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ALLOTMENT IN FIRST-PRICE AUCTIONS: AN EXPERIMENTAL INVESTIGATION

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Allotment in First-Price Auctions: An Experimental Investigation*

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Abstract

We experimentally study the effects of allotment — the division of an item into several units — in independent private value auctions. In particular, we compare a single-item, first-price auction with two equivalent treatments with allotment: a two-unit discriminatory auction and a setting in which subjects participate in two identical and simultaneous first-price auctions, each involving a single unit. We find that allotment mitigates overbidding, with this effect being more pronounced in the discriminatory auction. In the allotment treatments, most bidders submit different bids for identical units (bid spreading). Across treatments, the discriminatory auction is the least efficient.

JEL classification: H57, D44

Keywords: Allotment, multi-unit auction, discriminatory auction, first price auction, laboratory experiment.

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1 Introduction

In settings where an auctioneer wants to allocate a certain quantity of a divisible good to a number of bidders, a fundamental question is whether to sell the entire quantity as a single bundle or to allot it into several distinct units.

Answering this question is not easy, as there are arguments both for of and against allotment. In favor of allotment, it has been argued this practice can promote competition (and thereby enhance efficiency) by facilitating the participation of bidders with limited budget or capacity.[1] Furthermore, allotment is advisable when some bidders are interested in a few units only: these bidders are, *de facto,* excluded from the competition when the entire quantity is sold as a single bundle. Finally, in auctions involving risky items, such as financial assets (national bonds, shares of companies, derivatives), allotment allows pursuit of differentiation of idiosyncratic risks across investors.

However, while in principle allotment can be welfare improving, in fact it is not necessarily so: bidders may strategically respond to the presence of multiple units, leading in some cases to losses in efficiency and revenue. As the literature on multi-unit and multi-object auctions has shown, this strategic behavior may take a variety of forms, which depend on elements like the degree of substitutability or complementarity across units, or asymmetries across bidders. But is this the whole story? That is, beyond the factors above, does allotment *per se* matter in auction formats? Although the theory based on the standard assumption of risk neutrality suggests a negative answer, there is no empirical contribution yet that assesses the effects of allotment on bidding behavior and efficiency when all these confounding factors are controlled for.

The present paper aims to fill this gap by reporting results from an intuitive experimental design with three treatments. The first treatment consists of a standard single-unit, first-price, independent private value auction and represents our benchmark. In the other two treatments, we introduce allotment by mimicking two natural procedures adopted in the real world. In the second treatment, bidders compete in a two-unit discriminatory auction: each bidder places two

[1] The recent "Green Paper on the modernization of EU public procurement policy" points out that allotment can encourage small and medium enterprises (SMEs) to participate in public procurement auctions, whereby SMEs are considered of crucial importance for stimulating job creation, economic growth and innovation (European Union, 2011, p.29). In a similar vein, in designing auctions to allocate CO_2 emission allowances, allotment is recommended to avoid limits in participation (see European Union, 2010, p.23; see also the NYSERDA Report, 2007, p.35).

bids (so four bids are collected), the two highest bids are deemed winning and the bidder(s) who placed the winning bids is (are) assigned the units and pay the amount of his (their) winning bid(s). Concrete examples of this type of allotment setting include auctions involving financial assets, tradeable (emission) permits, electricity quotas, and raw materials. Finally, in the third treatment, bidders participate in two simultaneous first-price auctions, each involving a single unit. For instance, on eBay and other online platforms, public and private operators frequently administer simultaneous auctions for identical products.

Several aspects of our experimental design are intended to disentangle the pure effect of allotment from other confounding factors. First, all treatments are based on the same "pay-your-bid" principle. Second, in the two treatments with allotment, the two units have the same value and their sum (essentially) has the same distribution of that in the benchmark. Third, apart from how allotment is implemented, all other experimental features are kept constant across treatments. These aspects imply that all treatments are equivalent in terms of maximum attainable efficiency. Moreover, under the traditional assumption of risk neutrality, the predicted bids in the three formats should be the same.

Our experimental results can be summarised as follows. First, we observe overbidding — bidding above the risk neutral equilibrium — in all the treatments, with bids being more aggressive in the benchmark than in the two allotment treatments. Second, we detect bid spreading, i.e. different bids for the two units in the allotment formats. Third, in all our treatments, particularly when the allotment is introduced, a considerable number of units are inefficiently assigned to the subjects with the lowest valuation. Fourth, while no significant differences in bidders' surplus are detected across treatments, we find that the auctioneer's revenue is lower in the discriminatory auction than in the other two treatments.

Overall, the previous findings call into question the validity of the standard assumption of risk neutrality to account for bidders' behavior. In line with recent advancements in behavioral economics, we propose risk aversion and "joy of winning" as possible interpretations of our experimental evidence. We show that these behavioral hypotheses successfully account for bid spreading, as well as for the observed differences across treatments.

The rest of the paper is organised as follows. Section 2 briefly reviews the relevant literature. Section 3 presents the experimental design and the theoretical predictions under the standard assumption of risk neutral bidders. Section 4 reports the results of our experiment.

Section 5 discusses risk aversion and joy of winning as potential explanations for our puzzling results. Finally, Section 6 concludes with concrete applications and policy implications of our experimental findings.

2 Literature review

Although to date, the theoretical and experimental literature on auctions is vast, a thorough understanding of the behavioral effects of allotment is still missing. That selling many objects together rather than separately can generate relevant effects has been clear since the work of Palfrey (1983). The author theoretically shows that, with two bidders and under a second-price rule, allocating different goods as a single bundle is revenue superior (but inferior in terms of total welfare) to selling them through parallel auctions.[2]

Recently, Popkowski Leszczyc and Haubl (2010) empirically investigate into the research question advanced by Palfrey (1983). They report results from a field experiment based on a second-price rule: they show that when there is complementarity between goods, a bundle auction generates higher revenue than two parallel auctions for the components of the bundle; however, the opposite occurs when the goods are substitute. There are three main differences between these contributions and our paper. First, we do not introduce any complementarity/substitutability between goods; rather, we assume that the two goods are identical and that each bidder's valuation is the same for both units. This allows us to study the effects that allotment exerts on bidders' behavior. Second, these papers do not consider multi-unit auctions in which bidders place multiple bids, one for each item involved. Finally, despite its practical relevance, these two papers do not consider the "pay your bid" pricing rule.

Most of the interest of economists in selling mechanisms involving multiple goods has concentrated on multi-unit auctions, investigating the properties of properties of the standard formats (uniform-price, discriminatory and Vickrey and their open outcry counterparts).[3] In particular, the theory on discriminatory auctions (see Engelbrecht-Wiggans and Kahn, 1998) has shown that, when the units have decreasing marginal valuations, (i) bidders shade all bids and (ii) the difference between two bids placed by a bidder tends to be smaller than the difference between the corresponding valuations.

[2] The work of Palfrey has been subsequently extended by Chakraborty (1999).

[3] For a detailed survey, see Kagel and Levin (1995 and 2011).

From the experimental point of view, however, economists have mainly focused on uniform-price and Vickrey auctions, while less attention has been devoted to “pay-your-bid” formats. Indeed, to the best of our knowledge, only Engelmann and Grimm (2009) have experimentally analyzed results from discriminatory auctions. In their paper, the authors study bidders' behavior and efficiency in five different multi-unit auction formats: discriminatory, uniform-price sealed-bid, uniform-price open, Vickrey and Ausubel auctions. All the formats are characterized by independent private value and involve two symmetric bidders having flat demand for two homogeneous units. In their discriminatory auction, in sharp contrast to theoretical predictions under the (traditional) assumption of risk neutral bidders, the authors find a strong evidence of overbidding and bid spreading. In a companion paper (Grimm and Engelmann, 2005), they advance risk aversion and "joy of winning" as possible explanations for this puzzling evidence, finding empirical arguments that suggest the superiority of the latter behavioral artifact.[4]

We depart from the study by Engelmann and Grimm (2009) as we are interested in assessing the effects of allotment on bids and efficiency in alternative “pay-your-bid" auction formats. To this end, we compare results from the discriminatory auction with an equivalent (in value) single-unit first-price auction. Moreover, in order to check whether results depend on the specific form of allotment, we also consider an additional treatment in which bidders participate in two identical and simultaneous first-price auctions, each involving one unit.

Finally, our paper also relates to the literature on combinatorial auctions in which buyers can submit bids on either single units or a package. Combinatorial auctions are a useful allocation mechanism when units are characterized by the presence of synergies or if because of their budget capacity and dimension, there are bidders who are interested in subsets of goods only (Cramton et al., 2006). In these settings, if the items are allocated separately, while there is an incentive to place high bids to get the extra value associated with the synergy, a bidder may refrain from aggressive bidding in order to avoid exposure to losses in case she wins only a limited number of units. This phenomenon, called the *exposure problem*, may negatively affect the efficiency and revenue of the auction (Kagel and Levin, 2005; Katok and Roth, 2004). A combinatorial auction reduces the risk of exposure and allows for more efficient allocations, as recently shown by Chernomaz and Levin (2012).[5]

[4] Joy of winning is discussed in Section 5 below.

[5] In the Chernomaz and Levin (2012) setting, two local bidders (i.e., bidders interested in only one good)

We share with the literature on combinatorial auctions attention to the effects of introducing the possibility of allotting items. However, the two approaches cannot be directly compared as they present relevant differences in the strategy profiles they endorse. In combinatorial auctions, package bids compete with single-item bids in the same auction. In our study, instead, we compare results from an auction in which only package bids are allowed with those observed in two formats in which bidders can exclusively place separate bids.

3 Experimental design and predictions

In order to assess how allotment affects bids and efficiency, we compare results from a benchmark treatment in which bidders compete for a single item with those observed in two equivalent (in terms of experimental features and potential payoff) auction formats with multiple and identical units. For each treatment, we ran three sessions. Each session included 15 periods and involved 18 participants, for a total of 162 subjects.

3.1 Treatments

The benchmark treatment, $1A1U$, consists of a single-unit, first-price, independent private value auction with two bidders. Once pairs are formed, each bidder privately observes the value of an indivisible item. In each period, values are randomly and independently drawn from the same uniform distribution defined on the interval $[0, 200]$. All the characteristics of the distribution used to generate private values are common knowledge. After observing their values, bidders simultaneously place their bids. The bidder who places the highest bid wins the item and the earning is given by the difference between the assigned value and the winning bid. The bidder placing the lowest bid earns nothing.

In the second treatment, $1A2U$, in every period the two bidders compete in a two-unit discriminatory auction. To each bidder, the two units have the same value, which, in each period, is randomly and independently drawn from a uniform distribution defined on the interval $[0; 100]$. This implies that the distribution (in terms of mean and upper and lower bounds) of the sum of the private values in $1A2U$ coincides with that used in the benchmark, $1A1U$.

compete with one global bidder (i.e., a bidder interested in both goods and who can benefit from synergies) in two simultaneous (first-price) auctions and a combinatorial auction; their results highlight that if the value of the synergy is sufficiently high, the latter format can improve efficiency and revenue.

The rules of the auction are as follows: given her value, each bidder places two bids. Once the four bids are collected, the two units are assigned to the bidder(s) who placed the two highest bids. For any unit acquired, the bidder earns an amount given by the difference between her value and the corresponding winning bid. As in $1A1U$, if a bidder does not obtain any unit, she earns nothing.

In the third treatment, $2A1U$, allotment is introduced by letting the two bidders participate in two identical and simultaneous first-price auctions, each involving a single unit. Apart from this aspect, all the other experimental features are identical to those adopted in $1A2U$. Given her value, each bidder places two bids, one in each auction. In each of the two auctions, the highest bid wins the corresponding unit. Earnings are then computed as in $1A2U$.

3.2 Procedures

Upon their arrival, subjects were randomly assigned to a computer terminal. In all sessions, instructions were distributed at the beginning of the experiment and read aloud.[6] Before the experiment started, subjects were asked to answer a number of control questions to make sure that they understood the instructions as well as the consequences of their choices. When necessary, answers to the questions were privately checked and explained.

At the beginning of the experiment, the computer randomly formed three rematching groups of six subjects each. The composition of the rematching groups was kept constant throughout the session. At the beginning of every period, subjects were randomly and anonymously divided into pairs. Pairs were randomly formed in every period within rematching groups. Subjects were told that pairs were randomly formed in such a way that they would never interact with the same opponent in two consecutive periods.[7]

In every period, subjects were presented with two consecutive screens. On the first screen, subjects were informed about their private value and were required to place their bids (one in $1A1U$, two in $1A2U$ and $2A1U$). On the second screen, each subject was informed about the winning bid(s) as well as her payoff in the period. Payoff was expressed in points and accumulated over periods. Subjects started the experiment with a balance of 300 points to cover the possibility of losses. At the end of the experiment, the number of points obtained by

[6] Experimental instructions are included in Appendix C.

[7] The rematching protocol implemented in the experiment was intended to increase the number of independent observations per treatment.

a subject during the experiment was converted at an exchange rate of 1 euro per 100 points, and monetary earnings were paid in cash privately.

The experiment took place at the Experimental Laboratory of the University of Innsbruck in November 2011. Participants were mainly undergraduate students, recruited by using *ORSEE* (Greiner, 2004). The experiment was computerised using the z-Tree software (Fischbacher, 2007). On average, subjects earned 12.00 Euro in sessions lasting 45 minutes (including the time for instructions and payment). Before leaving the laboratory, subjects completed a short questionnaire containing questions on their socio-demographics and their perception of the experimental task.

3.3 Testable predictions: bids, spread and efficiency in the three treatments

In order to derive the testable predictions on bidding behaviour and efficiency in the three treatments, we develop a simple theoretical framework based on the assumption of risk neutral bidders. Although a systematic violation of risk neutrality has been extensively documented by the economic literature, it still constitutes the standard assumption in models studying the equilibrium properties of different auction formats. Moreover, as discussed in the next pages, the assumption of risk neutrality provides two very intuitive predictions in our experiment, namely equivalence (in terms of bids, total efficiency and auctioneer's revenue) across auction formats, and zero-spreading between the two bids placed by a bidder in the allotment treatments, $1A2U$ and $2A1U$.

In $1A1U$, the item auctioned off has a value V_i for bidder i. Values are private information, but it is commonly known that they are *i.i.d.* random variables with uniform distribution over the interval $[0, 200]$. After observing her own value, bidder i places her bid, a_i. The item is assigned to the bidder who places the highest bid; the winner has to pay her bid. Risk neutrality implies that bidder i's payoff is $V_i - a_i$ if she wins the auction, and zero otherwise.

In the allotment treatments, $1A2U$ and $2A1U$, the two bidders compete for two units: in $1A2U$ the two units are auctioned off in the same auction; in $2A1U$ bidders participate in two identical and simultaneous auctions, each involving a single unit. In both cases, each of the two units has value v_i for bidder i. Values (of a single unit) are *i.i.d.* random variables with uniform distribution over $[0, 100]$. Therefore, as in $1A1U$, the sum of the values of the two units, $2v_i$, is uniformly distributed over the interval $[0, 200]$.

In $1A2U$, bidder i places two bids, $b_{i,1}$ and $b_{i,2}$, with $b_{i,1} \geq b_{i,2}$ (i.e. $b_{i,1}$ and $b_{i,2}$ are the highest and lowest bids of i, respectively). Four bids are thus collected within a pair, the two highest bids win and are thus paid by the corresponding winner(s). Bidder i's payoff is $2v_i - b_{i,1} - b_{i,2}$ if she wins both units, $v_i - b_{i,1}$ if she wins only one unit, and zero otherwise.

In $2A1U$, bidder i places one bid, $c_{i,1}$, in the first auction and one bid, $c_{i,2}$, in the second. In each auction, the highest bid wins the unit and the corresponding bidder pays it. If bidder i wins both auctions, her payoff is thus $2v_i - c_{i,1} - c_{i,2}$; if she wins only the first (second) auction, her payoff is $v_i - c_{i,1}$ $(v_i - c_{i,2})$; if she wins neither auction, her payoff is null.

Under the assumption of risk neutral bidders, each of the three treatments admit a unique symmetric (Bayes-Nash) equilibrium in pure strategies. Given the equilibria in the three treatments, we can state the following testable predictions (proofs are in Appendix B):

$H1$ **Zero-spreading.** *In 1A2U and 2A1U, bidders place two identical bids for the two units.*

$H2$ **Bid equivalence.** *In all treatments, bids are equal to half the value assigned to that unit. Thus, for given private value, bids in 1A1U are equal to the sum of the two bids in 1A2U and 2A1U.*

$H3$ **Efficiency equivalence.**[8] *In all treatments, the units are allocated to the bidder with the highest private value. Thus, on average, the overall surplus is the same across treatments. Finally, in every treatment, the overall surplus is split equally between bidders and the auctioneer.*

4 Experimental Results

We organise our results as follows. First, we compare treatments with respect to the extent of overbidding by looking at both the (sum of the) bids and the proportion of subjects that bid above the risk neutral equilibrium level (henceforth, RN). Second, focusing on the two treatments with allotment, $1A2U$ and $2A1U$, we study bid spreading. Third, we investigate differences across treatments in overall efficiency, bidders' surplus and the auctioneer's revenue.

[8] Given that in the experiment, bidders' earnings are observable while their "true" surplus is not, throughout the paper we will adopt the convention common in this literature of measuring bidders' surplus with their monetary earnings (the difference between price paid and the valuation). The allocation will thus be considered efficient (i.e., it maximizes the sum of revenue and bidders' earnings) if and only if both units are allocated to the bidder with the highest valuation. Clearly, this definition of welfare is correct if bidders are risk-neutral, but it is not under the different specifications of their utility functions.

With the exception of the proportion tests, the non parametric tests discussed below are based on 27 independent observations (nine re-matching groups per treatment). Similarly, in order to account for potential dependence across periods, the estimated coefficients in the parametric regressions are based on standard errors clustered at the rematching group level.

4.1 (Over-)bidding behavior

Figure 1 shows the bids over all periods in the three treatments.

[*Figure 1* about here]

We observe overbidding in all our three treatments. In particular, the proportion of bids above the RN level is 89.14% in 1*A*1*U*, 79.38% in 1*A*2*U*, and 83.46% in 2*A*1*U*. The difference of overbids between 1*A*1*U* and each of the two treatments with allotment is highly significant according to a test of proportions (compared to 1*A*2*U*, $z = 5.389$, $p < 0.01$; compared to 2*A*1*U*, $z = 3.323$, $p < 0.01$). We also observe a significant difference in the proportions of overbids between 1*A*2*U* and 2*A*1*U* ($z = -2.108$, $p < 0.05$).

The following *Table 1* reports the Probit marginal effect estimates for the probability of overbidding in the three treatments.

[*Table 1* about here]

As shown in column (4), allotment reduces the probability of overbidding. Indeed, the coefficient of 1*A*2*U*&2*A*1*U* is negative and highly significant. Column (5) reveals that relative to 1*A*1*U*, the probability of overbidding in 1*A*2*U* is significantly lower. The treatment effect remains negative, although non significant (according to a two-sided test) in 2*A*1*U*. Finally, the difference between the coefficients of 1*A*2*U* and 2*A*1*U* is not significant ($\chi^2(1) = 1.57, p = 0.210$).

Moving to the magnitude of overbidding, *Table 2* reports results from parametric panel regressions to study the determinants of the (sum of the) bids in the three treatments.

[*Table 2* about here]

By looking at the first three columns of *Table 2*, the coefficient of $Value$ is significantly greater than 0.5 in 1*A*1*U* ($\chi^2(1) = 422.73$, $p < 0.01$), in 1*A*2*U* ($\chi^2(1) = 166.89$, $p < 0.01$)

and in $2A1U$ ($\chi^2(1) = 317.41$, $p < 0.01$).[9] As indicated by the coefficient of $1A2U\&2A1U$ in column (4), after controlling for the private value and the linear time trend, allotment significantly decreases the (sum of the) bids. Moreover, when we replace $1A2U\&2A1U$ with two distinct treatment dummies, we find that both $1A2U$ and $2A1U$ have the same sign, although (according to a two-sided test) it is significant in $1A2U$ only. The difference between the coefficients of $1A2U$ and $2A1U$ in column (5) is not significant ($\chi^2(1) = 0.38$, $p = 0.539$).[10] As a final observation, the (sum of the) bids significantly decrease(s) over periods as indicated by the coefficient of *Period*. We summarise the previous findings on overbidding as follows.

Result 1. *In all treatments, subjects bid above the RN level. Bids in* $1A1U$ *are greater than the sum of the bids in the allotment treatments,* $1A2U$ *and* $2A1U$*.*

4.2 Bid spreading and subjects' behavior in $1A2U$ and $2A1U$

In contrast to what predicted under risk neutrality, we find that 84.32% and 75.31% of the bidders' offers is in $1A2U$ and $2A1U$ respectively associated with bid spreading. *Figure 2* illustrates the bid spreading between bids per value in $1A2U$ and $2A1U$.

[*Figure 2* about here]

Over all periods, relative to the *RN* level, the bid spreading is 30.33% in $1A2U$ and 29.57% in $2A1U$. In both treatments, the relative spreading is significantly greater than 0 according to a (two-sided) Wilcoxon signed-rank test (in both treatments, $z = 2.667$, $p < 0.01$). Moreover, in 29.88% and 30.00% of offers made in $1A2U$ and $2A1U$, respectively, the relative spreading is greater than 40%. Finally, as highlighted by the previous figure, the size of the bid spreading is increasing in the assigned value.

In *Table 3*, we parametrically investigate the determinants of both the size and the probability of bid spreading in the two allotment treatments.

[*Table 3* about here]

[9] This result is confirmed by a (two-sided) Wilcoxon signed-rank test, which rejects the null hypothesis that the (the sum of the) bid(s) is equal to the *RN* level in the three treatments ($z = 2.666$, $p < 0.01$).

[10] In line with the previous results, a (two-sided) Mann–Whitney rank-sum test strongly rejects the null hypothesis that the mean of the bid(s) in $1A1U$ is the same of that in the treatments with allotment ($z = 2.160$, $p = 0.031$). Again, the negative effect of allotment on bids is stronger when $1A1U$ is compared to $1A2U$ ($z = 2.075$, $p = 0.038$), rather than when it is opposed to $2A1U$ ($z = 1.634$, $p = 0.102$ for the two-sided test). We do not find any significant difference between $1A2U$ and $2A1U$ ($z = -0.574$, $p = 0.566$).

There is a positive and highly significant correlation between the size of the bid spreading and the sum of the private values in the period. Interestingly, as shown by the coefficient of $Period$ in columns (1), (3) and (5), the size of the spreading either remains stable or increases over periods. This confirms that rather than converging to the level predicted under the assumption of risk neutrality, subjects persistently and intentionally choose to place different bids. As indicated by the coefficient of $2A1U$ in column (5), we do not observe significant differences in the size of the spreading between $1A2U$ and $2A1U$.[11] By looking at the probit marginal effect estimates in columns (2), (4) and (6), we find that the probability of bid spreading positively depends on the assigned value and increases over periods. Finally, column (6) shows that bid spreading is more likely to occur in $1A2U$ than in $2A1U$.

Given the results of bid spreading and overbidding, in *Table 4* we look at the size of both the highest and the lowest bids in the allotment treatments.

[*Table 4* about here]

Both the highest and the lowest bids observed in $1A2U$ and $2A1U$ are associated with overbidding. The coefficient of $Value$ in columns (1)-(4) is always significantly greater than 0.5 (for the highest bids: $\chi^2(1) = 372.00$, $p < 0.01$, in $1A2U$; $\chi^2(1) = 610.64$, $p < 0.01$, in $2A1U$; for the lowest bids: $\chi^2(1) = 30.39$, $p < 0.01$, in $1A2U$; $\chi^2(1) = 77.52$, $p < 0.01$, in $2A1U$). A (two-sided) Wilcoxon signed-rank test rejects the null hypothesis that the highest and lowest bid in both treatments are equal to the RN level (for the highest and lowest bids in $1A2U$ as well the highest bid in $2A1U$, $z = 2.667$, $p < 0.01$; for the lowest bid in $2A1U$, $z = 2.547$, $p = 0.011$). Moreover, by looking at columns (5) and (6), we do not find any significant difference between treatments in both the highest and the lowest bids.

Result 2. *In the allotment treatments,* $1A2U$ *and* $2A1U$*, we observe a persistent and large-in-size bid spreading. Both the size and the probability of bid spreading do not differ between* $1A2U$ *and* $2A1U$*. Finally, both the highest and lowest bids in* $1A2U$ *and* $2A1U$ *are associated with overbidding.*

By looking at *Table 1* and *Table 3*, we find that the difference in overbidding across auction formats is mainly caused by the lowest (rather the highest) bids in the allotment treatments.

[11] This result is confirmed by non parametric tests: a (two-sided) Mann–Whitney rank-sum test does not reject the null hypothesis that the spread in $1A2U$ is the same that in $2A1U$ ($z = 0.751$, $p = 453$).

Indeed, the coefficients attached to $Value$ in the two regressions based on the highest bids in $1A2U$ and $2A1U$ (columns (1) and (3) of *Table 4*) are similar to those observed in $1A1U$ (column (1) of *Table 2*). On the other hand, the coefficients of $Value$ in the regressions based on the lowest bids in $1A2U$ and $2A1U$ (columns (2) and (4) of *Table 4*) are substantially smaller than those observed in $1A1U$.[12]

The finding that subjects in $1A2U$ overbid with both their lowest and highest bids represents a remarkable difference from the results of Engelmann and Grimm (2009): in a discriminatory auction similar to our $1A2U$, they find that the lowest bid is, on average, below the *RN* level (i.e. underbidding). This discrepancy in results can be due to the different (re)matching protocol used in the two experiments. While the partner matching used in Engelmann and Grimm (2009) can induce subjects to collude over periods and thus to lower bids, the random rematching protocol implemented in our experiment makes collusion (virtually) impossible.

4.3 Efficiency in $1A1U$, $1A2U$ and $2A1U$

We now turn our attention to the level of efficiency achieved in the three treatments. Under the assumption of risk neutrality, the three treatments imply full allocative efficiency: the two units are assigned to the subject with the highest (sum of the) private value(s).

To investigate this issue, for each pair and in every period, we build three measures: (i) the *relative efficiency,* defined as the ratio between the achieved total welfare and the maximum possible welfare; (ii) the *relative auctioneer's revenue,* given by the ratio between the winning bid(s) and the maximum possible welfare; and (iii) the *relative bidders' surplus*, the ratio between the monetary payoff of the winning bidders and the maximum possible welfare. *Table 5* reports the estimates from GLS random effects models to compare relative efficiency, relative auctioneer's revenue, and relative bidders' surplus in the three treatments computed by averaging at the rematching group level.

[*Table 5* about here]

As shown by the first two columns of *Table 5*, relative efficiency is significantly higher in

[12] We run two additional panel regressions (with clustered standard errors). The dependent variable in the first (second) regression is equal to the unique bid in $1A1U$ and to the highest (lowest) bid in the two allotment treatments, $1A2U$ and $2A1U$. As controls, both regressions include the allotment dummy, $1A2U\&2A1U$, the value of one item and the time trend. The estimate of $1A2U\&2A1U$ is not significant in the first regression while it is negative and significant at the 1% level in the second regression.

1*A1U* than in the two allotment treatments. This result seems to be mainly driven by the more pronounced efficiency loss recorded in 1*A2U* relative to the other two treatments. By looking at column (2) of *Table 5*, we find a significant welfare loss in all the treatments. The relative efficiency in 1*A1U* (as measured by the constant term) is significantly smaller than 1 ($\chi^2(1) = 28.43$, $p < 0.01$). Similarly, the measures of relative efficiency in 1*A1U* and 1*A2U* (as expressed by the linear combination of the constant term with the estimate of the corresponding treatment dummy) are significantly lower than 1 (in 1*A2U*, $\chi^2(1) = 79.58$, $p < 0.01$; in 2*A1U*, $\chi^2(1) = 35.95$, $p < 0.01$). The loss of relative efficiency in all treatments is associated with allocative inefficiency: the percentage of winning bids placed by subjects with the lowest private value(s) is 10.67% in 1*A1U*, 29.85% in 1*A2U*, and 18.04% in 2*A1U*. Finally, the difference in the relative efficiency between 1*A2U* and 2*A1U* is highly significant ($\chi^2(1) = 8.19$, $p < 0.01$).[13]

Moving to the relative auctioneer's revenue and focusing on columns (3) and (4) of *Table 5*, we do not find significant differences between 1*A1U* and the two allotment treatments. However, when we replace the allotment dummy, 1*A2U*&2*A1U*, with two separate treatment dummies, we find that 1*A2U* is associated with the lowest relative auctioneer's revenue. Indeed, the coefficient of 1*A2U* in column (4) is negative and highly significant and the difference between the estimates of 1*A2U* and 2*A1U* is highly significant ($\chi^2(1) = 79.58$, $p < 0.01$).[14]

Focusing on columns (5) and (6) of *Table 4*, we find no significant differences in the relative bidders' surplus between treatments, either in the regressions (both the coefficients of 1*A2U* and 2*A1U* in column (6) as well as the estimate of 1*A2U*&2*A1U* are not significant; the difference between the coefficients of 1*A2U* and 2*A1U* in column (6) is not significant: $\chi^2(1) = 0.00$, $p = 0.992$), or by using non parametric tests (according to a two-sided Mann–Whitney rank-sum test, the difference in the relative bidders' surplus between 1*A1U* and the treatments with allotment is not significant, $z = -0.463$, $p = 0.643$; similarly, the difference between 1*A2U* and 2*A1U* is not significant, $z = 0.044, p = 0.965$).

Finally, because of overbidding, in all treatments the relative auctioneer's revenue is greater than the relative bidders' surplus. Indeed, for each treatment, a (two-sided) Wilcoxon signed-

[13] In line with these findings, a (two-sided) Mann–Whitney rank-sum test rejects the null hypothesis that the relative efficiency in 1*A1U* is the same as that in the two allotment treatments ($z = 2.057$, $p < 0.05$). Similarly, the test detects a significant difference in the relative efficiency between 1*A2U* and 2*A1U* ($z = -2.693$, $p < 0.01$) as well as between 1*A1U* and 1*A2U* ($z = 2.517$, $p < 0.05$). No significant differences are observed between 1*A1U* and 2*A1U* ($z = 1.015$, $p = 0.310$).

[14] A (two-sided) Mann–Whitney rank-sum test confirms the previous result as it rejects the null hypothesis that the relative auctioneer's revenues in 1*A1U* and 1*A2U* are equal ($z = 1.810$, $p = 0.070$).

rank test rejects the null hypothesis that the relative auctioneer's revenue is equal to the relative bidders' surplus ($z = 2.667$, $p < 0.01$).

Result 3. *In all treatments, we observe allocative inefficiency. Relative efficiency and relative auctioneer's revenue are higher in* $1A1U$ *and* $2A1U$ *than in* $1A2U$. *Treatments do not significantly differ in the relative bidders' surplus. Finally, because of overbidding, in all treatments the relative auctioneer's revenue is higher than the relative bidders' surplus.*

5 Discussion

Our findings question the validity of the assumption of risk neutrality to describe bidders' behavior at least for the following four considerations. First, in line with the existing experimental literature, we detect overbidding in all treatments. Second, allotment reduces overall bids, with this effect being more pronounced in the discriminatory auction, $1A2U$. Third, relative to the other two treatments, $1A2U$ is associated with the lowest allocative efficiency and the most pronounced reduction of auctioneer's revenue. Finally, in the allotment treatments, $1A2U$ and $2A1U$, we observe large and persistent bid spreading. In order to provide an interpretation of our experimental results, we develop in this section two simple theoretical frameworks based on the assumptions of risk aversion and joy of winning.

Risk aversion is certainly the most invoked explanation for overbidding and is the natural candidate for bid spreading in discriminatory auctions.[15] Under risk aversion, the utility function $u(\cdot)$ is strictly concave.[16] In $1A1U$, bidder i's payoff is $u(v_i - a_i)$ if she wins the auction and zero otherwise. In $1A2U$, bidder i's payoff is $u(2v_i - b_{i,1} - b_{i,2})$ if she wins both units, $u(v_i - b_{i,1})$ if she wins one unit only and zero otherwise. Finally, in $2A1U$, bidder i's payoff is $u(2v_i - c_{i,1} - c_{i,2})$ if she wins both auctions, $u(v_i - c_{i,1})$ if she wins only the first auction, $u(v_i - c_{i,2})$ if she wins only the second auction and zero otherwise.

As noted by Kagel (1995, p.525), "It is probably safe to say that risk aversion is one element, but far from the only element, generating bidding above the risk neutral Nash equilibrium."[17]

[15] Kagel and Levin (1993, 2011).

[16] We do not make any assumption on the particular form of the utility function. Hence, the results apply to any specific form of risk aversion. Moreover, they are also independent from the probability distribution of values.

[17] Kirchkamp et al. (2010), using a novel experimental design allowing to manipulate bidders' risk preferences through the number of income-relevant auctions, show that risk preferences explain overbidding by about 50%.

"Joy of winning" has been proposed as an alternative behavioral explanation of bidders' behavior in auction experiments.[18]

In $1A1U$, we introduce joy of winning by simply assuming that when bidder i wins the auction, she obtains an extra utility that is captured by a multiplicative parameter, w. In other words, the *perceived* value of the item conditional on winning the auction is wV_i, with $w > 1$. Therefore, bidder i's (perceived) payoff is $wV_i - a_i$ if she wins the auction, and zero otherwise. In $1A2U$ and $2A1U$, a bidder can win one or two units. We assume that, if a bidder wins both units, the second unit does not provide any additional source of utility.[19] Formally, in $1A2U$, bidder i's (perceived) payoff is $wv_i + v_i - b_{i,1} - b_{i,2}$ if she wins both units, $wv_i - b_{i,1}$ if she wins (only) one unit and zero otherwise. Similarly, in $2A1U$ bidder i's (perceived) payoff is $wv_i + v_i - c_{i,1} - c_{i,2}$ if she wins both auctions, $wv_i - c_{i,1}$ if she wins only the first auction, $wv_i - c_{i,2}$ if she wins only the second auction, and zero otherwise.

On the basis of the equilibrium analysis, the hypotheses of joy of winning and risk aversion produce the following alternative predictions that are in line with our experimental results (all the proofs are included in Appendix B).

$h0$ **Overbidding.** *In all treatments, bidders bid more than what is implied by risk neutrality. Both bids in the allotment treatments,* $1A2U$ and $2A1U$, *are associated with overbidding.*

Under joy of winning, bidders bid more aggressively because winning *per se* assigns a utility premium to the winner. Under risk aversion, bidders bid more aggressively as they are willing to give up some units of payoff in order to increase the probability of winning.

$h1$ **Bid spreading.** *In the allotment treatments,* $1A2U$ and $2A1U$, *bidders place two different bids for the two units.*

By the assumption of joy of winning, bidders perceive in the allotment treatments the first unit as more valuable than the second. Therefore, they bid more aggressively on the first rather

[18] Grimm and Engelmann (2005); Cox et al. (1988); Goeree et al. (2002); Roider and Schmitz (2012). Several other explanations of observed behavior in auctions have been proposed. Armantier and Treich (2009) consider misperception of probabilities. Furthermore, according to Engelbrecht-Wiggans (1989), Engelbrecht-Wiggans and Katok (2008, 2009), Filiz-Ozbay and Ozbay (2007), and Roider and Schmidt (2012), bidders overbid to prevent regret in case they do not win the auction. Notice that this missed-opportunity-to-win regret and joy of winning are strictly related. Finally, joy of winning has found increasing support from recent studies in neuroscience and psychology, according to which placing subjects into competitive environments can trigger a desire to win (see Delgado et al., 2008, and Malhotra, 2010, both of which adopt an auction setup).

[19] This assumption can be relaxed by attaching an extra utility also to the second item won; as long as this extra utility is lower than the one associated with the first item, the theoretical implications do not qualitatively change.

than on the second unit. Under risk aversion, bidders dislike risk: by bid spreading, a bidder increases the probability of winning (only) one unit. This is preferable to making two identical bids which would lead to a higher probability of either winning both units or none.

$h2$ **Difference in bids across treatments.** *Under joy of winning, bids in* $1A1U$ *are higher than the sum of the two bids in the allotment treatments,* $1A2U$ and $2A1U$*. In addition, the sum of the two bids in* $1A2U$ *coincides with that in* $2A1U$*.*

By assumption, in $1A2U$ and $2A1U$, the extra utility from winning is associated only with the first unit won. Therefore, in equilibrium, a bidder places one aggressive bid whose purpose is to obtain the (highly valued) first unit (which also incorporates joy of winning) and one moderate bid that is exclusively addressed to obtain the value of the second unit. Of course, bid spreading is not possible in $1A1U$ and bidders have a unique opportunity to obtain the value of the item as well as the extra utility from winning. Thus, the unique bid in $1A1U$ is greater than the sum of the two bids in the allotment treatments. A similar comparison is not possible under risk aversion as equilibrium bidding functions cannot be explicitly derived.

$h3$ **Difference in efficiency across treatments.** *In* $1A1U$*, the item is allocated to the bidder with the highest private value. In the allotment treatments,* $1A2U$ and $2A1U$*, allocative efficiency is not guaranteed. Moreover, under joy of winning, the auctioneer's revenue is higher in* $1A1U$ *than in the the allotment treatments.*

The difference in allocative efficiency between $1A1U$ and the allotment treatments is due to bid spreading. Indeed, in $1A2U$ and $2A1U$, it is possible that the highest bid placed by the bidder with the lowest private value exceeds the lowest bid of the bidder with the highest private value. Of course, this is not possible in $1A1U$ where the single item is always allocated to the bidder with the highest evaluation. The prediction that auctioneer's revenue is higher in $1A1U$ than in the two treatments with allotment, $1A2U$ and $2A1U$, is a direct consequence of $h2$. This holds under joy of winning, whereas no revenue ranking can be assessed under risk aversion.

It is worth noting that in our experiment we observe higher allocative efficiency in $2A1U$ than in $1A2U$. This difference is explained by the structure of the equilibria in the two treatments with allotment. As shown in Appendix B, in $1A2U$ there is a symmetric (Bayes-Nash) equilibrium in which each bidder places two different bids. Thus, in equilibrium, there is a positive probability that the lowest bid placed by the bidder with the highest value is defeated by the highest bid placed by the bidder with the lowest value. Differently, $2A1U$ admits two equilibria in pure strategies in which bidders must properly coordinate their bids in such a way that in each auction, the highest bid of one bidder competes with the lowest bid of the other bidder. Suppose that bidders have opposing expectations about which equilibrium will be played such that they mis-coordinate and place their highest bid in the first auction and their lowest bid in the second auction. In this case, the bidder with the highest private value wins both auctions and allocative efficiency is achieved. Given the absence of communication among bidders and the random re-matching protocol implemented in our experiment, allocative efficiency is (expected to be) higher in $2A1U$ than in $1A2U$.

As a final step, we investigate the empirical relevance of the assumptions of risk aversion and joy of winning by combining subjects' bids with the information from the post-experimental questionnaire. In particular, we draw from two questions: asking subjects to self-report their risk attitude and inquiring into their perception of the joy of winning during the experiment.[20]

Table 6 presents the proportions of subjects reported either being risk averse or enjoying winning for different levels of overbidding in the different treatments. In particular, for each subject, we first run a regression with the (sum of the) bid(s) as the dependent variable and the (sum of the) private value(s) and the constant term as controls. Given the coefficient of $Value$, each subject is classified according to the extent of overbidding. Then, for each of the four categories, *Table 6* shows the proportions of subjects who reported being risk averse or to enjoying winning.

[*Table 6* about here]

In *Table 6*, when we look at the pooled data, the proportions of subjects that reported either being risk averse or to enjoying winning increase with overbidding. Next, in *Table 7*, we re-run the panel regressions in *Table 2* by adding the two dummies from the questionnaire, RA (for risk aversion) and JoW (for joy of winning).

[20] The first question focuses on risk aversion. Subjects are asked to report on a 7-point scale (with 1 indicating risk aversion and 7 risk seeking) whether, in general terms, they are willing to take risks or try to avoid risks. The second question refers to the joy of winning. Subjects are asked whether they agree (on the basis of a 7-point scale, with 1 indicating strong disagreement and 7 strong agreement) with the statement that in a generic period of the experiment, winning (at least one of) the unit(s) was very important to them regardless of the corresponding monetary payoff.

[*Table 7* about here]

By looking at the regressions with pooled data (columns (4) and (5)), RA and JoW are both significant and have the expected sign. Robust evidence (although with different levels of significance) is found when we analyse each treatment separately (columns (1), (2) and (3)). Thus, subjects reporting to be either risk averse or to enjoy winning tend to place, on average, higher bids. Interestingly, relative to the results in *Table 2*, adding RA and JoW weakens the effect of allotment, as indicated by the lower significance of $1A2U\&2A1U$.

6 Conclusion

This paper reports results from a laboratory experiment designed to isolate the pure effect of allotment in auction settings. In particular, we compare results from a single-item, first-price, independent private value auction with those observed in two *equivalent* treatments with allotment: a two-unit discriminatory auction and a setting in which subjects participate in two identical and simultaneous first-price auctions, each involving a single unit. Apart from the allotment strategy, all remaining experimental features are kept constant across treatments.

In line with the existing experimental literature, we observe overbidding in all treatments. Surprisingly, allotment significantly mitigates overbidding, with this effect being more pronounced in the discriminatory auction. Furthermore, in both allotment treatments, we find a persistent tendency of subjects to place different bids for the two identical units auctioned. Bid spreading and differences in overbidding across treatments have important consequences on the efficiency of the auction formats. In particular, we detect a substantial loss of efficiency in the discriminatory auction relative to the other two treatments. Our findings reveal that the drop in efficiency in the discriminatory auction comes mostly at the cost of the auctioneer's revenue, while we do not observe any significant difference in bidders' surplus across treatments.

Our experimental results strongly reject the prediction of perfect equivalence of the three treatments as implied by the standard assumption of risk neutral bidders. Although discriminating between alternative explanations is beyond the scope of the present study, we have leaned on two of the most discussed and intriguing behavioral assumptions: risk aversion and joy of winning. Overall, the predictions derived under these assumptions successfully rationalise our experimental evidence.

With the usual caveats about generalizing results from laboratory experiments to complex real world situations, our study offers two important policy recommendations. First, in choosing the auction format to implement, the auctioneer should seriously take into account the effects that his choice exerts on bidders' behavior and, consequently, his revenue. As our experimental results reveal, allotment either reduces or leaves unchanged the auctioneer's revenue. Combined with the additional direct (administrative, organisational and managerial) costs generally associated with allotment, the previous consideration questions whether the allotment is an optimal revenue maximizing selling strategy. Second, in those cases in which allotment is required to pursue general and important social goals, our study highlights the superiority of multi-auction formats with respect to equivalent discriminatory auctions in enhancing efficiency.

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Appendix

A Figures and Tables

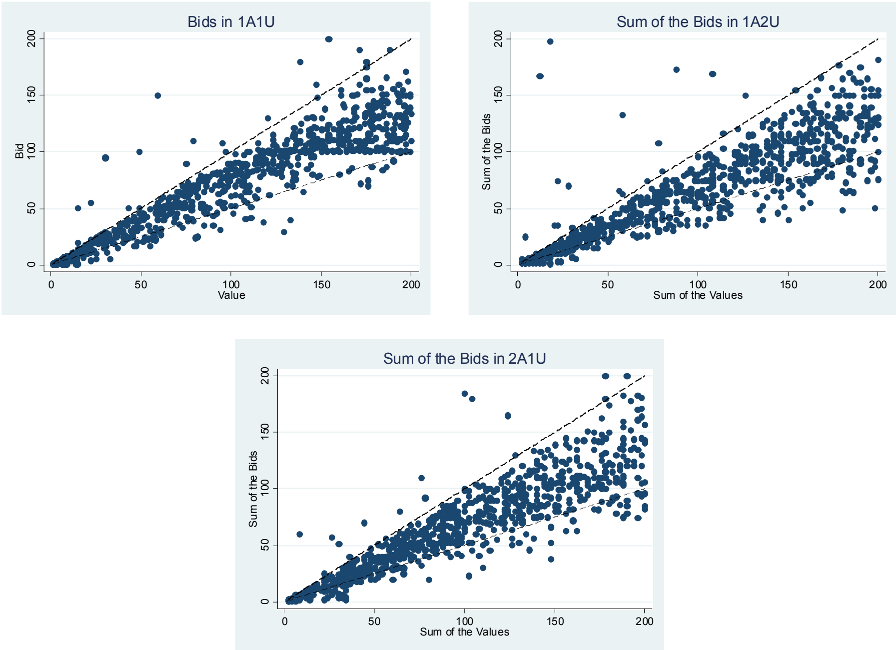


Fig. 1. (Sum of the) bids in 1A1U, 1A2U and 2A1U

Table 1. Probability of overbidding in $1A1U$, $1A2U$ and $2A1U$

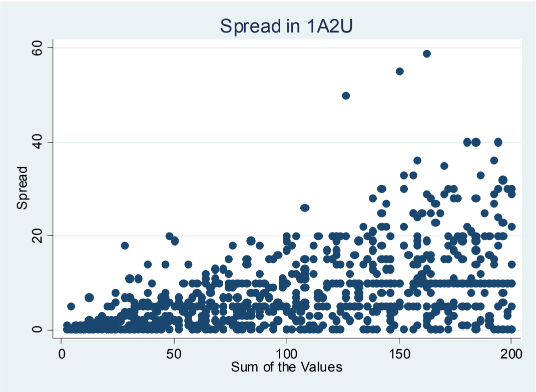
	$1A1U$	$1A2U$	$2A1U$	*Pooled*	
	(1)	(2)	(3)	(4)	(5)
$Value$	0.001^{***}	0.001	0.001^{***}	0.001^{***}	0.001^{***}
	$(1.8 * 10^{-4})$	(0.001)	$(1.8 * 10^{-4})$	$(2.2 * 10^{-4})$	$(2.2 * 10^{-4})$
$Period$	-0.001	$1.6 * 10^{-4}$	-0.003	-0.001	-0.001
	(0.003)	(0.004)	(0.003)	(0.002)	(0.002)
$1A2U\&2A1U$				-0.076^{***}	
				(0.028)	
$1A2U$					-0.109^{***}
					(0.032)
$2A1U$					-0.062
					(0.046)
lpl	-245.882	-407.405	-350.385	-1016.305	-1013.195
$Wald - \chi^2$	41.42	2.92	61.14	36.22	46.69
$p > \chi^2$	0.000	0.232	0.000	0.000	0.000
Obs.	810	810	810	2430	2430

Notes. This table reports Probit marginal effect estimates (robust standard errors clustered at the rematching group level in parentheses) over all periods. Columns (1)-(3) consider the three treatments, separately. Regressions in columns (4) and (5) are based on pooled data. The dependent variable is a dummy that takes a value of 1 if the (sum of the) bid(s) of the subject in the period is associated with over-bidding. $Value$ is the (sum of the) private value(s) assigned to the subject in the period. $Period$ is a linear time trend. $1A2U$ and $2A1U$ are treatment dummies. $1A2U\&2A1U$ is a dummy that takes a value of 1 if the treatment is either $1A2U$ or $2A1U$. Estimates remain unchanged when $Period$ is excluded from the regressions. Significance levels are denoted as follows: * p<0.1, ** p<0.05, *** p<0.01.

Table 2. (Sum of the) Bids in $1A1U$, $1A2U$ and $2A1U$

	$1A1U$	$1A2U$	$2A1U$	*Pooled*	
	(1)	(2)	(3)	(4)	(5)
$Value$	0.681^{***}	0.633^{***}	0.659^{***}	0.658^{***}	0.658^{***}
	(0.009)	(0.010)	(0.009)	(0.005)	(0.005)
$Period$	-0.494^{***}	-0.640^{***}	-0.800^{***}	-0.649^{***}	-0.649^{***}
	(0.117)	(0.135)	(0.111)	(0.070)	(0.070)
$1A2U\&2A1U$				-4.367^{***}	
				(2.120)	
$1A2U$					-5.218^{**}
					(2.478)
$2A1U$					-3.606
					(2.479)
$Constant$	8.182^{***}	9.136^{***}	9.224^{***}	11.793^{***}	11.790^{***}
	(2.294)	(2.372)	(2.221)	(1.902)	(1.923)
lrl	-3361.717	-3488.945	-3333.645	-10208.017	-10206.006
$Wald-\chi^2$	6014.09	3784.23	5430.88	14722.16	14721.77
$p>\chi^2$	0.000	0.000	0.000	0.000	0.000
Obs.	810	810	810	2430	2430

Notes. This table reports coefficient estimates (standard errors in parentheses) from two-way linear random effects models over all periods accounting for both potential individual dependency over repetitions and dependency within rematching group. The dependent variable is the (the sum of the) bid(s) placed by the subject in the period. The other remarks of Table 1 apply.



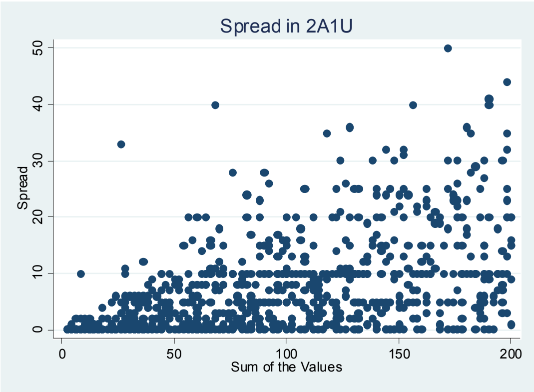


Fig. 2. Bid spreading in $1A2U$ and $2A1U$

Table 3. Bid spreading in $1A2U$ and $2A1U$

	$1A2U$		$2A1U$		*Pooled*	
	Size	*Prob.*	*Size*	*Prob.*	*Size*	*Prob.*
	(1)	(2)	(3)	(4)	(5)	(6)
Value	0.071***	0.002***	0.068***	0.003***	0.070***	0.002***
	(0.004)	$(2.6 * 10^{-4})$	(0.004)	$(3.5 * 10^{-4})$	(0.003)	$(2.1 * 10^{-4})$
Period	0.055***	0.007***	0.117***	0.008***	0.086***	0.007***
	(0.049)	(0.003)	(0.050)	(0.003)	(0.035)	(0.002)
$2A1U$					−0.101	−0.084**
					(0.794)	(0.036)
Constant	0.041		−0.333		−0.098	
	(0.749)		(0.874)		(0.676)	
$lrl(lpl)$	−2653.633	−314.564	−2678.307	−401.947	−5327.147	−716.958
$Wald - \chi^2$	367.03	27.59	294.99	109.54	659.62	96.19
$p > \chi^2$	0.000	0.000	0.000	0.000	0.000	0.000
Obs.	810	810	810	810	2430	2430

Notes. Columns (1), (3) and (5) report coefficient estimates (standard errors in parentheses) from two-way linear random effects models over all periods accounting for both potential individual dependency over repetitions and dependency within rematching group. The dependent variable in these regression is the absolute value of the difference of the two bids placed by the subject in the period. Columns (2), (4) and (5) report Probit marginal effect estimates (robust standard errors clustered at the rematching group level in parentheses) over all periods. The dependent variable is a dummy that takes a value of one if the subject places two different bids in the period. The other remarks of Table 1 apply.

Table 4. Highest and lowest bids in $1A2U$ and $2A1U$

	$1A2U$		$2A1U$		*Pooled*	
	Highest	*Lowest*	*Highest*	*Lowest*	*Highest*	*Lowest*
	(1)	(2)	(3)	(4)	(5)	(6)
Value (1 *unit*)	0.704***	0.562***	0.728***	0.592***	0.715***	0.576***
	(0.011)	(0.011)	(0.009)	(0.010)	(0.007)	(0.008)
Period	−0.292***	−0.348***	−0.342***	−0.459***	−0.317***	−0.403***
	(0.069)	(0.074)	(0.057)	(0.064)	(0.045)	(0.049)
$2A1U$					0.683	0.785
					(1.206)	(1.401)
Constant	4.596***	4.543***	4.451***	4.769***	4.202***	4.290***
	(1.163)	(1.320)	(1.027)	(1.332)	(0.988)	(1.131)
lrl	−2945.333	−3006.303	−2790.894	−2891.461	−5752.116	−5903.973
$Wald - \chi^2$	4444.91	2475.37	6229.13	3261.75	10314.65	5610.32
$p > \chi^2$	0.000	0.000	0.000	0.000	0.000	0.000
Obs.	810	810	810	810	2430	2430

Notes. This table reports report coefficient estimates (standard errors in parentheses) from both two-way linear random effects model over all periods accounting for both potential individual dependency over repetitions and dependency within rematching group. The dependent variable in columns (1), (3) and (5) is the highest bid of the subject in the period. The dependent variable in columns (2), (4) and (6) is the lowest bid of the subject in the period. *Value* (1 *unit*) refers to the private value assigned to one unit, only. All the other remarks of Table 1 apply.

Table 5. Relative efficiency in $1A2U$ and $2A1U$

	RE		RAR		RSB	
	(1)	(2)	(3)	(4)	(5)	(6)
$1A2U$&$2A1U$	-0.013^{***}		-0.038		0.015	
	(0.006)		(0.024)		(0.023)	
$1A2U$		-0.022^{***}		-0.055^{***}		0.015
		(0.006)		(0.027)		(0.027)
$2A1U$		-0.004		-0.021		0.015
		(0.006)		(0.027)		(0.027)
$Period$	0.002^{***}	0.002^{***}	-0.008^{***}	-0.008^{***}	-0.009^{***}	-0.009^{***}
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
$Constant$	0.967^{***}	0.967^{***}	0.798^{***}	0.798^{***}	0.171^{***}	0.171^{***}
	(0.007)	(0.007)	(0.021)	(0.021)	(0.021)	(0.021)
$Wald - \chi^2$	12.70	22.17	56.83	58.45	60.13	60.11
$p > \chi^2$	0.002	0.000	0.000	0.000	0.000	0.000
$Obs.$	405	405	405	405	405	405

Notes. This table reports report coefficient estimates from GLS random effects models (robust standard errors in parentheses clustered at the rematching group level). The dependent variable in columns (1) and (2), is the mean of the relative efficiency of the rematching group in the period. The dependent variable in columns (3) and (4), is the mean of the relative auctioner's unit of the goods of the rematching group in the period. The dependent variable in columns (5) and (6), is the mean of the relative bidders'surplus of the rematching group in the period. All the other remarks of Table 1 apply.

Table 6. Risk aversion and joy of winning in explaining overbidding

	$1A1U$			$1A2U$			$2A1U$			*Pooled*		
Overbidding	obs.	RA	JoW	obs.	RA	JoW	obs.	RA	JoW	obs.	RA	JoW
b(Value)≤0.5	0	0	0	11	0.182	0.364	7	0.286	0	18	0.222	0.222
0.5<b(Value)≤0.625	16	0.375	0.313	13	0.385	0.154	13	0.385	0.077	42	0.381	0.191
0.625<b(Value)≤0.75	29	0.379	0.276	18	0.444	0.278	19	0.526	0.211	66	0.439	0.258
0.75<b(Value)	9	0.777	0.222	12	0.5	0.333	15	0.333	0.267	36	0.5	0.278

Notes. This table shows the proportions of subjects that report either to be risk averse or to agree with the "joy of of winning" statement for different levels of over-bidding. RA is a dummy that takes a value of 1 if the subject reports to be risk averse (1, 2, 3 in the first question). JoW takes a value of 1 if the subject agrees with the "joy of of winning" statement (5, 6, 7 in the second question).

Table 7. Bids, risk aversion and joy of winning in $1A1U$, $1A2U$ and $2A1U$

	$1A1U$	$1A2U$	$2A1U$	*Pooled*	
	(1)	(2)	(3)	(4)	(5)
$Value$	0.681***	0.633***	0.660***	0.658***	0.658***
	(0.009)	(0.010)	(0.009)	(0.005)	(0.005)
RA	4.835**	6.217*	1.055	4.695**	4.664**
	(2.353)	(3.758)	(3.264)	(1.840)	(1.841)
JoW	2.467	4.530	10.847***	4.825**	5.015**
	(2.589)	(4.090)	(4.184)	(2.115)	(2.126)
$Period$	−0.493***	−0.640***	−0.800***	−0.649***	−0.649***
	(0.117)	(0.135)	(0.111)	(0.070)	(0.070)
$1A2U$&$2A1U$				−3.883*	
				(2.050)	
$1A2U$					−4.870**
					(2.382)
$2A1U$					−2.878
					(2.394)
$Constant$	5.363**	5.497*	7.014***	8.396***	8.352***
	(2.545)	(3.019)	(2.710)	(2.115)	(2.127)
lrl	−3355.838	−3482.530	−3325.905	−10199.444	−10197.308
$Wald - \chi^2$	6020.51	3788.57	5442.95	14739.69	14739.89
$p > \chi^2$	0.000	0.000	0.000	0.000	0.000
Obs.	810	810	810	2430	2430

Notes. This table reports coefficient estimates (standard errors in parentheses) from two-way linear random effects models over all periods accounting for both potential individual dependency over repetitions and dependency within rematching group. The other remarks of Tables 1 and 5 apply.

B Proofs of the predictions under risk neutrality, joy of winning and risk aversion

B.1 Risk neutrality

Under risk neutrality, the symmetric equilibrium of $1A1U$ is well known: each bidder bids according to $a^{RN}(V) = \frac{1}{2}V$.[21]

It is trivial to show that, in $1A2U$ and in $2A1U$, every bidder makes two identical bids according to the same equilibrium bidding function used in a single-unit first-price auction, namely:[22]

$$b_1^{RN}(v) = b_2^{RN}(v) = c_1^{RN}(v) = c_2^{RN}(v) = \frac{1}{2}v.$$

B.2 Joy of winning

B.2.1 Equilibrium bids in $1A2U$

Consider one bidder with value v drawn from a uniform distribution over the interval $[0, \bar{v}]$. Let us denote by b_1 and b_2, with $b_1 \geq b_2$, her highest and lowest bids, respectively. The expected payoff of this bidder is

$$\Pi(b_1, b_2; v) = (wv - b_1)F(b_2^{-1}(b_1)) + (v - b_2)F(b_1^{-1}(b_2))$$

where $b_1(\cdot)$ and $b_2(\cdot)$ are, respectively, the highest and the lowest bids of the opponent and $F(z) = z/\bar{v}$, for $z \in [0, \bar{v}]$.

If the optimal bids (b_1^*, b_2^*) are such that $b_1^* > b_2^*$ (we will verify this condition in the sequel), then they must satisfy the following first order (necessary) conditions:

$$\begin{cases} (wv - b_1^*)(b_2^{-1}(b_1^*))' = b_2^{-1}(b_1^*), \\ (v - b_2^*)(b_1^{-1}(b_2^*))' = b_1^{-1}(b_2^*). \end{cases} \tag{1}$$

In a symmetric equilibrium, it must be that $b_1^* = b_1(v)$ and $b_2^* = b_2(v)$. The first order

[21] For convenience, in the rest of this Appendix bidders' indexes are omitted, unless necessary.

[22] We do not provide a formal proof of this result. However, it can be easily derived from the model under joy of winning hypothesis, by letting w tend to 1.

conditions for a symmetric equilibrium $(b_1(v), b_2(v))$ are thus:

$$\begin{cases} (wv - b_1(v))(b_2^{-1}(b_1(v)))' = b_2^{-1}(b_1(v)), \\ (v - b_2(v))(b_1^{-1}(b_2(v)))' = b_1^{-1}(b_2(v)). \end{cases}$$

This system of differential equations, together with the boundary conditions[23] $b_1(0) = b_2(0) = 0$ and $b_1(\bar{v}) = b_2(\bar{v}) = \bar{b}$, defines a symmetric equilibrium $(b_1^{JoW}(v), b_2^{JoW}(v))$.[24] It is convenient to write the system above in terms of inverse bidding functions. To this end, let us define $\phi_1 = b_1^{-1}$ and $\phi_2 = b_2^{-1}$. The system of differential equations becomes:

$$\begin{cases} (w\phi_1(b) - b)\phi_2'(b) = \phi_2(b), \\ (\phi_2(b) - b)\phi_1'(b) = \phi_1(b). \end{cases}$$

Together with the boundary conditions $\phi_1(0) = \phi_2(0) = 0$ and $\phi_1(\bar{b}) = \phi_2(\bar{b}) = \bar{v}$, the previous system defines a symmetric equilibrium. Using standard techniques, one obtains the following solutions in terms of inverse bidding functions:

$$\phi_1^{JoW}(b) = \frac{2b}{w(1 - Cb^2)}, \quad \phi_2^{JoW}(b) = \frac{2b}{1 + Cb^2},$$

where $C = (w^2 - 1)/w^2\bar{v}^2$. Going back to the direct bidding functions, the solution is

$$b_1^{JoW}(v) = \frac{\sqrt{1 + Cw^2v^2} - 1}{Cwv}, \quad b_2^{JoW}(v) = \frac{1 - \sqrt{1 - Cv^2}}{Cv}.$$

That $b_1^{JoW}(v) > b_2^{JoW}(v)$, for all $v \in (0, \bar{v})$, will be shown below.

B.2.2 Equilibrium bids in $2A1U$

The expected payoff of a bidder with value v who bids c_1 in the first auction and c_2 in the second is:

$$\begin{aligned} \Pi(c_1, c_2; v) =& (wv - c_1)F(c_1^{-1}(c_1)) + (wv - c_2)F(c_2^{-1}(c_2)) \\ &- (w - 1)vF(\min(c_1^{-1}(c_1); c_2^{-1}(c_2))), \end{aligned}$$

[23] The boundary conditions follow from a simple weak dominance argument.

[24] Provided that, $b_1^{JoW}(v) > b_2^{JoW}(v)$, for all $v \in (0, \bar{v})$.

where $c_1(\cdot)$ and $c_2(\cdot)$ are her opponent's bidding function in the first and the second auction, respectively.

We first show that a symmetric equilibrium in which both bidders bid $c_1(v)$ in the first auction and $c_2(v)$ in the second auction does not exist. More generally, we show that for a bidder it is never optimal to place two bids c_1 and c_2 such that $c_1^{-1}(c_1) = c_2^{-1}(c_2)$. Now, let c_1^* and c_2^* be the optimal bids for a bidder and suppose that $c_1^{-1}(c_1^*) = c_2^{-1}(c_2^*)$. The expected payoff is not differentiable at (c_1^*, c_2^*). However, at that point, left and right partial derivatives are well defined. For (c_1^*, c_2^*) to be optimal, the left partial derivatives must be nondecreasing and the right partial derivatives must be non-increasing (otherwise, a small increase or decrease in bids would be profitable). This corresponds to require that

$$\begin{cases} (wv - c_1^*)(c_1^{-1}(c_1^*))' - c_1^{-1}(c_1^*) - (w-1)v(c_1^{-1}(c_1^*))' \geq 0, \\ (wv - c_1^*)(c_1^{-1}(c_1^*))' - c_1^{-1}(c_1^*) \leq 0, \end{cases}$$

which cannot be satisfied simultaneously since the left hand side of the first equation is strictly lower than the left hand side of the second.

As a consequence, the optimal bids (c_1^*, c_2^*) must be such that $c_1^{-1}(c_1^*) \neq c_2^{-1}(c_2^*)$. Suppose, without loss of generality, that $c_1^{-1}(c_1^*) > c_2^{-1}(c_2^*)$. The expected payoff is now differentiable at (c_1^*, c_2^*). Thus, the first order conditions are

$$\begin{cases} (wv - c_1^*)(c_1^{-1}(c_1^*))' = c_1^{-1}(c_1^*), \\ (v - c_2^*)(c_2^{-1}(c_2^*))' = c_2^{-1}(c_2^*). \end{cases}$$

Notice the perfect analogy with the first order conditions for optimal bids in $1A2U$ (see system 1): if (b_1^*, b_2^*) respectively are the highest and the lowest equilibrium bids in $1A2U$ when the opponent's highest and lowest bidding functions are $(b_1(\cdot), b_2(\cdot))$, then, in $2A1U$, b_1^* is the optimal bid in the first auction and b_2^* is the optimal bid in the second auction when the opponent respectively bids according to $b_2(\cdot)$ in the first auction and to $b_1(\cdot)$ in the second auction. Formally, there is an equilibrium of $2A1U$ in which bidding functions are:[25]

$$\begin{cases} c_{1,1}^{JoW}(v_1) = b_1^{JoW}(v_1),\ c_{1,2}^{JoW}(v_1) = b_2^{JoW}(v_1), \\ c_{2,1}^{JoW}(v_2) = b_2^{JoW}(v_2),\ c_{2,2}^{JoW}(v_2) = b_1^{JoW}(v_2). \end{cases}$$

In this equilibrium, bidder 1 respectively places the highest bid in the first auction and the lowest bid in the second, while bidder 2 makes the opposite. There is also a specular equilibrium in which bidder 1 respectively places the lowest bid in the first auction and the highest bid in the second, while bidder 2 makes the opposite:

$$\begin{cases} c_{1,1}^{JoW}(v_1) = b_2^{JoW}(v_1),\ c_{1,2}^{JoW}(v_1) = b_1^{JoW}(v_1), \\ c_{2,1}^{JoW}(v_2) = b_1^{JoW}(v_2),\ c_{2,2}^{JoW}(v_2) = b_2^{JoW}(v_2). \end{cases}$$

B.2.3 Bid spreading in $1A2U$ and $2A1U$

Given that the equilibrium bidding functions are the same in $1A2U$ and $2A1U$, we only need to show bid spreading in $1A2U$, that is, $b_1^{JoW}(v) > b_2^{JoW}(v)$, for all $v \in (0, \bar{v})$, or

$$\frac{\sqrt{1 + Cw^2v^2} - 1}{Cwv} > \frac{1 - \sqrt{1 - Cv^2}}{Cv}, \forall v \in (0, \bar{v}).$$

By multiplying both sides by Cwv and rearranging, we obtain $\sqrt{1 + Cw^2v^2} + w\sqrt{1 - Cv^2} > w+1$. By simple algebra, this condition reduces to $w^2 - 1 - Cw^2v^2 > 0$, or $(w^2-1)(1-v^2/\bar{v}^2) > 0$, which is satisfied for all $v \in (0, \bar{v})$.

B.2.4 Overbidding

Showing that, in $1A1U$, $a^{JoW}(V) > a^{RN}(V)$ for all $V \in (0, \bar{V}]$ is trivial.

To have overbidding in $1A2U$ (and in $2A1U$), it must be that $b_2^{JoW}(v) > b_2^{RN}(v)$, namely:

$$\frac{1 - \sqrt{1 - Cv^2}}{Cv} > \frac{1}{2}v, \forall v \in (0, \bar{v}].$$

By simple algebra, the previous condition reduces to $C^2v^4 > 0$, which is satisfied for all $v \in (0, \bar{v}]$.

[25] In $2A1U$, since there is an asymmetry in bids in the two auctions, we have to re-introduce bidders' indexes. Thus, $c_{i,1}^{JoW}(v_i)$ and $c_{i,2}^{JoW}(v_i)$, indicate the equilibrium bidding functions of bidder i in $2A1U$.

B.2.5 Efficiency in $1A1U$, inefficiency in $1A2U$ and $2A1U$

Allocative efficiency in $1A1U$ is implied by the symmetric nature of the equilibrium and the fact that equilibrium bidding functions are strictly increasing. As a consequence, the bidder with the highest private value places the highest bid and wins the auction. In both $1A2U$ and $2A1U$, the bidder with the highest private values wins for sure one unit, as her highest bid is greater than both opponents' bids. However, there is a strictly positive probability that the bidder with the lowest private values wins one unit with her highest bid. Indeed, notice that an inefficient allocation of the second unit occurs if either $v_1 > v_2$ but $b_2^{JoW}(v_1) < b_1^{JoW}(v_2)$, or $v_1 < v_2$ but $b_1^{JoW}(v_1) > b_2^{JoW}(v_2)$. The ex-ante probability of an inefficient allocation of the second unit is the probability that one of these events occurs, namely:

$$\int_0^{\bar{v}} \left[F\left((b_2^{JoW})^{-1}(b_1^{JoW}(v_2))\right) - F\left((b_1^{JoW})^{-1}(b_2^{JoW}(v_2))\right)\right] f(v_2)dv_2.$$

The previous expression is strictly positive. Indeed, since $b_2^{JoW}(.) < b_1^{JoW}(\cdot)$, we have $(b_2^{JoW})^{-1}(\cdot) > (b_1^{JoW})^{-1}(\cdot)$ and $(b_2^{JoW})^{-1}(b_1^{JoW}(\cdot)) > (b_1^{JoW})^{-1}(b_2^{JoW}(\cdot))$.

B.2.6 Bidders bid more in $1A1U$ than in $1A2U$ and $2A1U$

We show that, for $V = 2v$, $a^{JoW}(V) > b_1^{JoW}(v) + b_2^{JoW}(v)$, for all $v \in (0, \bar{v}]$. It is sufficient to show that $a^{JoW}(V) > 2b_1^{JoW}(v)$, namely:

$$\frac{1}{2}wV = wv > 2\frac{\sqrt{1 + Cw^2v^2} - 1}{Cwv}.$$

By simple algebra, the previous condition reduces to $C^2w^4v^4 > 0$, which holds for all $v \in (0, \bar{v}]$.

B.2.7 Auctioneer's revenue is higher in $1A1U$ than in $1A2U$ and $2A1U$

This implication follows from the previous result. Indeed, suppose that bidders' private values are v' and v'', with $v' \geq v''$, in the allotment treatments, while they are $V' = 2v'$ and $V'' = 2v''$ in $1A1U$. The auctioneer's revenue in $1A1U$ is simply $a^{JoW}(V')$. The auctioneer's revenue in $1A2U$ and $2A1U$ are either $b_1^{JoW}(v') + b_2^{JoW}(v')$ if $b_2^{JoW}(v') > b_1^{JoW}(v'')$ or $b_1^{JoW}(v') + b_1^{JoW}(v'')$ if $b_2^{JoW}(v') < b_1^{JoW}(v'')$. In the former case, we have already shown that $a^{JoW}(v') > b_1^{JoW}(v') +$

$b_2^{JoW}(v')$. In the latter case, since $v' \geq v''$, we have that $2b_1^{JoW}(v') \geq b_1^{JoW}(v') + b_1^{JoW}(v'')$. However, by previous results, $a^{JoW}(V') > 2b_1^{JoW}(v')$, which implies that $a^{JoW}(V') > b_1^{JoW}(v') + b_1^{JoW}(v'')$.

B.3 Risk Aversion

B.3.1 Equilibrium bids in $1A2U$

We characterise a symmetric equilibrium in differentiable, strictly increasing strategies. Using the same notation as before, in $1A2U$, the expected payoff of a bidder with value v is:

$$\Pi(b_1, b_2; v) = u(2v - b_1 - b_2)F(b_1^{-1}(b_2)) + u(v - b_1)[F(b_2^{-1}(b_1)) - F(b_1^{-1}(b_2))],$$

where $u(\cdot)$ is a strictly increasing and strictly concave function, with $u(0) = 0$.

If the optimal bids (b_1^*, b_2^*) are such that $b_1^* > b_2^*$ (we will verify this condition in the sequel), then they must satisfy the following first order necessary conditions:

$$\begin{cases} -u'(2v - b_1^* - b_2^*)F(b_1^{-1}(b_2^*)) - u'(v - b_1^*)\left[F(b_2^{-1}(b_1^*)) - F(b_1^{-1}(b_2^*))\right] \\ +u(v - b_1^*)f(b_2^{-1}(b_1^*))(b_2^{-1}(b_1^*))' = 0, \\ -u'(2v - b_1^* - b_2^*)F(b_1^{-1}(b_2^*)) + \\ +\left[u(2v - b_1^* - b_2^*) - u(v - b_1^*)\right]f(b_1^{-1}(b_2^*))(b_1^{-1}(b_2^*))' = 0. \end{cases} \tag{2}$$

In a symmetric equilibrium, it must be that $b_1^* = b_1(v)$ and $b_2^* = b_2(v)$. Thus, the first order conditions for a symmetric equilibrium are:

$$\begin{cases} -u'(2v - b_1(v) - b_2(v))F(b_1^{-1}(b_2(v))) - u'(v - b_1(v))[F(b_2^{-1}(b_1(v))) \\ -F(b_1^{-1}(b_2(v)))] + u(v - b_1(v))f(b_2^{-1}(b_1(v)))(b_2^{-1}(b_1(v)))' = 0, \\ -u'(2v - b_1(v) - b_2(v))F\left(b_1^{-1}(b_2(v))\right) \\ +\left[u(2v - b_1(v) - b_2(v)) - u(v - b_1(v))\right]f\left(b_1^{-1}(b_2(v))\right)\left(b_1^{-1}(b_2(v))\right)' = 0. \end{cases}$$

This system of differential equations, together with the boundary conditions $b_1(0) = b_2(0) = 0$ and $b_1(\bar{v}) = b_2(\bar{v}) = \bar{b}$ defines a symmetric equilibrium $(b_1^{RA}(v), b_2^{RA}(v))$.[26]

[26] Provided that, $b_1^{RA}(v) > b_2^{RA}(v)$, for all $v \in (0, \bar{v})$.

B.3.2 Equilibrium bids in $2A1U$

The expected payoff of a bidder with value v in $2A1U$ is:

$$\Pi(c_1, c_2; v) = u(v - c_1)F(c_1^{-1}(c_1)) + u(v - c_2)F(c_2^{-1}(c_2))$$
$$+[u(2v - c_1 - c_2) - u(v - c_1) - u(v - c_2)]F(\min(c_1^{-1}(c_1); c_2^{-1}(c_2))$$

Suppose that, as in a symmetric equilibrium, the optimal bids c_1^*, c_2^* are such that $c_1^{-1}(c_1^*) = c_2^{-1}(c_2^*)$. Notice that, at this point, the expected payoff is not differentiable. However, it admits right and left partial derivatives. In particular, the right partial derivatives must be non-positive and the left partial derivatives must be nonnegative (otherwise, any increase or decrease in bids would be profitable):

$$\frac{\partial \Pi(c_1^*, c_2^*)}{\partial c_1^+} \leq 0, \quad \frac{\partial \Pi(c_1^*, c_2^*)}{\partial c_1^-} \geq 0;$$

$$\frac{\partial \Pi(c_1^*, c_2^*)}{\partial c_2^+} \leq 0, \quad \frac{\partial \Pi(c_1^*, c_2^*)}{\partial c_2^-} \geq 0;$$

Let us focus on the first two conditions (by symmetry, the same holds for the other two conditions), which are:

$$-u'(2v - c_1^* - c_2^*)F(c_2^{-1}(c_2^*)) + u(v - c_1^*)f(c_1^{-1}(c_1^*))(c_1^{-1}(c_1^*))' \leq 0 \tag{3}$$

$$-u'(2v - c_1^* - c_2^*)F(c_1^{-1}(c_1^*)) + [u(2v - c_1^* - c_2^*) - u(v - c_2^*)]f(c_1^{-1}(c_1^*))(c_1^{-1}(c_1^*))' \geq 0 \tag{4}$$

Since $c_1^{-1}(c_1^*) = c_2^{-1}(c_2^*)$, the first term of (3) coincides with that of (4). Let us concentrate on the second term. Since $u(\cdot)$ is strictly concave, the incremental ratio $[u(z + h) - u(z)]/h$ is strictly decreasing in z. Therefore, we have

$$u(2v - c_1^* - c_2^*) - u(v - c_2^*) < u(v - c_1^*).$$

But then, (3) and (4) cannot simultaneously hold. This means that the optimal bids (c_1^*, c_2^*) are such that $c_1^{-1}(c_1^*) \neq c_2^{-1}(c_2^*)$. In particular, they imply that there are no symmetric equilibria.

Now, suppose that the optimal bids (c_1^*, c_2^*) are such that $c_1^{-1}(c_1^*) > c_2^{-1}(c_2^*)$. The expected payoff is now differentiable at (c_1^*, c_2^*) and the first order conditions are:

$$\begin{cases} -u'(2v - c_1^* - c_2^*)F(c_2^{-1}(c_2^*)) - u'(v - c_1^*)\left[F(c_1^{-1}(c_1^*)) - F(c_2^{-1}(c_2^*))\right] \\ +u(v - c_1^*)f(c_1^{-1}(c_1^*))(c_1^{-1}(c_1^*))' = 0 \\ -u'(2v - c_1^* - c_2^*)F(c_2^{-1}(c_2^*)) + [u(2v - c_1^* - c_2^*) - u(v - c_1^*)] * \\ *f(c_2^{-1}(c_2^*))(c_2^{-1}(c_2^*))' = 0 \end{cases} \tag{5}$$

Notice the perfect analogy with the first order conditions for optimal bids in $1A2U$ (see system 2). Therefore, in $2A1U$ there is an equilibrium in which:

$$\begin{cases} c_{1,1}^{RA}(v_1) = b_1^{RA}(v_1),\ c_{1,2}^{RA}(v_1) = b_2^{RA}(v_1), \\ c_{2,1}^{RA}(v_2) = b_2^{RA}(v_2),\ c_{2,2}^{RA}(v_2) = b_1^{RA}(v_2). \end{cases}$$

In this equilibrium, bidder 1 respectively places the highest bid in the first auction and the lowest bid in the second, while bidder 2 makes the opposite. As under joy of winning, there is also a specular equilibrium in which bidder 1 respectively places the lowest bid in the first auction and the highest bid in the second, while bidder 2 makes the opposite:

$$\begin{cases} c_{1,1}^{RA}(v_1) = b_2^{RA}(v_1),\ c_{1,2}^{RA}(v_1) = b_1^{RA}(v_1), \\ c_{2,1}^{RA}(v_2) = b_1^{RA}(v_2),\ c_{2,2}^{RA}(v_2) = b_2^{RA}(v_2). \end{cases}$$

B.3.3 Bid spreading in $1A2U$ and $2A1U$

Given the analogy between $1A2U$ and $2A1U$ in terms of equilibrium bids, we focus on $1A2U$ and show that, if $(b_1^{RA}(v), b_2^{RA}(v))$ constitutes a symmetric equilibrium in differentiable strategies, then it must be that $b_1^{RA}(v) > b_2^{RA}(v)$, for all $v \in (0, \bar{v})$.

Consider a bidder with value $v \in (0, \bar{v})$ who finds optimal to bid $b_1^* = b_2^* = b^*$. Then, it must be the case that deviations are non-profitable, namely:

$$\frac{\partial \Pi(b^*, b^*)}{\partial b_1} \leq 0, \quad \frac{\partial \Pi(b^*, b^*)}{\partial b_2} \geq 0.$$

In a symmetric equilibrium, $b_1^* = b_1(v)$, $b_2^* = b_2(v)$, and since $b_1^* = b_2^* = b^*$, then $b_1(v) =$

$b_2(v) = b^*$. The conditions above reduce to

$$b_2'(x) \geq \frac{u(v - b^*)}{u'(2v - 2b^*)} \frac{f(v)}{F(v)}$$

and

$$b_1'(x) \leq \frac{u(2v - 2b^*) - u(v - b^*)}{u'(2v - 2b^*)} \frac{f(v)}{F(v)}.$$

Since $u(\cdot)$ is strictly concave and $u(0) = 0$, we have that, for $y > 0$, $2u(y) > u(2y)$. Therefore,

$$\frac{u(v - b^*)}{u'(2v - 2b^*)} \frac{f(v)}{F(v)} > \frac{u(2v - 2b^*) - u(v - b^*)}{u'(2v - 2b^*)} \frac{f(v)}{F(v)},$$

which implies that $b_2'(v) > b_1'(v)$. Thus, whenever the two bids are equal, the bidding function corresponding to the lowest bid is steeper than that of the highest bid. However, this is not possible under the assumption that $b_1(\cdot)$ and $b_2(\cdot)$ are differentiable.

B.3.4 Overbidding

It is well known that $a^{RA}(V) > a^{RN}(V)$. Let us prove overbidding in $1A2U$ and $2A1U$. Starting from $1A2U$, suppose that the opponent bids according to generic (strictly increasing) bidding strategies, $b_1(\cdot)$ and $b_2(\cdot)$. Under risk neutrality, the first order conditions that define the optimal lowest bid b_2^* of a bidder with value v is

$$(v - b_2^*) f(b_1^{-1}(b_2^*))(b_1^{-1}(b_2^*))' = F(b_1^{-1}(b_2^*)) \tag{6}$$

Now, suppose that under risk aversion, the bidder bids according to b_2^* defined by (6). The partial derivative of her expected payoff with respect to b_2 evaluated at b_2^* is

$$-u'(2v - b_1 - b_2^*)F(b_1^{-1}(b_2^*)) + [u(2v - b_1 - b_2^*) - u(v - b_1)]f(b_1^{-1}(b_2^*))(b_1^{-1}(b_2^*))'.$$

By using (6), the last expression can be written as

$$\frac{u(2v - b_1 - b_2^*) - u(v - b_1)}{v - b_2^*} - u'(2v - b_1 - b_2^*),$$

which is strictly positive. Indeed, notice that the first addend is the incremental ratio of $u(\cdot)$ from $(v - b_1)$ to $(2v - b_1 - b_2^*)$. The second term is the derivative of $u(\cdot)$ evaluated at $(2v - b_1 - b_2^*)$.

Since $u(\cdot)$ is strictly concave, the first term is strictly greater than the second. This shows that, under risk aversion, for any possible bidding strategy adopted by the opponent, a bidder has an incentive to increase her lowest bid with respect to the corresponding equilibrium level under risk neutrality.

B.3.5 Efficiency in $1A1U$, inefficiency in $1A2U$ and in $2A1U$

The argument is identical to the one used under joy of winning. Inefficiency in $1A2U$ and in $2A1U$ follows directly from bid spreading.

C Experimental instructions

In what follows, we present the instructions given to participants in the three treatments. Instructions were originally written in German. We first present the part of *Instructions* that is common to all treatments (*[All treatments]*) and, then, the part of *Detailed Instructions* which is specific of each to the three treatments (*[1A1U], [1A2U], [2A1U]*).

Instructions - [All treatments]

Welcome. Thanks for participating in this experiment. If you follow the instructions carefully you can earn an amount of money that will be paid in cash at the end of the experiment. During the experiment you are not allowed to talk or communicate in any way with the other participants. If you have questions raise your hand and one of the assistants will come and answer it. The rules that you are reading are the same for all participants.

General rules

For showing up on time, you will receive 4 euro. During the experiment you will receive points. At the end of the experiment, the total number of points you have accumulated will be converted into Euro at the rate of 100 points = 3 euro. Your final payment will be composed of the show-up fee of 4 euro plus the amounts of money that you will earn during the experiment. Your final payment will be paid to you in cash immediately at the end of the experiment.

Detailed Instructions - [$1A1U$]

In this experiment you will participate in one auction that involves one unit of a hypothetical good. If you acquire the unit, the experimenter will purchase that unit at its resale value. Your resale value will be communicated to you before the beginning of the auction. This value will be randomly drawn from an interval between 0 and 200 points, with every number in this interval having the same probability of being drawn.

In total, two persons will participate in the auction. Thus, in addition to you, there is another participant who will also want to acquire the unit involved in the auction. Exactly like you, the other participant will be given a resale value before the beginning of the auction. This value will be, again, randomly drawn from an interval between 0 and 200 points, with every number in this interval having the same probability of being drawn. The resale values of the two participants in the auction are therefore drawn independently of each other. Thus, it is likely that you and the other participant will be given different resale values. During the auction, you will not be informed about the resale value of the other participant, nor will the other participant be informed of your resale value.

Rules of the auction

The unit of the good involved in the auction will be auctioned off according to the following rules. You, as well as the other participant, will place one bid for the unit. The bid will be equivalent to the number of points that you are willing to pay to acquire the unit. Given the choices of the two participants, the highest bid wins the unit involved in the auction. This means that you will acquire the unit if you place the highest bid in the auction. In the case of identical bids, the winning bid will be randomly determined. If you acquire the unit, your earnings in points will be given by the difference between the resale value and your winning bid. If you do not acquire the unit, you will earn nothing. Note that you may also generate losses if you acquire the unit by bidding more than the resale value. Eventual losses will be subtracted from your total earnings in points.

Example. The following table reports the two hypothetical bids placed by the two participants in the auction. The two bids are ranked in order from the highest to the lowest. The participant placing the highest bid wins the unit and pays 87 points to acquire that unit.

Auction

Rank	*Bid*
1	87
2	53

Repetitions of the experimental task

The experiment consists of 15 periods. In each period you will participate in an auction involving one unit. In each period, you and the other participant will be given new resale values drawn according to the previous rules. In each period, you will be randomly re-matched with another participant in such a way that you will never interact with the same opponent in two consecutive periods. The experiment is anonymous, meaning that you will not be told who the other group member is. At the end of each period, the computer will show your bid, the winning bid and how many points you have obtained in that auction.

Detailed Instructions - [$1A2U$]

In this experiment you will participate in one auction that involves two units of a hypothetical good. For each unit you acquire, the experimenter will purchase that unit at its resale value. Your resale value will be communicated to you before the beginning of the auction. This value will be randomly drawn from an interval between 0 and 100 points, with every number in this interval having the same probability of being drawn. Finally, the two units in the auction have the same resale value.

In total, two persons will participate in the auction. Thus, in addition to you, there is another additional participant who will also want to acquire the two units involved in the auction. Exactly like you, the other participant will be given a resale value for each of the two units before the beginning of the auction. Also for the other participant, the two units are assigned the same resale value. The resale value will be, again, randomly drawn from an interval between 0 and 100 points, with every number in this interval having the same probability of being drawn. The resale values of the two participants in the auction are therefore independently drawn from each other. Thus, it is likely that you and the other participant will be given different resale values. During the auction, you will not be informed about the resale values of the other participant, nor will the other participant be informed of your resale values.

Rules of the auction

The two units of the good involved in the auction will be auctioned off according to the following rules. You and the other participant will each place two bids, one for each unit. Each bid will be equivalent to the number of points that you are willing to pay to acquire the corresponding unit. Given the choices of the two participants, the two highest bids win the two units involved in the auction. This means that you will acquire one unit if you place one of the two highest bids in the auction. Similarly, you will acquire both units if you place the two highest bids in the auction. In case of identical bids, the winning bids will be randomly determined. For each unit you acquire, your earnings in points will be given by the difference between its resale value and your winning bid. If you do not acquire any unit, you will earn nothing. Note that you may also generate losses if you acquire a unit by bidding more than its resale value. Eventual losses will be subtracted from your total earnings in points.

Example. The following table reports the four hypothetical bids placed by the two participants in the auction. The four bids are ranked in order from highest to lowest. The participant placing the highest bid wins the first unit and pays 87 points to acquire that unit. The participant placing the second highest bid wins the second unit and pays 77 points to acquire that unit.

Auction

Rank	*Bid*
1	87
2	77
3	66
4	53

Repetitions of the experimental task

The experiment consists of 15 periods. In each period, you will participate in an auction involving two units. In each period, you and the other participant will be given new resale values drawn according to the previous rules. In each period, you will be randomly re-matched with another participant in such a way that you will never interact with the same opponent in two consecutive periods. The experiment is anonymous, meaning that you will not be told who the other group member is. At the end of each period, the computer will show your bids, the two winning bids in the auction and how many points you have obtained in the auction.

Detailed Instructions - $[2A1U]$

In this experiment you will participate in two simultaneous auctions, each involving one unit of a hypothetical good. For each unit you acquire, the experimenter will purchase that unit at its resale value. Your resale value will be communicated to you before the beginning of the auction. This value will be randomly drawn from an interval between 0 and 100 points, with every number in this interval having the same probability of being drawn. Finally, the two units in the two auctions have the same resale value.

In total, two persons will participate in the two auctions. Thus, in addition to you, there is another participant who will also want to acquire the two units involved in the two auctions. Exactly like you, the other participant will be given the resale value of each of the two units before the beginning of the two auctions. Also for the other participant, the units in the two auctions are assigned the same resale value. The resale value will be, again, randomly drawn from an interval between 0 and 100 points, with every number in this interval having the same probability of being drawn. The resale values of the two participants in the two auctions are therefore independently drawn from each other. Thus, it is likely that you and the other participant will be given different resale values. During the two auctions, you will not be informed about the resale values of the other participant, nor will the other participant be informed of your resale values.

Rules of the auctions

The two units of the good involved in the two auctions will be auctioned off according to the following rules. You and the other participant will each place two bids, one in each of the two auctions. Each bid will be equivalent to the number of points that you are willing to pay to aquire the unit in the corresponding auction. Given the choices of the two participants, in each auction, the highest bid wins the corresponding unit. This means that you will acquire one unit if you place the highest bid in one of the two auctions. Similarly, you will acquire two units if you place the highest bids in both auctions. In case of identical bids, the winning bids will be randomly determined. For each unit you acquire, your earnings in points will be given by the difference between its resale value and your winning bid. If you do not acquire any unit, you will earn nothing. Note that you may also generate losses if you acquire a unit by bidding more than its resale value. Eventual losses will be subtracted from your total earnings in points.

Example. The following two tables report the bids placed by the two participants in the two auctions. In each auction, the two bids are ranked in order from highest to lowest. The participant placing the highest bid in the first auction wins the first unit and pays 87 points to acquire that unit. The participant placing the highest bid in the second auction wins the second unit and pays 77 points to acquire that unit.

Auction A		*Auction B*	
Rank	*Bid*	*Rank*	*Bid*
1	87	1	77
2	53	2	66

Repetitions of the experimental task

The experiment consists of 15 periods. In each period you will participate in two simultaneous auctions, each involving one unit. In each period, you and the other participant will be given new resale values drawn according to the previous rules. In each period you will be randomly re-matched with another participant in such a way that you will never interact with the same opponent in two consecutive periods. The experiment is anonymous, meaning that you will not be told who the other group member is. At the end of each period, the computer will show your bids, the winning bids in the two auctions and how many points you have obtained in each of the two auctions.