Addressing the Demand Risk in PPP for the FIFA World CupÔ Stadiums: A Real-Options Approach

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ABSTRACT

In this paper, we discuss the demand risk in Public Private Partnerships for football stadiums for the FIFA World Cup. Given that uncertainties are present in the venture, we use real options theory in two distinct scenarios. In the first case, besides modeling a single stadium construction situation, we test the model in a case study of a stadium to be built for the 2014 FIFA World Cup. In the second case, we show that demand risks can be mitigated when several stadiums are built under central coordination by the main government. In both cases we assume that government investments must be minimized without harming private sector attractiveness. As well as our instrumental contribution to public and private managers, we expect to contribute to sports management and economics literature by showing the role of real options theory in the provision of sports facilities.

Key words: PPP, real options, football stadiums

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INTRODUCTION

Governments throughout the world have been exerting much energy to attract mega-sports events such as the Olympic Games and FIFA World Cup™. Indeed, based on allegedly successful experiences there is a shrewd wisdom that such events may represent a unique opportunity for urban development and an important source of revenues too. Not surprisingly, several governments have financed the necessary infrastructure, including the building and the refurbishment of sports facilities.

However, some researchers posit that the economic impacts of major events are questionable and that the expenditure is higher than the effective gains (along these lines see Matheson & Baade, 2005). Common sense suggests that infrastructure investments (roads, airports and so on) should be led by the government and investment in sports facilities should be allocated to private investors wherever possible; but when we observe the behavior of countries that have hosted the FIFA World Cup™ over the past few decades, we detect massive public investments in football stadiums[1]. This in particular may cause some surprise because there are certain risks that could be allocated to private operators and are assumed by the government in the end.

Nevertheless, as there are tangible and hard to measure intangible benefits associated with the promotion of the FIFA World Cup™, governments and private companies frequently share the responsibility of making such investments, which is normally in the sense of a Public-Private Partnership (PPP). Consequently, governments always play some role in terms of supporting or coordinating the construction of sports facilities.

As with other public utilities, investment in football stadiums is subject to several risks. We aim to discuss the demand risk issue in PPP to football stadiums in this paper. This is because attendance at matches may be dependent on the performance of the football clubs that use the

[1] We use the expression "football" to designate the most popular sport practiced in the world, which is managed by FIFA.

stadium; and this is a risk that a private operator cannot control. We assume in our analysis that government investments must be minimized while at the same time preserving the attractiveness for the private sector. In order to deal with demand risks, we use the real options theory given that uncertainties are present. We consider two distinct scenarios.

First, we model a situation in which a local government is the responsible for running the PPP bidding for a single stadium construction. We apply real options theory in a case study based on data from one of the host cities of the 2014 FIFA World Cup™ to be held in Brazil, where the local government has promoted a PPP tender for a new football stadium. Based on stadium project information and on 31 years of attendance records of the two major local football clubs we calculate the demand volatility and simulate alternatives for the financial viability of the stadium. Assuming a stochastic process for the demand we use the Monte Carlo simulation to calculate the probability distribution of the Net Present Value (NPV). Our analyses show that uncertainties associated with attendance at football matches require investments in commercial activities around the stadium to mitigate risk and to make the PPP viable with minimum investment on the part of the government. In addition, we calculate the insurance premium in case of a slump in stadium attendance, which is an essential aspect to attract investors.

Second, we model a scenario where a given central government is responsible for the coordination of the financing and risk management of the football stadiums. We show that due to the diversification effect when several stadiums are involved, the uncertainties in the set of investments can be mitigated because the risk of a fall in attendance at one stadium is likely to be compensated by a rise in the number of fans elsewhere (e.g., when a club does not attract fans due to bad performance it is possible to have the corresponding losses balanced with the superior performance of the rival teams). As a consequence the resulting insurance premium may be lower than the sum of the individual insurance premiums. This is very likely to occur when a given country is organizing the FIFA World Cup™ and football stadiums need to be built or refurbished in a short time.

Therefore, with our paper we expect to provide additional insights for policy makers and private agents involved with investment decisions in sports facilities. Besides informing public policy on an instrumental basis, our paper may, from the theoretical point of view, contribute to sports management and economics literature by showing the role of real options theory in the provision of sports facilities within a public and private partnership setting. In our contribution, we highlight some conditions where PPP in football stadiums may be socially acceptable and some possible mechanisms that governments can make use of to attract private investments to football stadium construction or refurbishment.

Our paper is structured as follows. First, in the context of Public-Private Partnerships (PPP) we discuss the existing risks in the building and operation of football stadiums and the respective allocation criteria. Then, after a brief presentation on the pertinence of the real options approach in infrastructure investment decisions, we model the conditions in which a local government can minimize expenditure for the taxpayer without harming private sector enthusiasm. In this section, we also calculate the insurance premium for several levels of minimum coverage according to the chosen NPV. Following this, we address the outcomes of central government coordination in the provision of new football facilities. The last section concludes.

PPP AND RISK ALLOCATION IN FOOTBALL STADIUMS

Football stadiums present some characteristics in common with other public goods in that they can generate benefits related to consumer surplus through the fans who attend games, positive externalities to non-attenders, increased community visibility and enhanced community image (Siegfried & Zimbalist, 2000; Crompton, 2004). Following this reasoning, some football stadiums are publicly-owned such as San Siro in Milan and the Rome Olympic

Stadium. Indeed, in recent decades several stadiums have been subsided from public sources in the United States (Noll & Zimbalist, 1997; Matheson & Baade, 2006).

On the other hand, there is an intense debate on the opportunity costs of building new stadiums as the public funds used could be diverted to other projects that could bring more value to society (Matheson & Baade, 2004). Further, considering that the financial viability of investments in football stadiums has a strong correlation with the revenues of the home team (Rebeggiani, 2006), there would be no reason for public funding if the local club makes financial gains. In this case, the subsidies for stadiums are likely to result in an increase in club revenues and players' salaries (Matheson & Baade, 2006). The trend of increasing costs in the team's payroll normally results in a lack of income to cover stadium construction costs and that is why few facilities have been built with the financial support of clubs or private investors (Siegfried & Zimbalist, 2000).

Although government financial support for football stadiums is a controversial subject, on closer inspection we see that public funds have been widely used to this end. In this matter, government and private investors might share the investment and the associated risks associated with football stadiums in a Public-Private Partnership (PPP)[2]. In this sense, in order to assure public interest i.e., minimization of government participation without constraining private investments; it is necessary to take into account issues related to risk sharing between investors and public entities and the related allocation criteria (Martimort & Pouyet, 2008).

However, what are the risks associated with the construction and operation of football stadiums? The Lille Council of Local Municipalities (LCMU) in France pointed out that the main risks can be grouped in *preliminary risks*—those related to the possibility of the

[2] Although there are several definitions for PPP, in this paper we consider this contractual mode as an arrangement involving a public authority and one or more private entities to provide a certain service. The amount of risks and investments can be either defined by the government or subject to negotiation among the parties involved.

inexistence of acceptable offers and to the objection of bidding procedures; *conception and construction risks*— which apply to the definition of requirements, legal authorizations to initiate the construction and also the construction risks; *financial risks*—associated to fluctuations in interest and inflation rates; and *exploitation risks*—which cover the sports performance risks of the host team and also the errors in the ex-ante estimation of the operation and maintenance costs (LCMU 2006).

A second question arises: how should such risks be allocated? If we assume the classical assumption in which government (principal) is risk neutral and private operator (agent) is risk averse, the risk should be allocated to the party best able to manage it or to the agent able to bear the risk at the lowest cost (Oudot, 2007). Although in some cases it is the agent who is best able to control the risks, the agents may not be able to handle some risks in a low cost way (Medda, 2007). Notwithstanding, the allocation of the majority of the existing risks associated to football stadiums is straightforward. For instance, the consequences of problems related to the bidding process must be absorbed by governments. The risk associated to the construction of the facilities is likely to be engulfed by the responsible party for performing such tasks, which is normally the private contractor in a Build-Operate and Transfer (BOT) public-private agreement (Ping, Akintoye, Edwards & Hardcastle 2005). At the same line, financial risks linked to the search for funding, negotiations with banking organizations and to variations in the future interest rates can be split between public and private agents. Governments must also assume the risks of building the 'rules of the game', i.e., the legal and regulatory devices that allow public and private partnerships (Grimsey & Lewis, 2004).

However, the allocation of demand risk in PPP contracts remains a critical issue (Iossa & Martimort, 2009). When we set our sights on football stadiums, we observe that demand risks are associated to sports performance. Indeed, football is often unpredictable and the number of fans of a given team is highly correlated with club performance (Madalozzo & Villar, 2009). In practice, low performance standards are likely to undermine returns on investments previously made because bad results may imply a lower number of ticket sales (Forrest & Simmons, 2002), less media coverage, a fall in sponsorship revenue and a fall in sales of club licensed products (Borland & Macdonald, 2003). In this situation, risk-averse investors are expected to behave parsimoniously in terms of investing in the football business; mainly within a scenario of credit rationing and pressures for liquidity (Greenwald & Stiglitz, 1990). Furthermore, "winner's course" may arise because the expected value can not be captured in the future due to demand uncertainties (Thaler, 1998).

Demand uncertainties give rise to problems in the calculations of bids submitted for infrastructure auctions in general, and it is one of the main causes of project failure in concessions (Vassalo, 2006). Therefore, as the risks associated to the host club performance may thwart private investment in PPP for football stadiums, governments can consider the partial bearing of risks associated with demand fluctuations. In order to devise the proper participation of the government and the limits of private participation in the presence of demand uncertainty it is necessary to deal with the probability of risk materialization. Real options theory can be helpful in this matter.

REAL OPTIONS THEORY AND RISK DEMAND IN FOOTBALL STADIUMS

The advantages of real options approach over the net present value (NPV) rule for investment decisions under uncertainty are well known, mainly when they involve sunk costs and flexibilities as such as regarding the time of investing. Arrow and Fisher (1974) and Henry (1974) discuss the valuation of the option to postpone the decision to invest in a project with sunk costs. Dixit and Pindyck (1994) discuss three types of uncertainty: economic uncertainty, technical uncertainty and uncertainty in the strategy. These dimensions play a leading role in the option of switching unprofitable projects to profitable ones and even abandoning unprofitable projects, which is particularly important in the context of infrastructure investment as costs that cannot be effortlessly recouped if the expected scenario deteriorates.(Guasch, 2004). Real options theory deals with these constraints and can help the modeling of investment decisions under uncertainty where revenues of a given venture may vary according to their probability distribution (Defilipi, 2004).

The valuation of real options may be achieved by binomial tree models, dynamic programming or contingent claim analysis. The binomial tree model is presented in Cox, Ross and Rubinstein (1979). Dynamic programming models typically involve a stochastic problem that may be solved by the Bellman equation (Merton 1973). Contingent claim analysis may deal with solutions as presented by Black and Scholes (1973) for financial option problems (for further discussions see Dixit and Pindyck 1994).

Some advances in real options theory apply to public and private partnerships. These aspects might be invoked in situations where government participation is required to reduce risks to private agents—such as roads (Brandão & Saraiva, 2008)— or in cases where choices between single and multiple operators in infrastructure concessions (see Defilipi 2004, for an example in the port sector). Blank, Baidya & Dias (2009) also discuss real options in public private partnership in the case of a toll road concession. Armada, Pereira & Rodrigues (2008) evaluate optimal incentives that an investor will need to take the decision of investing in conditions of constrained growth in an airport. The possible applications of real options theory in other sectors are vast and may include football stadiums as we discuss in this work.

The model

We use the binomial tree model (Cox, Ross & Rubinstein 1979) to evaluate the investment in the construction of a football stadium with high demand uncertainty. In line with the previous discussion, it is worth emphasizing that even with a positive net present value, achieved by an estimated average demand of fans, investors may not be stimulated to invest due to the uncertainty associated with the football business and the volume of funds required. We assume that the government will be the hedge provider offering a guarantee of a minimum NPV[3] in order to stimulate the private sector to make the necessary investment in the football stadium.

First of all the deterministic NPV based on an average demand of football audience has to be calculated. The Monte Carlo simulation with the NPV as a function of the average and volatility of demand helps to estimate the probability distribution of the NPV (Suttinon & Nasu 2010; Cortazar 2004; Geske & Shastri 2004).[4] The binomial tree is built based upon the probability distribution of the NPV. After this the guarantee of minimum demand can be evaluated. Using such an approach also makes it possible to evaluate strategies for risk mitigation through the building of other facilities near the stadium such as a shopping center, cinemas, convention centers and so on. The choice for suitable investments may help to reduce uncertainty and minimize the costs of the guarantees that might be provided by the government. The discussion addressed in this paper tries to valuate the reduction of the risks in the investment, when one adds a new business to the project.

The model determines the minimum demand guarantee that the government could provide to the private sector to reduce demand uncertainty in the investment. This kind of guarantee is similar to a put option. The investor may manage the risk of the investment according to his risk appetite buying this put option and therefore paying a premium to the hedge provider. We consider that government would act as the hedge provider that a) receives the option premium; or, b) gives the guarantee free to the investor as an incentive to start the project as soon as possible. We assume that the NPV follows a geometric Brownian motion (Brandão & Cury 2006; Copeland & Antikarov 2001) as following:

$$dNPV = mdt + sdZ \qquad (1)$$

[3] This guarantee may be achieved by agreements on the minimum annual demand or minimum cash flow.
[4] Average and volatility of customer demand may be estimated from historical data.

Where *μ* is the average grow rate of the *NPV* (discount rate used in the *NPV* calculation, for instance), *s* is the volatility of the stochastic process of the *NPV* and *dZ* is the standard Wiener process. In the binomial model the *NPV* may reach one of two states in the next time interval: *NPV x u* or *NPV x d*, with *u > 1*; *d < 1*. It may be shown that:

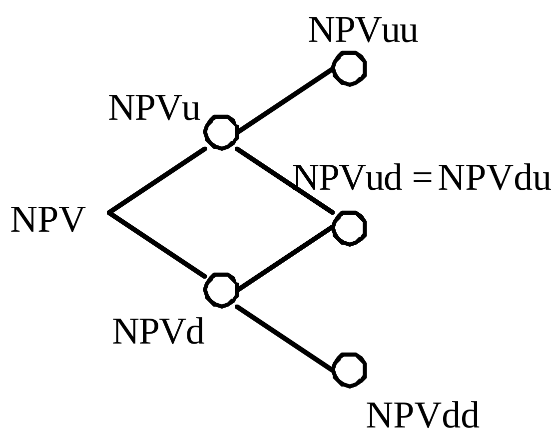
$$u = e^{s\sqrt{\Delta t}} \ , \ d = 1/u \ , \ \ p = \frac{e^{r\Delta t} - d}{u - d} \qquad (2)$$

Where *r* is the risk free rate and *p* is the risk neutral probability of the state *NPV x u* in the time interval *?t*, while *(1 - p)* is the probability of the state *NPV x d*. The expected *NPV* after a period *T* may be given by:

$$E[NPV_T] = p \times u \times NPV + (1 - p) \times d \times NPV = NPVe^{rT} \qquad (3)$$

Figure 1 shows the dynamic of the *NPV*.

Figure 1- Binomial Tree



It is worth highlighting that the NPV is calculated based upon cash flow. Therefore, it is possible to assume that each NPV in Figure 1 is associated with a particular cash flow. Similar to Brandão and Cury (2006) we assume that for a given instant of time the weight of a specific discounted cash flow in the deterministic NPV is the same to all other NPV in the binomial tree. As all the NPVs are known in the binomial tree, it is possible to calculate the cash flow for each node of the tree.

$$NPV = \sum_{k=1}^{N} \frac{C_k}{(1+i)^k} \quad (4)$$

$$w_k = \frac{C_k / (1+i)^k}{NPV} \quad (5)$$

The investor may mitigate risks by buying an insurance of a minimum level for the NPV and this insurance may be calculated by the binomial tree. This evaluation is achieved by imposing constraints for the cash flows at the nodes of the tree (estimated as a fraction $?_k$ of the NPV on the tree) that go below the minimum level and by calculating the new NPV. The difference between the new NPV and the deterministic NPV gives the value of the insurance, i.e. the value of the put option. The put option is valuated based on the constraint applied as a fraction of the deterministic cash flow. It means that a given cash flow may not be lower than a fraction of its corresponding deterministic cash flow. As it is possible to estimate a new cash flow for each node in the binomial tree, it is possible to calculate the new NPV in a recursive formula given by:

$$NPV_{t-1} = \left(p \times NPV_{u,t} + (p-1) NPV_{d,t}\right) \times e^{-r\Delta t} + FC_{t-1} \quad (6)$$

Where $NPV_{u,t}$ and $NPV_{d,t}$ are the net present value for period t in the upper and lower nodes respectively. FC_{t-1} is the cash flow in $t-1$, which may be the original node cash flow or the minimum guaranteed cash flow.

Next, we apply the model in a case study concerning a host city of the 2014 FIFA World Cup™ 2014.

CASE STUDY: A MODEL APPLICATION IN THE FONTE NOVA FOOTBALL STADIUM

This section presents an application of the real options model in the rebuild of the Fonte Nova football stadium in Salvador, Bahia, Brazil (one of the host cities of the 2014 FIFA World Cup™). The data used in our study come from one of the projects analyzed by the local government during the bidding process.

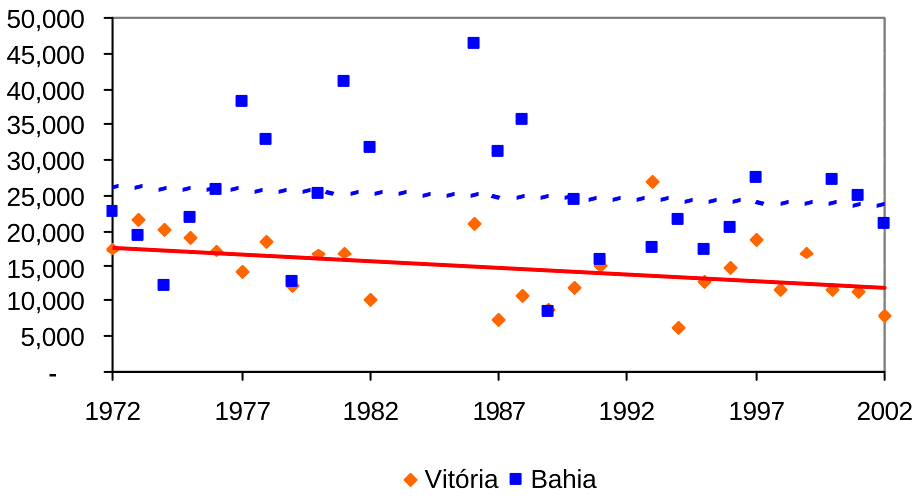
In this exercise, we aim to evaluate the local government guarantees to reduce private sector risks in the investment. The guarantee is evaluated as a put option where the private sector is long (the private agent buys the option or receives it for free as an incentive to invest as soon as possible) and the state government is short (the government sells the option or gives it as an incentive to the private sector to invest as soon as possible). We also analyze the alternative of complementary investments near the stadium in order to reduce uncertainty and consequently the guarantee costs. These complementary businesses are shopping centers, cinemas, a theater, hotels etc. We use the data of one of the proposals submitted to the PPP tender organized by the Bahia State government in 2008 (see data used in our calculations in Annex 1).

The first step in the option valuation is the calculation of the net present value volatility. The investment is divided in two blocks. One of them is the investment in the stadium and in the basic infrastructure nearby. The risk associated with these investments is s_S. The other block concerns the investment in the complementary business with risk s_E. Considering the investments in both blocks (stadium, basic infrastructure and alternative business) the risk of the entire complex (s_{CFN}) is given by:

$$s_{CFN} = \sqrt{X_E^2 s_E^2 + X_S^2 s_S^2 + 2 \times X_E s_E X_S s_S r} \qquad (7)$$

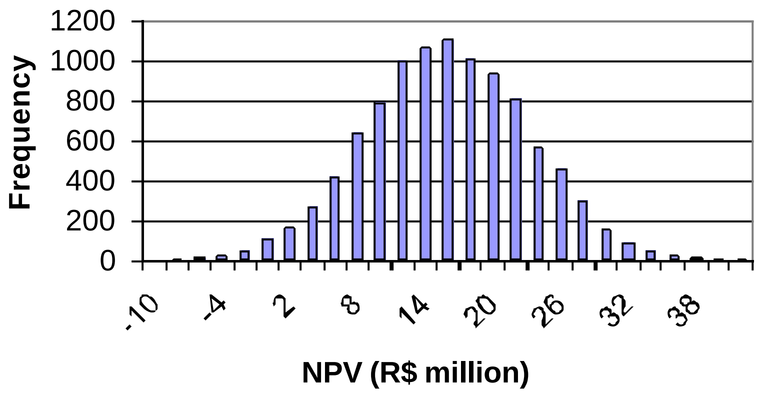
Where the correlation between the two blocks is r , and X_E and X_S are the weights invested in each of the two blocks. In order to forecast the risk associated to the net present value of the investment in the stadium, s_S, an estimate of the average and standard deviation of the demand for matches at the stadium by the two most important clubs in the city (Bahia and Vitória). Figure 2 shows the historical annual average attendance rates for each of the clubs from 1972 to 2002. The standard deviation of the variations in the demand presented in the annex II was 37.7%.

Figure 2 – Annual average match attendance (National League matches)



We consider that the stadium capacity is bounded. For this reason, we assume that the fans attendance follows a Brownian motion.[5] A 10.000 Monte Carlo simulation based on the data presented in annex I provides the probability distribution of the NPV using a Matlab code. The standard deviation of the NPV distribution presented in Figure 3 is 47.7%.

Figure 3 – NPV distribution based on Monte Carlo simulation



[5] Brandão and Saraiva (2007) use a similar approach to investigate an investment under uncertainty in a highway concession. They use an unbounded geometric Brownian motion for the demand that was not used here because the stadium capacity is bounded. Another difference is that the underlining asset here is the NPV and not the outcomes as in their work. The use of the outcomes makes the valuation of the option a more complex process as it is necessary to make a link between outcomes and a risk asset traded in the market in order to use the real option model. The authors use a similar approach as used in Hull (2003) and Irwin (2003) with some differences regarding the approach of capturing the risk premium of the project.

In order to evaluate the risk in the investment in the complementary businesses, s_E, the standard deviation of the GDP growth of the state of Bahia from 1986 to 2007 was used. We simulate the risk management alternatives by using a shopping center investment. Table 1 summarizes the data used to evaluate risk management for the investments. The risk of the investment in the stadium and in the shopping center was evaluated considering the range of possible values for the correlation of the returns (-1 to 1). Based on this range the standard deviation of the set of investments fluctuates from 35.6% to 37.2%. If we consider a zero correlation scenario we obtain a standard deviation of 36.4%.

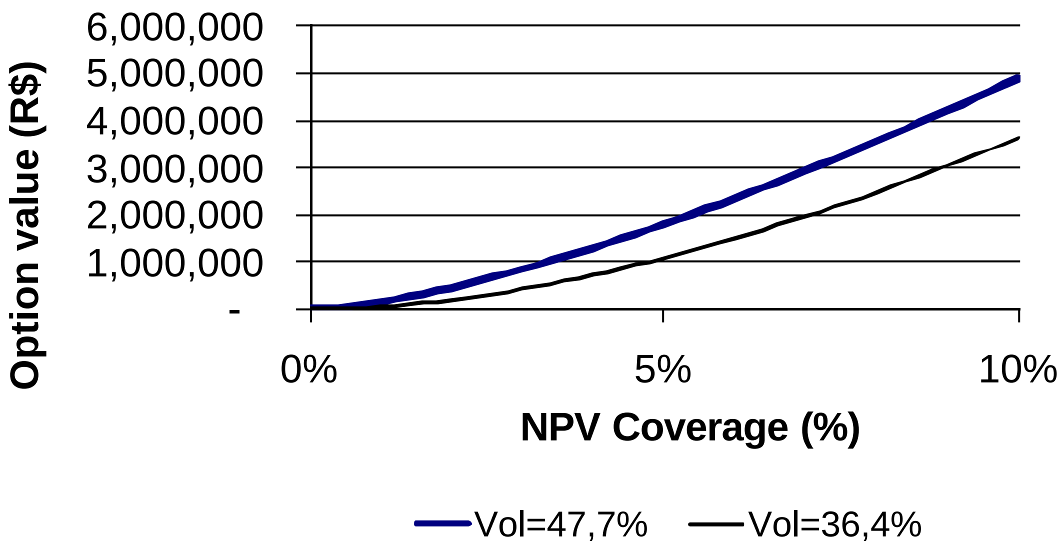
Table 1 – Data used to estimate volatility of investment in the stadium and shopping centre

	Initial investment (R$ million)	% Investiment	Returns STD	NPV (R$ million)
Stadium	265.3	76.4%	47.7%	14.9
Shopping	82.0	23.6%	3.20%	27.4
Total	347.4	100.0%		42.3

Based on the available data we evaluate the price of the option considering that the government of Bahia State shall guarantee a minimum demand on the stadium defined as a percentage of the NPV coverage[6]. It is important to highlight that risks associated to the stadium are higher than those associated to the shopping center investment and as can be seen in Table 1 investment in both will obviously reduce the volatility of the revenues. The two investment alternatives, i.e. the single stadium or the stadium plus the shopping center were evaluated based on the real option framework using the binomial tree approach. Figure 4 presents the results of this valuation.

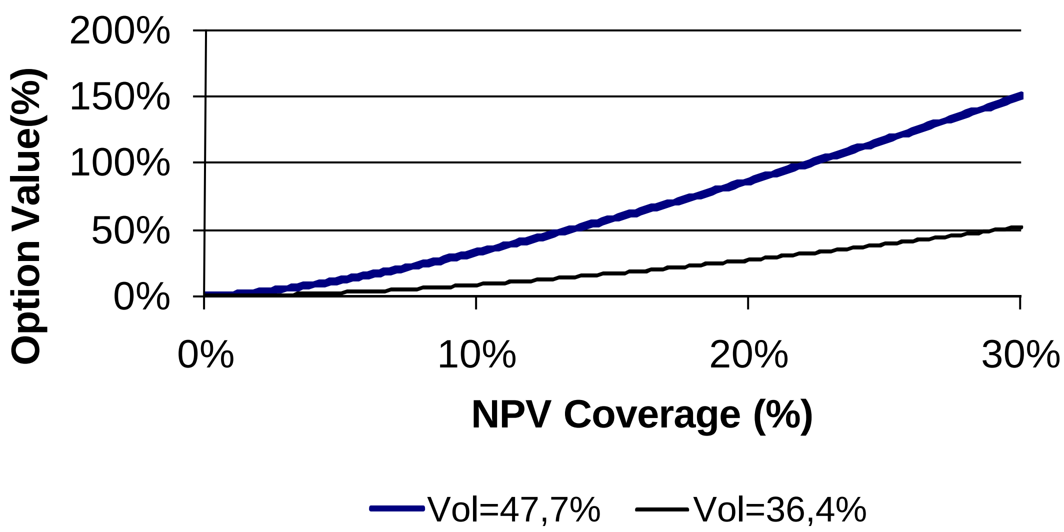
Figure 4 – Value of the option (R$) x Coverage of the NPV

[6] In order to build the binomial tree for the guaranteed investment the constrained cash flow corresponds to a fixed fraction of the original . This fraction is what we are calling the NPV “coverage”. .



As the investments are not the same Figure 5 shows the price of the option divided by the NPV.

Figure 5 – Value of the option (%NPV) x Coverage of the NPV



It is worth pointing out that investing in the shopping center reduces volatility and also the insurance premium. The difference between the two curves in both figures above shows the value of the option to invest in the shopping center too which would result in a reduction in the insurance premium of investing only in the stadium (the upper curve). In other words, it captures the value of an additional option in the project as the government may also allow the investor to have the concession on new business in the stadium neighborhood which may reduce the risks associated to the cash flow. This is an important issue in this case as the stadium demand uncertainties may discourage investors. Figure 5 shows that in the case of the stadium without a shopping center the insurance premium is very high if the investor tries to cover a high value of the NPV. The choice of the coverage level is a function of the investor's degree of risk aversion. Note that this choice may be used as the criteria that will determine the level of the incentive the investor may require from the government to invest as soon as possible. It must be emphasized that host governments are put under strong pressure by FIFA to avoid delays in stadium construction; and this may facilitate opportunistic behavior on the part of the private operators. The fear of retaliation in case of non-compliance of FIFA requirements may push governments to offer additional guarantees; thus decreasing the level of investment from private operators.

Major events like the FIFA World Cup normally involve the building or refurbishment of several stadiums in a tight time frame. In this case, central government coordination may be preferable as we show next.

BUILDING MULTIPLE FOOTBALL STADIUMS: THE ROLE OF THE CENTRAL GOVERNMENT AND RISK MANAGEMENT

Instead of local governments being responsible for providing the proper guarantees to private investors, central governments may assume the role of coordinating the guarantees in sports facilities, mainly when major events are to be organized. In this case, the national government could provide a central mechanism of assurance which from a risk management perspective seems more feasible when compared to the alternative of having local governments struggling to provide such guarantees and compete among themselves for central government funds normally in a non-optimal competition.

In the approach discussed before the local government may guarantee a minimum cash flow to the private sector if a slump in attendance occurs. Such a reduction may arise if local football clubs perform poorly in a specific period. Nevertheless, it is possible that the demand in a given state (or province, or any other local government unit) is not perfectly positively correlated to the demand in another one. This may occur because a certain local team performs badly in a year and is relegated while another football club in another state performs very well and wins the championship. In other words it is possible that a given state government has to compensate the private sector in a given period while another state does not. Thinking individually each state will need to build up a buffer of capital or a special fund to afford private sector compensation. It is intuitive that this arrangement is not optimal from a risk management perspective as the sum of capital of individual local governments is higher than the buffer required by a central government. It does not mean that the value of the insurance premium is lower but only that the required capital for risk management is lower. Such rationale is the same for car insurance or any other kind of insurance. Suppose a specific firm is set up to provide insurance for a single individual. Despite the fact that the value of the premium is a small amount of the value of the car the firm will need to build up a high buffer. If this firm insures several individuals the size of the buffer is not necessarily the sum of individual's buffer. Suppose we have a portfolio with several options. The value of each option is a function of the price of the underlying (in this case the NPV); the value of the strike (in this case, as a function of the minimum cash flow); the volatility of the returns of the underlying; the time and the interest rate. All these variables change over time as does the value of the portfolio. Using this type of portfolio it is possible to prove the following proposition:

Proposition: Central government may constitute a buffer of capital that is lower than the sum of individual buffers of the state governments due to the effect of diversification.

Proof: RiskMetricks (1996)[7] presents the delta-gama approach to evaluate risk of a portfolio of options. The methodology starts with a Taylor series expansion of the value of the option (V_t):

$$V_{t+n} \approx V_t + d\left(NPV_{t+n} - NPV_t\right) + 0.5g\left(NPV_{t+n} - NPV_t\right)^2 + q\left(t_{t+n} - t_t\right) \quad (8)$$

Where:

$$d = \frac{\partial V}{\partial NPV}, \; g = \frac{\partial^2 V}{\partial NPV^2} \text{ and } q = \frac{\partial V}{\partial t}$$

The *delta* captures the change in the value of the option due to changes in the price of the underlying. The *gama* is a measure of the changes in the *delta* due to changes in the price of the underlying, i.e., the second derivative. The *teta* of the option measures the changes of the value of the option due to change in time. Equation 8 may be written as:

$$\frac{V_t}{NPV_t}\left(\frac{V_{t+n} - V_t}{V_t}\right) = d\left(\frac{NPV_{t+n} - NPV_t}{NPV_t}\right) + 0.5gNPV_t\left(\frac{NPV_{t+n} - NPV_t}{NPV_t}\right)^2 + \frac{q}{NPV_t}\left(t_{t+n} - t_t\right) \quad (9)$$

Define:

$$R_V = \frac{V_{t+n} - V_t}{V_t}, \; R_{NPV} = \frac{NPV_{t+n} - NPV_t}{NPV_t}, \; n = t_{t+n} - t_t \;, \; h = \frac{NPV_t}{V_t},$$

$$\tilde{d} = dh \text{ and } \tilde{g} = ghNPV_t$$

Note that R_V is the return on the value of one option, R_{NPV} is the return of the underlying, n is the number of periods of time and h is a measure of leverage as it is the ratio between the asset price and the value of the option.

Based on Riskmetrics (1996) we have:

$$s^2_{NPV,t} = \tilde{d}^{'} M \tilde{d} + 0.5trace\left[\left(\tilde{g} M\right)^2\right] \quad (10)$$

[7] The RiskMetrics™ is one of the most popular systems for market risk management. Market risk literature also has a vast number of examples of RiskMetrics model evaluation.

Where M is the covariance matrix of the NPV's. s is the volatility and the subscript ' means the operation of transposing a vector since $\tilde{d}$ is a vector of the *deltas* multiplied by the h 's. So, it is easy to show that:

$$\tilde{d}' M \tilde{d} = \sum_{i=1}^{Kstates} \tilde{d}_i^2 s_i^2 + \sum_{i=1}^{Kstates} \sum_{\substack{j=1 \\ j \neq i}}^{Kstates} \tilde{d}_i \tilde{d}_j s_{i,j}$$

And that:

$$trace\left[\left(\tilde{g} M\right)^2\right] = \sum_{i=1}^{Kstates} \tilde{g}_i^2 s_i^4 + \sum_{i=1}^{Kstates} \sum_{\substack{j=1 \\ j \neq i}}^{Kstates} \tilde{g}_i \tilde{g}_j \left(s_{i,j}\right)^2 \quad (11)$$

Where $s_{i,j}$ is the covariance between investment in the stadiums *i* and *j*. The equations above show that regarding covariance matrix the highest value of $s^2_{NPV,t}$ is achieved if all the investments are perfectly positively correlated. If not, the variance of the portfolio will be lower, as we want to prove.

CONCLUSION.

It is clear that investment in sports facilities such as football stadiums is subject to uncertainty which requires appropriate tools. In spite of real options models being a well known approach for dealing with investments under uncertainties, it is not an easy task to apply such models in practical situations. In fact, real option models require the definition of a stochastic behavior for a variable in order to estimate the probability distribution for an underlying asset. Furthermore, it is important to identify all the options and flexibilities in a given investment. Therefore the work behind real options is far more complex than that of traditional approaches, such as the Net Present Value. Nevertheless, real options may shed additional light on essential aspects of the viability of public-private partnerships in the construction of sports facilities, namely addressing critical risks that may undermine the feasibility of the venture and the role of the government in attracting private actors to invest at same time minimizing tax payer expenditure.

In this vein, our paper contributes to sports management and economics literature by discussing a real options application in the context of football stadiums. In particular the paper suggests a way for the government to assure some guarantees for private sector in the presence of demand risk, which is a major issue in PPP in general. This alternative is the concession of investments near the stadiums that may reduce the uncertainty in the cash flow based on the diversification effect. In this paper we show that the aggregation of a shopping center in the investment may reduce uncertainties and may also reduce the risk premium. The real option approach allows the valuation of this option that may be given to private investors to stimulate them to invest.

We also discuss the possible effects of a fund coordinated by central governments to manage the guarantees of the several cities that are to host the FIFA World Cup™ matches. We demonstrate that a central fund will be more efficient from a risk management perspective than several funds coordinated by local governments. This is certainly a central issue for the governments who assume the responsibility for organizing such events.

The tight schedule for performing the necessary interventions forces the governments to stimulate private investment as soon as possible. As the FIFA quality standards and FIFA deadlines are widely known, private investors may bargain in order to mitigate their share in the investment, therefore increasing the government participation. Collusion among bidders, imperfect competition and government capture are some factors that may contribute to socially undesired outcomes as the tax payers burden is likely to increase.

The economic benefits from major sports events are controversial. It will be difficult to support the organization of events if the risks in investment are not allocated in a fair way. This signals precautions mainly in developing countries where the social cost of opportunity is huge as the number of structural problems to solve remains very high. Future studies can address this subject.

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Annex I – Data for NPV calculation

Item	Investiments (R$)
Stadium	136,907,000
Shopping Center	82,000,000
External area of the stadium	46,109,800
External public area	77,459,050
Roads	4,871,400
Total	347,347,250

Inflation = 4,5%

Discount rate = 13%

Risk free rate = 9%

Leverage = 60% of the investment

Borrowing interest rate = 7%

Maturity of the loan = 20 years

Concession period = 35 years

Ticket prices of the national championship = R$ 20

Ticket prices of the local championship = R$ 10

Ticket prices for special games = R$ 15

Net revenues shows = R$ 6.800.000

Net revenues with publicity = R$ 5.000.000

Net revenues with bars and restaurants = R$ 3.000.000

Net revenues with parking = R$ 1.400.000

Maintenance costs = R$ 2.000.000

Estimated cash flow

Year	Cash flow
2009	-27787780.00
2010	-30705496.90
2011	-33623213.80
2012	-36540930.70
2013	-39458647.60
2014	26424409.78
2015	18301823.14
2016	20676796.94
2017	23132385.11
2018	25672215.29
2019	28300078.38
2020	31019935.86
2021	33835927.47
2022	36752379.25
2023	39773811.91
2024	42904949.58
2025	46150729.00
2026	49516309.04
2027	53007080.74
2028	56628677.70
2029	60386987.08
2030	64288160.92
2031	68338628.14
2032	72545106.93
2033	76914617.82
2034	81454497.24
2035	96009285.90
2036	100329703.77
2037	104844540.43
2038	109562544.75
2039	114492859.27
2040	119645037.94
2041	125029064.64
2042	130655372.55
2043	136534864.32

Annex II – Annual average of Bahia and Vitoria matches in the city of Salvador

Year	Demand
1972	40,086
1973	41,255
1974	32,336
1975	40,808
1976	42,809
1977	52,148
1978	51,421
1979	24,506
1980	42,040
1981	57,792
1982	41,889
1983	33,748
1984	22,180
1985	41,497
1986	67,478
1987	38,534
1988	46,327
1989	17,091
1990	36,162
1991	30,595
1992	7,728
1993	44,645
1994	27,964
1995	30,126
1996	35,302
1997	46,176
1998	11,461
1999	16,963
2000	38,717
2001	36,102
2002	29,044