu+r)(T+\tau)})r + \beta(1 - e^{-r(T+\tau)})(\nu + r)} \geq \frac{\theta_H - \theta_L}{r + \nu} \mathcal{R}
\]
On closer expectation, we can see that this is a rearrangement of the expression $W_H(T, R) \leq 0$.

If $s \geq T$ and $\theta(t) = \theta_L$, then the temptation to deviate is as before: the player will receive a higher instantaneous payoff, but will set $s$ to 0. To check that the temptation to deviate is not sufficiently great, we simply need to confirm that $W_L(T, R)$ is indeed defined according to equation (7). We can show this as follows:

$$W_L(T, R) = W(\theta_L) - \frac{1 + e^{-rT}}{r} R - e^{-r(T+\tau)} \left[ W(\theta_L) - \beta(1 - e^{-\nu(T+\tau)})(W(\theta_L) - W(\theta_H)) \right]$$

$$= W(\theta_L)(1 - e^{-r(T+\tau)}) - \frac{1 - e^{-rT}}{r} R + \beta(e^{-r(T+\tau)} - e^{-(r+\nu)(T+\tau)})(W(\theta_L) - W(\theta_H))$$

$$= (W(\theta_L) - W(\theta_H))(1 - e^{-r(T+\tau)}) + e^{-r(T+\tau)} - e^{-(r+\nu)(T+\tau)}(W(\theta_L) - W(\theta_H))$$

$$= (W(\theta_L) - W(\theta_H))(1 - e^{-(r+\nu)(T+\tau)})$$

$$= \frac{1 - e^{-(r+\nu)(T+\tau)}}{(1 - \beta)(1 - e^{-\nu(T+\tau)})(R - \theta_L R) - (1 - e^{-rT}) R}$$

If $s < T$ or $R \neq R(T)$, deviating means sharing any investment rather than expropriating it. As before, this is clearly not optimal because expropriating brings a higher flow payoff than sharing, and their action does not affect future payoffs at all.

Finally, we need to consider whether the government has an incentive to deviate in their choice of R. Given player F’s equilibrium strategy, any other choice of $R$ will result in them not investing. Hence, since there is no reason for the government to postpone investment, they will always offer $R = R(T)$.

This therefore concludes our proof that the equilibrium is perfect Bayesian. We now need to show that each condition is necessary for the episodic-investment equilibrium to be perfect Bayesian.

First, suppose that $T$ does not meet condition (8). In this case, player G will wish to expropriate when the state of the world is high, and therefore the equilibrium is not perfect Bayesian.
Second, suppose that $T$ does not meet condition (9). In this case, player G will wish to share when it is the low type, and therefore the equilibrium is not perfect Bayesian.

Each condition is therefore necessary, and hence the combination of these conditions is both necessary and sufficient for the existence of a perfect Bayesian episodic-investment equilibrium with non-investment length $T$.

\[ \square \]

Proof of Proposition 3. Let us first show that there exists a weakly renegotiation proof perfect Bayesian episodic investment equilibria with non-investment length $T$ for all $T$ that satisfy the conditions in the proposition.

If the conditions hold, then Proposition 2 tells us that there exists a perfect Bayesian episodic-investment equilibrium with non-investment length $T^*$. We then simply need to show that there is such an equilibrium that is weakly renegotiation proof. We therefore need to check that the two players would not at any point like to agree to change $s$ - the last time that the government expropriated.

We consider an equilibrium where player G’s strategy is to offer player F a contract with rate of return $R(s)$ if $s \geq T$ and offer no contract if $s < T$.

First, let us consider whether either player would wish to renegotiate from a state $s$ to a state $s'$ when $s \geq T$. If $s' \geq T$, then this simply represents another state where investment is happening at a rate of return $R(T)$, and hence there is no reason to move to it. If $s' < T$, then this will result in postponing investment, and hence will be worse for the government. Hence both players do not have the incentive to renegotiate when $s \geq T$. If $s < T$, then clearly player F will not wish to renegotiate to a state $s' > s$ since this would bring forward investment, resulting in the rate of return $R$ not being sufficient to compensate the firm for the risk it was taking. Furthermore, player F would not wish to renegotiate to a state $s' < s$ since this would again simply postpone investment. Therefore the posited equilibrium is weakly renegotiation proof.

We now proceed to show that the equilibria described in the proposition are the only weakly renegotiation proof equilibria that involve some investment.

Since we are considering an equilibrium where there is some investment, and player F is behaving optimally given their beliefs, they must believe, at some point, the probability
of the government expropriating is less than or equal to the value that gives an expected payoff of zero. Since the equilibrium is perfect Bayesian, this belief must be based on the equilibrium strategy of player G. Hence player G’s equilibrium strategy involves sharing for at least one type for some $R$ that it will offer. Since this is player G’s equilibrium strategy, it must be a best response. However, sharing is strictly dominated as a strategy in the stage game, and hence there must be an incentive to share based upon the equilibrium strategy for future plays of the game. Therefore, in equilibrium, player G must receive an expected lower future payoff after they have expropriated, as compared to if they had shared. In particular, this lower future payoff must come about because player F will at some point not invest when they would have had there not been expropriation.

Let us first consider the case where for some state player F invests and both types of player G share (as in the full-investment equilibria). In this case, there is clearly an incentive for both player F and player G to renegotiate when they are in the ‘no-investment’ phase. In particular, they would both like to change to a state where there is investment and sharing, since here they both receive strictly expected payoffs. There are therefore no WRP perfect Bayesian equilibria that involve both types sharing for some state. In particular, no full-investment equilibria are WRP.

Let us now the case where player G’s actions are dependent on her type. Since the non-investment resulting from expropriation harms the government when the world is high as least as much as when it is low, if the government wishes to expropriate when the state of the world is high it will wish to do so when the world is low also. Hence there cannot be a situation where the low type shares and the high type expropriates. The only possibility therefore for the investment state is one where the low type expropriates and the high type shares (as in the episodic-investment equilibria). In this case, player F’s beliefs of player G’s type will be updated in the way described in the proof of Proposition 2.

We have therefore established that any WRP PBE consists of the high type sharing and the low type expropriating following investment. Without loss of generality, let us assume that player F invests at a time $s$ after the world was known to be low. If there is no expropriation between time $s$ and $s' > s$, then in any WRP equilibria there must be investment throughout the period $[s, s']$, since otherwise both parties would wish to renegotiate to state
s. Hence player F’s strategy must consist of investing if and only if the government has not expropriated within a length of time $T$, for some $T$. In other words, any WRP PBE must be an episodic-investment equilibria as defined above.

It now only remains to show that episodic-investment equilibria with non-investment length $T$ outside the range given in the proposition are either not weakly renegotiation proof or not perfect Bayesian. Proposition 2 tells us that any equilibrium with non-investment length $T < T^A$ is not perfect Bayesian, so we need only be concerned with those with $T^B > T \geq T^A$. In such an equilibrium, the government is offering a return of $R(T)$ to the firm at time $T$ after it last expropriated. Instead, it could deviate an instant $\epsilon$ by not offering anything until $T + \epsilon$, whereupon it could offer a lower rate rate of return $R(T + \epsilon)$. Supposing that this was accepted by the firm (i.e. player F invests), the payoff from not deviating in such a way would be

$$W(T) - e^{-r\epsilon}(e^{-\beta\nu \epsilon}W(T + \epsilon) + (1 - e^{-\beta\nu \epsilon})W(\theta_H))$$

where $W(T)$ is the payoff at state $T$ when there has been investment and the state of the world is high, and $W(\theta_H)$ is the payoff when there has been investment and the state of the world is low (which is independent of the time since expropriation). From equation (18) we have that

$$W(T) = \frac{1}{\nu \beta + r} \left[ (\overline{R} - \theta_L R(T)) + \nu \beta W(\theta_H) \right]$$

Hence expression (23) can be written as

$$W(T) - W(T + \epsilon) + W(T + \epsilon)(1 - e^{-(r+\beta \nu)\epsilon}) - e^{-r\epsilon}(1 - e^{-\beta \nu \epsilon})W(\theta_H)$$

since we are only concerned with the sign of this expression, we can divide it by $\epsilon$ and then take the limit as $\epsilon \to 0$. Doing this gives us

$$-W'(T) + W(T)(r + \beta \nu) - \beta \nu W(\theta_H)$$
and then after substituting in for \( W(T) \) from (24) we have

\[
= \frac{\theta_L}{\nu \beta + r} \frac{dR}{dT}(T) + \bar{R} - \theta_L R(T)
\]

Differentiating \( R(T) \) gives

\[
\nu(1 - \beta) \left( e^{-\nu(T+\tau)} \right) e^{-r\tau} R(T) - \frac{1}{r + \nu \beta} (1 - \beta) \left( 1 - e^{-\nu(T+\tau)} \right) e^{-r\tau} \frac{1}{r + \nu \beta} \frac{dR}{dT} = 0
\]

Hence

\[
\frac{\theta_L}{\nu \beta + r} \frac{dR}{dT} = -\frac{\theta_L}{(1 - \beta) \left( 1 - e^{-\nu(T+\tau)} \right)^2} \frac{1 - e^{-r\tau}}{r e^{-r\tau}}
\]

which is clearly increasing in \( T \). Since \(-R(T)\) is also increasing in \( T \) and \( T < T^B \), the expression is negative. Hence the payoff from not deviating is negative, and player G would prefer to postpone offering a rate of return to the firm. If the firm would accept an offer of \( R(T + \epsilon) \) at \( T + \epsilon \), the equilibrium is therefore not perfect Bayesian. However, if the firm would not accept such an offer, the equilibrium is not weakly renegotiation proof, since since both players would prefer to renegotiate to the state \( T \) at \( T + \epsilon \). Hence there are no WRP perfect Bayesian equilibria with \( T < T^B \). This concludes our proof.

\(\square\)

Proof of Proposition 4. Suppose that the condition is met. Consider a WRP PBE shown to exist in Proposition 3 with non-investment length \( T > T^* \). At time \( T^* \) after expropriation, player G would prefer for investment to take place at \( T^* \) than \( T \) since \( T^* > T^B \), which is the optimal value of \( T \) for player G. Moreover, player F receives an expected discounted payoff of 0 in either case, and therefore the two states are Pareto ranked, i.e. renegotiation will take place and we move to the equilibrium with non-investment length \( T^* \). This is also trivially better than any WRP equilibrium with no investment, and hence the episodic investment equilibrium with non-investment length \( T^* \) is the unique strongly renegotiation proof perfect Bayesian equilibrium.

If \( W_R(T, \bar{R}) \geq 0 \), then from Proposition 2 there are no WRP perfect Bayesian equi-
libria with investment, and hence the equilibrium with no investment is trivially strongly renegotiation proof.

Proof of Proposition 5. The proof is clearly by and large identical to that of Proposition 4, so here we simply present a sketch proof of the differences. Let us consider each equation in turn to examine how it does or does not differ from those in Proposition 4.

We now have $W_{CP}^L(T^A, \bar{R}_L) = 0$ in place of $W_L(T^A, \bar{R}) = 0$. In Proposition 4, this equation represented the value of $T$ at which the government is indifferent between expropriating and not doing so. In the case of uncertain costs, there will clearly now be two such equations - i.e. in equilibrium we require that the government does not expropriate in the high state of the world neither when costs are high nor when they are low. We therefore need to calculate the incentive to deviate in both scenarios.

Let $W_{CP}^H(T, \bar{R}_L)$ be the relative payoff from sharing rather than deviating when $\theta = \theta_H$ and $\bar{R} = \bar{R}_L$. By modifying the expression for $W_H(T, \bar{R})$ above appropriately, we have:

$$
W_{CP}^H(T, \bar{R}_L) = -\frac{1 - e^{-r(T+\tau)}}{r}(\beta \theta_H + (1 - \beta)\theta_L)R(T) + \frac{e^{-r\tau} - e^{-r(S+\tau)}}{r}\bar{R}_L
$$

$$
+ \frac{e^{-r(S+\tau)} - e^{-r(T+\tau)}}{r}E[\bar{R}] - 1 - \frac{e^{-(r+\nu)(T+\tau)}}{r + \nu}(1 - \beta)(\theta_H - \theta_L)R(T)
$$

Similarly, for $W_{CP}^H(T, \bar{R}_H)$ we have:

$$
W_{CP}^H(T, \bar{R}_H) = -\frac{1 - e^{-r(T+\tau)}}{r}(\beta \theta_H + (1 - \beta)\theta_L)R(T) + \frac{e^{-r\tau} - e^{-r(S+\tau)}}{r}\bar{R}_H
$$

$$
+ \frac{e^{-r(S+\tau)} - e^{-r(T+\tau)}}{r}E[\bar{R}] - 1 - \frac{e^{-(r+\nu)(T+\tau)}}{r + \nu}(1 - \beta)(\theta_H - \theta_L)R(T)
$$

In order to examine which of these two equations will form the binding constraint, we calculate the difference, i.e.

$$
W_{CP}^H(T, \bar{R}_L) - W_{CP}^H(T, \bar{R}_H) = \frac{e^{-r\tau} - e^{-r(S+\tau)}}{r}(\bar{R}_H - \bar{R}_L)
$$

Clearly this expression is always negative. It is less tempting to share when returns are low, because losing the investment is less costly. So the binding state will be when $\bar{R} = \bar{R}_H$, since we need to have the government wish to expropriate when needs are high for both values of
Now let us consider the expressions $W^{CP}_L(T, R_L)$ and $W^{CP}_L(T, R_H)$. Again, modifying the expression for $W_L(T, \overline{R})$ above appropriately we have

$$W^{CP}_L(T, R_H) = \frac{1 - e^{-(r+\nu)(T+\tau)}}{(1 - \beta)(1 - e^{-(r+\nu)(T+\tau)})r + \beta(1 - e^{-r(T+\tau)})(\nu + \tau)} \cdot \left[ (1 - e^{-r(S+\tau)})(R_H - \theta_L R(T)) + (e^{-r(S+\tau)} - e^{-r(T+\tau)})(E[R] - \theta_L R(T)) \right] - (1 - e^{-rT})$$

and

$$W^{CP}_L(T, R_L) = \frac{1 - e^{-(r+\nu)(T+\tau)}}{(1 - \beta)(1 - e^{-(r+\nu)(T+\tau)})r + \beta(1 - e^{-r(T+\tau)})(\nu + \tau)} \cdot \left[ (1 - e^{-r(S+\tau)})(R_L - \theta_L R(T)) + (e^{-r(S+\tau)} - e^{-r(T+\tau)})(E[R] - \theta_L R(T)) \right] - (1 - e^{-rT})$$

Again, we calculate the difference:

$$W^{CP}_L(T, R_L) - W^{CP}_L(T, R_H) = \frac{1 - e^{-(r+\nu)(T+\tau)}}{(1 - \beta)(1 - e^{-(r+\nu)(T+\tau)})r + \beta(1 - e^{-r(T+\tau)})(\nu + \tau)} \cdot \left[ -(1 - e^{-rT})(R_H - R_L) \right]$$

So, again, it is less tempting to share when returns are low, because losing the investment is less costly. So the binding condition will be the case case when $\overline{R} = \overline{R}_L$, since we require that the low-needs government shares in both cases.
Proof of Proposition 6. We proceed as in the proof above. In this case, we have:

\[
W_H^{PC}(T, \overline{R}_H) = -\frac{1 - e^{-r(S+\tau)}}{r}(\beta \theta_H + (1 - \beta)\theta_L)R_H(T) \\
+ \frac{e^{-r(S+\tau)} - e^{-r(T+\tau)}}{r}(\beta \theta_H + (1 - \beta)\theta_L)\mathbb{E}[R(T)] \\
+ \frac{e^{-r\tau} - e^{-r(S+\tau)}}{r}R_H + \frac{e^{-r(S+\tau)} - e^{-r(T+\tau)}}{r}\mathbb{E}[R]
\]

Subtracting one from the other gives us:

\[
W_H^{PC}(T, \overline{R}_H) - W_H^{PC}(T, \overline{R}_L) = -\frac{1 - e^{-r(S+\tau)}}{r}(\beta \theta_H + (1 - \beta)\theta_L)(R_H(T) - R_L(T)) \\
+ \frac{e^{-r\tau} - e^{-r(S+\tau)}}{r}(\overline{R}_H - \overline{R}_L) \\
+ \frac{1 - e^{-(r+\nu)(S+\tau)}}{r + \nu}(1 - \beta)(\theta_H - \theta_L)(R_H(T) - R_L(T))
\]

This we can see is ambiguously signed, and hence either constraint might be the binding one.
in equilibrium.

Let us now consider the cases for when $\theta = \theta_L$:

\[
W^{PC}_L(T, \bar{R}_H) = 1 - e^{-(r + \nu)(T + \tau)} \cdot (1 - \beta)(1 - e^{-(r + \nu)(T + \tau)})r + \beta(1 - e^{-r(T + \tau)})(\nu + r) \\
\triangleq \left[ (1 - e^{-r(S + \tau)})(\bar{R}_H - \theta_L R_H(T)) + e^{-r(S + \tau)} - e^{-r(T + \tau)}(E[R] - \theta_L E[R(T)]) - (1 - e^{-r\tau})\bar{R}_H \right]
\]

and

\[
W^{PC}_L(T, \bar{R}_L) = 1 - e^{-(r + \nu)(T + \tau)} \cdot (1 - \beta)(1 - e^{-(r + \nu)(T + \tau)})r + \beta(1 - e^{-r(T + \tau)})(\nu + r) \\
\triangleq \left[ (1 - e^{-r(S + \tau)})(\bar{R}_L - \theta_L R_L(T)) + e^{-r(S + \tau)} - e^{-r(T + \tau)}(E[R] - \theta_L E[R(T)]) - (1 - e^{-r\tau})\bar{R}_L \right]
\]

Subtracting one from the other gives

\[
W^{PC}_L(T, \bar{R}_H) - W^{PC}_L(T, \bar{R}_L) = \frac{1 - e^{-(r + \nu)(T + \tau)}}{(1 - \beta)(1 - e^{-(r + \nu)(T + \tau)})r + \beta(1 - e^{-r(T + \tau)})(\nu + r)} \\
\triangleq \Delta \bar{R} \left[ (1 - e^{-r(S + \tau)})(1 - \theta_L) - (1 - e^{-r\tau}) \right] (26)
\]

Again, the sign of this expression is ambiguous, and hence either of the terms above might form the binding expression.

\begin{proof}
Comparing equations (6) and (11), it is clear that $\hat{R}(T)$ in the cost-plus case is equal to $E[R(T)]$ in the price-cap case. Hence, comparing (10) and (12), we can see that $T^B$, the optimal value of $T$ as far as the government is concerned, will be the same, i.e. $T^{BP}_C = T^{BP}_P$. It is therefore only necessary to compare the values $T^A$ in the two cases, the value at which the low-need government is indifferent between expropriating and sharing.

In the cost-plus case, we have shown that it is when $\bar{R} = \bar{R}_L$ that expropriation is most tempting, and therefore this forms the binding constraint. However, in the price cap case which value of $\bar{R}$ forms the binding constraint is ambiguous, and hence we need to compare $W^{CP}_L(T, \bar{R}_L)$ with both $W^{PC}_L(T, \bar{R}_L)$ and $W^{PC}_L(T, \bar{R}_H)$. Beginning with $W^{PC}_L(T, \bar{R}_L)$, we

\end{proof}
\[
W^\text{CP}_L(T, \bar{R}_L) - W^\text{PC}_L(T, \bar{R}_L) = \frac{1 - e^{-(r+\nu)(T+\tau)}}{(1 - \beta)(1 - e^{-(r+\nu)(T+\tau)})r + \beta(1 - e^{-r(T+\tau)})(\nu + r)} \\
\quad \times (1 - e^{-r(S+\tau)})\theta_L(R(T) - R_L(T))
\]

This is clearly negative. Hence, if the low state is the binding state under the price-cap regime, then a price-cap does better because it gives lower return to firm in this case. Now let us consider the case where the high state is the binding state for price cap:

\[
W^\text{CP}_L(T, \bar{R}_L) - W^\text{PC}_L(T, \bar{R}_H) = \frac{1 - e^{-(r+\nu)(T+\tau)}}{(1 - \beta)(1 - e^{-(r+\nu)(T+\tau)})r + \beta(1 - e^{-r(T+\tau)})(\nu + r)} \\
\quad \times \left[(1 - e^{-r(S+\tau)})\theta_L(R_H(T) - R(T)) - (e^{-r\tau} - e^{-r(S+\tau)})(\bar{R}_H - \bar{R}_L)\right] \\
= \frac{1 - e^{-(r+\nu)(T+\tau)}}{(1 - \beta)(1 - e^{-(r+\nu)(T+\tau)})r + \beta(1 - e^{-r(T+\tau)})(\nu + r)} \\
\quad \times [\theta_L - \theta^*_L]
\]

Where \(\theta^*_L\) is as defined by equation (13). Hence, if \(\theta_L\) large, this is positive, and the cost-plus regime is superior. It now only remains to show that, if \(\theta_L \geq \theta^*_L\), the binding state of the world under the price cap regime is when \(\bar{R} = \bar{R}_H\). From equation (26) we can see that the term which determines the binding state is \((1 - e^{-r(S+\tau)})(1 - \theta_L) - (1 - e^{r\tau})\). Now, if \(\theta_L \geq \theta^*_L\), we have

\[
\theta_L \geq \frac{e^{-r\tau} - e^{-r(S+\tau)}}{1 - e^{-r(S+\tau)}} \frac{\bar{R}_H - \bar{R}_L}{R_H(T) - R(T)}
\]

Rearranging therefore tells us that

\[
1 - \theta_L \geq 1 - \frac{e^{-r\tau} - e^{-r(S+\tau)}}{1 - e^{-r(S+\tau)}} \frac{\bar{R}_H - \bar{R}_L}{R_H(T) - R(T)}
\]
Hence

\[
(1 - e^{-r(S+\tau)})(1 - \theta_L) \leq (1 - e^{-r(S+\tau)}) - (e^{-r\tau} - e^{-r(S+\tau)}) \frac{R_H - R_L}{R_H(T) - R(T)} < (1 - e^{-r(S+\tau)} - (e^{-r\tau} - e^{-r(S+\tau)}) = 1 - (e^{-r\tau})
\]

Which therefore gives us that the expression \((1 - e^{-r(S+\tau)})(1 - \theta_L) - (1 - e^{-r\tau})\) will be negative and hence it is the case that \(R = R_H\) will be the binding condition. This thus concludes our proof.

\[\square\]

Proof of Proposition 2. Since \(R_1 > \frac{R_1 + R_2}{2} > R_2\), it is clear that also \(W_L(T, R_1) > W_L(T, R_2)\). Hence if there exists a value of \(T\) satisfying \(W_L(T, R_2) = 0\), then clearly there is also a value of \(T\) satisfying \(W_L(T, \frac{R_1 + R_2}{2})\), and hence the range of parameters for which such a value of \(T\) exists increases. A similar logic applies to show \(T^*_2 \geq T^*_0 \geq T^*_1\).

\[\square\]

References


