Public-private contracting under limited commitment: from fixed to state-dependent duration

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Preliminary version

Abstract

A government delegates a build-operate-transfer project to a private firm. At the contracting stage, parties are uncertain about the operating cost. The firm can propitiate the realization of a low cost by exerting effort while building the facility; once this is in place, it learns the true cost and begins to operate. Under limited commitment, some party, as dissatisfied with the attained payoff, may renege on the contract during the operation phase. With regards to frameworks of this kind, Danau and Vinella (2012) study the financial structure of the project for which the *fixed-term* contract that stipulates the efficient allocation is enforced. In this paper, we broaden the analysis to a *flexible-term* contract, allowing for the duration to be conditioned on the cost realization. We identify the benefits that can be derived and delimit the circumstances under which they become available. We highlight that, by extending (resp. shortening) the contract when the cost is low (resp. high) and inducing more or less dispersion in the distribution of the firm's profits, it is possible to lessen the more serious between the moral-hazard and the enforcement problem, if not both.

Keywords: Fixed- and flexible-term contract; state-dependent duration; moral hazard; adverse selection; full and limited commitment; enforcement

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1 Introduction

Consider the following story. A government delegates a build-operate-transfer project to a private firm. At the contracting stage, they do not yet know whether the operating cost will be low (the favourable state) or high (the unfavourable state). The firm can propitiate the realization of the low cost by exerting some non-contractible but costly effort while building the facility. Once the latter is in place, the firm learns the realized cost (the true state) privately and begins to produce. Two evolutions can be envisaged thereafter. When parties fully commit to their obligations (*full commitment*), the contract is executed till the termination date agreed upon. When parties do not commit (*limited commitment*), either the government or the firm, being dissatisfied with the resulting payoff, reneges on the contract; then, a new negotiation takes place, leading either to a new agreement or to break-up of the partnership.

This story closely mirrors a huge number of episodes in public-private contracting, those with the renegotiation epilogue being far more frequent. Although they occur mostly in developing countries (Banerjee *et alii* [1], Estache and Wren-Lewis [8], Guasch [10], Guasch *et alii* [11] - [12]), they are also widespread in transition economies (Brench *et alii* [2]) and even in developed countries (Gagnepain *et alii* [9]). The massive reliance on partnerships with private partners and, above all, the pervasiveness of renegotiation phenomena naturally open the broad question of how contracts that discipline public projects delegated to private firms should be designed. This question is especially delicate in limited-commitment frameworks, in which contracts must be self-enforcing for the targeted outcomes to be attained.

While addressing this multifaceted issue, one primary aspect to consider is the role that the financial structure plays in the concerned projects, and the way in which it could be used as a commitment device to propitiate contract enforcement when parties do not commit. This aspect is studied in Danau and Vinella [4]. They find that, under full commitment, the exact mix of public and private (own and borrowed) funds is irrelevant, whereas a requirement is imposed by the moral-hazard problem on the duration of the contract. Under limited commitment, the way in which funds are combined and quantified does prove core to warrant enforcement of the contract that entails the efficient outcome (*i.e.*, efficient production and surplus extraction) and yet, through this channel, new requirements may add up to the choice of the contractual length.

From these findings, it is clear that duration is another aspect of utmost relevance in public-private contracting. In spite of this, it is still under-explored in the economic literature. Danau and Vinella [4] account for it focusing on a *fixed-term* contract, that has the same duration no matter the true cost. Although this is insightful, it yields only a partial picture of how duration could be used to accomplish advisable contractual outcomes and/or promote commitment. For a more instructive investigation a wider approach should be adopted, beyond the fixed-term paradigm.

In this paper, we strive to make progress in the study of the duration of contracts stipulated between governments and private partners, particularly in limited-commitment environments. At this aim, we broaden the previous analysis allowing for duration to be conditioned on the cost realization. This is referred to as a *flexible-term* contract. A comparison of our results with those obtained in the earlier study unveils the attainments of state-dependent duration in the frameworks at stake. It is, of course, not surprising that having more flexibility in the choice of duration is potentially useful. Nevertheless, it is important to identify the exact benefits that can be derived and delimit the specific circumstances under which they become available, and all the more that flexibility in duration does not result in one sole contractual option.

The idea of conditioning the duration of the contract on the realized state has previously appeared in the literature on public-private contracting with regards to contexts of *limited enforcement*, in which only the firm does not commit. Referring to situations where, as in our setting, the outcome of the delegated project is uncertain at the contracting stage, Engel *et alii* [6] - [7] put forward the flexible-term contract as a tool to prevent the firm from seeking a new negotiation *ex post*.[1] They propose that duration and, indirectly, profits be modulated across possible states so that the firm obtains exactly the same payoff no matter the state that is realized, eventually. A compensation scheme so structured imposes no risk on the firm. This enables the government to persuade the partner to honour the contract even when operating conditions come out to be unfavourable.[2]

In our work, while endorsing the idea of tieing the duration of the contract to the realized state, we push the analysis further to explore the possibilities that this yields in a more complex environment. As compared to the papers aforementioned, additional complexity to our framework comes from two core circumstances. First, moral hazard arises at the construction stage and adverse selection as soon as the facility is in place. Because of this, even the "ideal" full-commitment world is here far from perfect. Second, moving away from that world, lack of commitment concerns not only the firm but also the government *i.e.*, limited enforcement comes along with *non-commitment*.[3] Both sources of complexity are far from innocent, and their implications must be well understood for the motivation and the reach of our study to be fully appraised.

Both moral hazard and adverse selection require differentiating the (present value of the stream of future) profits between states. A first divergence, thus, arises with respect to the contractual solution proposed by Engel *et alii* [6] - [7]. That is, in the setting that we consider,

[1] In Engel *et alii* [6] - [7], uncertainty concerns market demand rather than production cost.

[2] In a subsequent work (Engel *et alii* [5]), the ultimate benefit of the flexible-term contract is identified in that it removes the need to inefficiently deplete public funds at the aim of inducing private firms involved in public projects to stick to the contract and operate even in bad states.

[3] See Estache and Wren-Lewis [8] for the labels reported in the text.

the firm cannot be assigned the same compensation in the two states. Then, not surprisingly in the presence of moral hazard, for any given level of dispersion induced in the profit distribution, the contract must last *sufficiently long* for the firm to enjoy the benefits delivered by the initial effort, and longer the higher that level.[4] However, the need to account for the two information problems at once may make it difficult to induce an adequate level of profit dispersion and, consequently, to pick an adequate duration. On the one hand, the harsher moral hazard, the more risk must be imposed on the firm to prevent shirking, the more dispersed the compensation scheme must be made. On the other hand, the firm can be persuaded to release information only if profits do not diverge too much between states. Albeit this latter requirement is relaxed, to some extent, as the contract is extended, when moral hazard is especially harsh, it may yet be impossible to design a compensation scheme and select a duration that trigger information release, together with effort provision, without leaving any rent.

Moreover, under limited commitment, it might be necessary to *shorten* the contract to make the partnership viable. This is due to the circumstance that, under the contract, the firm is compensated "as time passes by," whereas, should the partnership break down, the government would incur a loss of reputation/credibility that is higher the earlier the interruption. Hence, parties' *ex-post* payoffs, under the contract and in the alternative event of a new negotiation, depend heavily upon the contractual length. The need to shorten the contract to favour enforcement contrasts with the need to extend the contract enough over time to prevent shirking *ex ante*. In extreme cases, it might be impossible to identify a suitable duration, that reconciles the enforcement concern with the moral-hazard problem.

The potential for these two conflicts (between moral hazard and adverse selection, the former; between moral hazard and lack of commitment, the latter), which are both linked to how the duration of the contract is set, naturally suggests that having some more flexibility in the choice of duration could enhance contractual achievements, on the one hand, and facilitate implementation, on the other. Our study confirms that these improvements are at hand, indeed, and shows how they are attained.

Everything boils down to the level of dispersion that is induced in the firm's profits and to the way in which this is related to the termination date. It turns out that, by letting the contract last *longer* (resp. *shorter*) when a *low* (resp. *high*) cost is realized, rather than imposing a fixed duration, it is possible to either *enlarge* or *narrow* the profit wedge that the firm is faced with. One or the other outcome is achieved, depending upon how the compensation scheme is exactly drawn. Specifically, if the wedge is to be enlarged, then the compensation must be calibrated on the duration of the contract in the bad state, which is shorter. Otherwise, it must be calibrated on the duration in the good state, which is longer. The two options may be useful

[4] Compare Iossa and Martimort [14], for instance, on this point.

in different contexts.

To begin with, put commitment concerns aside and concentrate on moral hazard and adverse selection. Recall that, while the former only introduces a lower bound to the admissible wedge, the latter also imposes an upper bound. When moral hazard is especially serious, the two bounds are not compatible. Here is where setting a longer duration in the good state and enlarging the profit wedge can prove helpful. Actually, we show that this strategy does relax the upper bound that adverse selection imposes on the wedge. Hence, the two information problems are more easily reconciled. It means that the efficient outcome is possibly attained even when moral hazard is so harsh that this outcome would not be at hand with a fixed duration.

Now add up commitment concerns. As these are crucial, in general, in the frameworks of our interest, it is essential to account for the consequences that making profits more dispersed has in terms of contract enforceability. Lack of commitment concerning both the firm and the government, there are two requirements, related to the duration of the contract, that must be satisfied for the latter to be sustainable (compare Danau and Vinella [4]). First, the firm must be sufficiently wealthy to be able to contribute at least the minimum amount of resources for which it prefers honouring the contract, even when it operates at a high cost and obtains low profits, rather than foregoing the quota of those same resources that would remain locked in the project. Second, the government must bear a sufficiently high loss of reputation/credibility when the partnership breaks down that it prefers sticking to the contract, even when the firm operates at a low cost and grasps high profits. Importantly, the firm's necessary endowment is bigger the longer the duration of the contract; in turn, the loss incurred by the government is higher the longer the residual contractual period. Differentiating duration between states to make the compensation scheme riskier for the firm means further decreasing profits when the cost is high and further raising them when the cost is low. Intuitively, this may tighten both the requirement on the firm's endowment and that on the government's loss. In fact, the former event is escaped, provided that duration is set shorter in the bad state. The latter is or not escaped depending upon how sharply the loss increases with the residual period in the good state. When it increases very sharply, extending the contract in the good state to enlarge the profit wedge even facilitates enforcement. It means that this contractual strategy yields also a second benefit, in that particular case.

Of course, this is not true in other limited-commitment situations, where imposing more risk on the firm, which involves raising its compensation in the favourable state, exacerbates the government's reluctance to abide by the contract when that state is realized. To lessen this problem, the wedge should be narrowed, instead. That is, the second contractual option mentioned above should be adopted. Because, as we said, informational asymmetries impede that the firm be assigned the same payoff in both states, here narrowing the wedge never means nullifying it. However, even if it were unnecessary to impose any risk for the firm not to exploit the informational advantage, hence the profit wedge could be decreased to zero, the firm's incentive to renege on the contract would not be removed. As stopping the partnership beforehand and switching to a new operator is costly to the government, the firm might still be able to extract some surplus by inducing the government to a new negotiation. It follows that, in our framework, the flexible-term contract that Engel *et alii* [6] - [6] put forward would not be an effective tool to prompt enforcement on the firm's side, even if information problems were absent.

Just as enlarging the profit wedge helps with moral hazard *ex ante* but may make contract enforcement more difficult, symmetrically, narrowing the wedge makes the contract more viable *ex post* but may weaken the incentives to exert effort at the construction stage. This points to the conclusion that, while yielding advantages, flexible-term contracts are yet not free from shortcomings. In fact, this looks rather natural in complex environments, where coexistence of a number of (potentially conflicting) problems leaves little room for one-for-all recipes.[5] It is, nonetheless, understood that there is no duty to follow one or the other strategy, or to make, at all, the contractual term flexible. Depending upon how serious information and enforcement problems are, in relative and absolute terms, one is free to select the option that appears more appropriate, or even to stick to the fixed-term contract, if the latter already secures the targeted outcome.

1.1 Outline

The remainder of the article is organized as follows. In section 2, we describe the model. In section 3, we consider the full-commitment scenario and highlight a first benefit of state-dependent duration. In section 4, we turn to the limited-commitment framework and describe the renegotiation game. In section 5, analysing contract enforcement under limited commitment, we emphasize further attainments of state-dependent duration. Section 6 briefly concludes. Mathematical details are relegated to an appendix.

As the framework that we represent and the story that we tell echo those in Danau and Vinella [4], the part of analysis that is drawn thereof is reported synthetically. We only insist on the aspects that are core to a comprehensive understanding of the very contribution of the current study. Interested readers are advised to refer to the previous paper for further details

[5] The finding that flexible-term contracts may have undesired effects echoes previous results of the literature. With regards to situations where uncertainty about operating conditions is *not* dissipated at the outset of the operation phase, Danau [3] shows that the expected duration of a flexible-term contract yielding to the firm its reservation payoff is longer than the (certain) duration of a fixed-term contract yielding to the firm that same payoff (in expectation), and that this effect becomes more pronounced as uncertainty increases over time.

on the conceptual approach, the analytical setup and the relation to the pertaining literature.

2 The model

We consider the contractual relationship between a government G and a private firm F for the realization of a project that consists in two tasks, namely the construction of a facility and the provision of a good (or service) to the collectivity. The project unfolds over two stages. At the first stage, which takes place at date 0, the facility is financed and built (the *construction phase*). At the second stage, which begins soon thereafter and lasts till date $T > 0$, the facility is used to provide the good (the *operation phase*). At date T, the contract ends and the infrastructure is transferred to G, which then runs the activity, say, through a public firm.

Technology, production, consumer surplus, demand At time 0, F bears the sunk cost $I > 0$ and exerts effort $a \in \{0, 1\}$ to build the facility. Effort yields the disutility $\psi(a)$, with $\psi(0) = 0$ and $\psi(1) = \psi > 0$. It is unobservable to both G and third parties and cannot be contracted upon. At each instant $\tau \in (0, T)$, F provides $q \geq 0$ units of the good incurring a marginal cost θ and a fixed cost K. Exerting effort $a = 1$ propitiates the realization of a low marginal cost. As a return from production, F receives a transfer t from G and collects revenues $p(q)q$ on the market. Consumption of q units of the good yields instantaneous gross surplus $S(q)$, such that $S' > 0$, $S'' < 0$, $S(0) = 0$ and the Inada's conditions are satisfied. Consumers cannot store the good and transfer consumption to future periods. The output produced at some given τ is entirely consumed at that same date and sold on the market at price $p(q) \equiv S'(q)$. This defines the inverse demand function. Once the investment is made, technology and demand parameters remain constant for the whole duration of the project.

Information structure The contract between G and F is signed, the investment I made, the effort a exerted and the disutility $\psi(a)$ borne *ex ante i.e.*, when the value of θ is unknown to either party. At the contracting stage, it is commonly known that θ will be either low (θ_l) or high (θ_h) with probabilities ν_1 and $1 - \nu_1$ if $a = 1$, ν_0 and $1 - \nu_0$ if $a = 0$. Let $\Delta\nu = \nu_1 - \nu_0 > 0$, meaning that exerting effort at the construction stage makes the realization of θ_l more likely. We denote $\Delta\theta = \theta_h - \theta_l > 0$ the degree of uncertainty about the value of θ. Once the facility is in place, F observes the state of nature $i \in \{h, l\}$ privately and begins to produce.

Project financing To finance the investment, F injects an amount $M \in [0, E]$ of own funds, E denoting its resource endowment, and borrows $C \geq 0$ on the credit market. G makes an up-front transfer $t_0 \in \mathbb{R}$ to F such that

$$M + C + t_0 = I. \tag{1}$$

When $t_0 < 0$, the project is entirely financed with private funds and the contribution of F includes a fee for being awarded the contract.

2.1 Payoffs under complete information

Suppose, for a while, that not only F but also G knows the effort provided in construction as well as the marginal cost of production. We present the parties' payoffs in this environment, for some given value of θ observed at the outset of the operation phase.

The payoff of F Let $d \geq 0$ be the repayment that F makes to the lender L at each instant $\tau \in (0, T)$ in return for the amount of money C received initially. For the given θ, F obtains the instantaneous operating profit $\pi = t + p(q)\,q - (\theta q + K) - d$. Further denoting r the discount rate, the present value at date τ of the whole stream of profits till date T is given by

$$\Pi_\tau = \int_\tau^T \pi e^{-r(x-\tau)} dx.$$

The payoff of F is the net present value of the project:

$$\widetilde{\Pi} = \Pi_0 - (M + \psi(a)).$$

The payoff of G G is a benevolent government that aims at maximizing the discounted consumer surplus generated under both private and public management, net of the market expenditures and the social cost of transferring resources from taxpayers to producer. To finance the transfers, G needs to raise distortionary taxes. Each transferred euro requires collecting $1 + \lambda$ euros from taxpayers, with $\lambda > 0$. The imperfections of the taxation system are taken not to vary over time, hence λ to remain constant. The discounted benefit of G over the period (τ, T) is formulated as

$$V_\tau = \int_\tau^T w(q) e^{-r(x-\tau)} dx - (1 + \lambda)(\Pi_\tau + D_\tau),$$

with $w(q) \equiv S(q) + \lambda p(q) q - (1 + \lambda)(\theta q + K)$ and $D_\tau = \int_\tau^T de^{-r(x-\tau)} dx$ the value of the debt of F at date τ. The credit market is competitive and populated by a large number of lenders, each facing zero outside opportunity, so that $D_0 = C$. Accordingly, the benefit of G under private management is expressed as

$$\widetilde{V}=\int_0^T w(q)e^{-rx}dx-(1+\lambda)\left(\Pi_0+I-M\right).$$

No additional investment is necessary to continue the activity after the end of the contract and the production technology is related to the inner characteristics of the facility. Once this is in place, the marginal cost of production is the same no matter who runs the activity. Under these circumstances, at date T, the optimized return of G from public management is equal to $\int_T^\infty w^* e^{-r(y-T)}dy$, where $w^*\equiv w(q^*)$ and q^* is the output level that maximizes $w\left(q\right)$. This is defined by the Ramsey-Boiteux condition

$$\frac{p(q^*)-\theta}{p(q^*)}=\frac{\lambda}{1+\lambda}\frac{1}{|\varepsilon(q^*)|},\tag{2}$$

with $\varepsilon(q)\equiv\left(dp(q)/dq\right)q/p(q)$ the price elasticity of market demand when quantity is q. Given θ, the overall payoff of G is formulated as

$$W=\widetilde{V}+\int_T^\infty w^*e^{-ry}dy.$$

Optimized payoffs Under complete information, G requires that F both exert effort in construction, provided that this is desirable, and produce the output level pinned down by (2), at which $w\left(q\right)$ attains the largest value, without leaving any surplus. For the given value of θ :

$$\Pi_0^*=M+\psi\quad\text{and}\quad W^*=\int_0^\infty w^*e^{-ry}dy-(1+\lambda)\left(I+\psi\right).$$

2.2 Contracts

Two contracts are involved in the partnership. One regulates the relationship between G and F, the other that between F and L.

2.2.1 The contract between G and F

G makes a take-it-or-leave-it offer to F. First, this specifies the financing triplet (M,C,t_0) that will be devoted to fund the investment at date 0. Second, to address the adverse-selection problem, the Revelation Principle can be invoked and attention restricted to direct mechanisms under which F releases private information. At this aim, the menu of allocations and termination dates $\left\{\left(q_l,t_l;T_l\right);\left(q_h,t_h;T_h\right)\right\}$ is included in the offer, with q_i the quantity to be produced and t_i the transfer to be made at each instant τ till date T_i in the event that the realized cost is θ_i. That is, G conditions the instantaneous allocation and the overall duration on the realized state

$i \in \{l, h\}$, to be publicly revealed by the report that F will deliver at the outset of the operation phase. From now on, the subscript i will be appended to all variables that are contingent on the realized state. Assuming that exertion of effort is desirable from the viewpoint of G, the latter faces also a moral-hazard problem.[6] Provided effort has a stochastic impact on θ_i, conditioning the allocation on the state allows G to address this concern as well.

2.2.2 The credit contract

Consistently with the deal made with G, the credit contract stipulates the amount of money C that F is to borrow from L and invest in the project at date 0. Additionally, the contract fixes the repayment d_i that, in state $i \in \{l, h\}$, F will make to L at each instant of the operation phase till date T_i. This is set to yield neither a surplus nor a loss *i.e.*,

$$\mathbb{E}_i [D_{i,0}] = \mathbb{E}_i \left[d_i \frac{1 - e^{-rT_i}}{r} \right] = C. \tag{3}$$

3 Full commitment

We begin by characterizing the optimal contract between G and F in a framework where they both commit to their reciprocal obligations and, on top of that, F commits to its obligations *vis-à-vis* L. The optimal credit contract is pinned down accordingly. To perform the analysis, we proceed to a standard change of variables and refer to the pair of discounted cumulated profits $\{\Pi_{l,0}, \Pi_{h,0}\}$, rather than to the pair of instantaneous transfers $\{t_l, t_h\}$.

3.1 Fixed duration

As a first step, we shortly recall what happens when the contract has a fixed duration $T_l = T_h \equiv T$. In this context, G makes the contractual offer $\{(M, C, t_0)\,; (q_l, \Pi_{l,0})\,, (q_h, \Pi_{h,0})\,; T\}$ and the instantaneous debt repayment is such that $\mathbb{E}_i [d_i] = rC/ \left(1 - e^{-rT}\right)$.

[6] As usual, effort provision is desirable as long as the expected gain from effort exceeds the cost of inducing effort. At the Ramsey-Boiteux quantities, this means that $\mathbb{E}_i [w_i^*] - \widetilde{\mathbb{E}}_i [w_i^*] > r\psi$, where $\mathbb{E}_i$ (resp. $\widetilde{\mathbb{E}}_i$) is the expectation operator over the two states l and h, corresponding to $a = 1$ (resp. $a = 0$).

To pin down the whole set of variables optimally, G solves the following programme:

$$\underset{\{(M,C,t_0);(q_l,\Pi_{l,0}),(q_h,\Pi_{h,0});T\}}{Max} \mathbb{E}_i\left[W_i\right]$$

subject to (1) as well as

$$\Pi_{i,0} \geq \Pi_{i',0} + \int_0^T \Delta\theta q_{i'} e^{-rx} dx,\ \forall i \neq i' \in \{l,h\} \tag{4a}$$

$$\Pi_{l,0} - \Pi_{h,0} \geq \frac{\psi}{\Delta\nu} \tag{4b}$$

$$\mathbb{E}_i\left[\Pi_{i,0}\right] \geq M + \psi. \tag{4c}$$

In this programme, (4a) is the incentive-compatibility constraint whereby a firm of type θ_i not be tempted to choose the quantity-profit pair designed for a firm of type $\theta_{i'}$; (4b) is the moral-hazard constraint whereby F not be tempted to shirk at the construction stage; (4c) is the participation constraint, implicitly taking the best outside opportunity of F to be zero.

Take the disutility of effort to be so small that

$$\psi \leq \Delta\nu\Delta\theta\frac{q_l^*}{r}. \tag{5}$$

Then, the quantity is optimally pinned down equal to the efficient (Ramsey-Boiteux) level q_i^* for each possible cost value θ_i. The expected profit is set to saturate (4c) so as to retain all surplus *ex ante i.e.*, $\mathbb{E}_i\left[\Pi_{i,0}^*\right] = M + \psi$. The pair of optimal profits satisfying (4a) and (4b) is any pair

$$\Pi_{l,0}^*(z) \equiv M + \psi + (1-\nu)\int_0^T \Delta\theta z e^{-rx} dx \tag{6a}$$

$$\Pi_{h,0}^*(z) \equiv M + \psi - \nu\int_0^T \Delta\theta z e^{-rx} dx \tag{6b}$$

determined by picking the "sharing rule" (z,T) such that $z \in Z \equiv \left[\max\left\{\frac{r\psi}{\Delta\nu\Delta\theta}, q_h^*\right\}, q_l^*\right]$ and, accordingly,

$$T \geq \underline{T}(z) \equiv \frac{1}{r}\ln\frac{\Delta\nu\Delta\theta z}{\Delta\nu\Delta\theta z - r\psi}, \tag{7}$$

which means letting the contract last long enough to ensure that the effort of F is compensated.

As long as (5) holds, G can solve the adverse-selection problem by inducing a profit wedge that is not smaller than required to also solve the moral-hazard problem at least when the contract lasts forever. Under this circumstance, any duration complying with (7) allows G to reconcile the two information issues. At the limit, when (5) is satisfied as an equality, only an infinitely long duration makes the job. By contrast, when (5) is violated, there is clearly no way to motivate F to exert effort by means of the compensation scheme in (6a) and (6b).

Under (5), G reaps, in expectation, the same net benefit that it would obtain from the project if the realization of θ were publicly observed at the outset of the operation phase *i.e.*,

$$\mathbb{E}_i\left[W_i^*\right] = \int_0^\infty \mathbb{E}_i\left[w_i^*\right] e^{-rx} dx - (1+\lambda)(I+\psi).$$

This result is attained no matter how exactly M, C and t_0 are mixed to fund the project, once it is ensured that (1) holds.

For the sake of shortness, we denote $\Psi \equiv \left\{(M, C, t_0); (q_l^*, \Pi_{l,0}^*(z)), (q_h^*, \Pi_{h,0}^*(z)); T\right\}$ the fixed-term contract that stipulates the efficient allocation for some $z \in Z$, with T and (M, C, t_0) fulfilling (7) and (1), respectively.

3.2 State-dependent duration

We now turn to explore the framework of our interest, in which the duration of the contract is conditioned on the realized state of nature. As we said, in this environment, G makes the contractual offer $\{(M, C, t_0); (q_l, \Pi_{l,0}; T_l), (q_h, \Pi_{h,0}; T_h)\}$. Constraints in the programme of G are as before except for (4a), which is replaced by

$$\Pi_{i,0} \geq \Pi_{i',0} + \int_0^{T_{i'}} \Delta\theta q_{i'} e^{-rx} dx, \ \forall i \neq i' \in \{l, h\}. \tag{8}$$

We stressed that, under full commitment, the optimal fixed-term contract does secure the best available outcome as long as ψ is small enough that there exists a sharing rule $z \in Z$ under which F can be motivated to exert effort. That is, (5) must hold. Let us stick to this case, for the time being. Then, there is clearly no gain to expect, under full commitment, from conditioning duration on the true cost. Not surprisingly, the quantity is still optimally pinned down equal to the efficient level q_i^* for each possible cost value. Surplus is still entirely retained from F *ex ante.* The exact capital structure does not matter in the attainment of the largest payoff $\mathbb{E}_i\left[W_i^*\right]$. There is, nonetheless, something new in the contract, as we now illustrate.

The novelty resides in the pair of optimal profits satisfying (4b) and (8), which is any pair

$$\Pi_{l,0}^*(z_j, T_j) \equiv M + \psi + (1-\nu_1)\Delta\theta \int_0^{T_j} z_j e^{-rx} dx \tag{9a}$$

$$\Pi_{h,0}^*(z_j, T_j) \equiv M + \psi - \nu_1 \Delta\theta \int_0^{T_j} z_j e^{-rx} dx \tag{9b}$$

defined by picking the sharing rule (z_j, T_j), $j \in \{l, h\}$, such that $z_j \in Z_j \equiv \left[\max\left\{\frac{r\psi}{\Delta\nu\Delta\theta}, q_j^{\min}\right\}, q_j^{\max}\right]$, where

$$q_j^{\min} \equiv \begin{cases} q_h^* \frac{1-e^{-rT_h}}{1-e^{-rT_l}} & \text{if} \quad j=l \\ q_h^* & \text{if} \quad j=h \end{cases} \quad \text{and} \quad q_j^{\max} \equiv \begin{cases} q_l^* & \text{if} \quad j=l \\ q_l^* \frac{1-e^{-rT_l}}{1-e^{-rT_h}} & \text{if} \quad j=h \end{cases},$$

and, accordingly,

$$T_j \geq \underline{T}(z_j) \equiv \frac{1}{r} \ln \frac{\Delta\nu\Delta\theta z_j}{\Delta\nu\Delta\theta z_j - r\psi}. \tag{10}$$

Two observations are in order. First, the state j to which the sharing rule refers does not need to coincide with the state i to actually materialize.[7] Consequently, the restriction imposed by (10) does not need to concern the duration of the contract in the actual state. Second, for G to attain the largest payoff in this environment, it must be the case that

$$\psi \leq \Delta\nu\Delta\theta \frac{q_j^{\max}}{r}, \tag{11}$$

at least for some $j \in \{l, h\}$. Comparing (11) with (5), it becomes clear that, when the best outcome is not at hand with a fixed duration (*i.e.*, (5) is violated), it can still be with a state-dependent duration, provided that an appropriate choice of termination dates is made. That is, it should be the case that $T_l > T_h$. This suggests that, although no specific benefit has been detected so far, it may well be advantageous to make duration contingent on the cost realization and, consequently, have the opportunity of inducing a convenient ranking of durations across states. In fact, as it will become apparent in a moment, a first benefit arises already under full commitment, precisely ensuing from the slack that (11) yields over (5). Other benefits (together with limits) emerge under limited commitment, and will be presented at a later stage.

From now on, we denote $\Psi_j \equiv \left\{(M, C, t_0);\ (q_l^*, \Pi_{l,0}^*(z_j, T_j); T_l),\ (q_h^*, \Pi_{h,0}^*(z_j, T_j); T_h)\right\}$, $j \in \{l, h\}$, the contract that stipulates the efficient allocation with state-dependent duration for some $z_j \in Z_j$, T_j fulfilling (10) and (M, C, t_0) fulfilling (1).

3.2.1 Promoting effort provision with sharing rule (z_h, T_h)

To be prepared to investigate the attainments that are at hand with state-dependent duration, as a preliminary step, it is helpful to clarify what exactly happens with the compensation scheme when the termination date is not forced to be the same in the two states. We hereafter provide some hints on this aspect, relegating mathematical details to Appendix A.1.

Start from situations in which (5) holds and the compensation scheme is designed as in Ψ. The wedge that can be created between profits to solve both the adverse-selection and the moral-hazard problem ranges from a minimum of $\Delta\theta \max\left\{\frac{r\psi}{\Delta\nu\Delta\theta}, q_h^*\right\} \frac{1-e^{-rT}}{r}$ to a maximum of $\Delta\theta q_l^* \frac{1-e^{-rT}}{r}$, T being fixed according to (7). First consider raising duration in the good state.

[7] It is precisely to make this distinction that we use the subscript j to index the sharing rule and the subscript i to denote the state of nature.

Then, by picking the sharing rule (z_h, T_h), the profit wedge can be enlarged to a maximum size of $\Delta\theta q_h^{\max} \frac{1-e^{-rT_h}}{r}$. Next consider decreasing duration in the bad state. Then, by picking the sharing rule (z_l, T_l), the profit wedge can be narrowed to a minimum size of $\Delta\theta \max\left\{\frac{r\psi}{\Delta\nu\Delta\theta}, q_l^{\min}\right\} \frac{1-e^{-rT_l}}{r}$. Of course, duration can be raised in the good state and reduced in the bad state *at the same time.*[8] Even so, in the former case, shortening the contract in state h would not lead to violate the requirement contained in (10) for $j = h$, provided this is weaker the higher z_h. In the latter, the restriction imposed by (10) for $j = l$ is tightened by the decrease in z_l and, yet, this would be matched by the contract extension prescribed in state l.

Now focus on situations in which (5) is violated. Then, it is not possible to find a termination date T for which, when profits (6a) and (6b) are assigned, shirking is prevented. Actually, by sticking to that compensation scheme, G fails to impose enough risk on the firm, for the latter is not motivated to exert effort. To circumvent this difficulty, the spread in the profit distribution is to be enlarged. Provided that (11) holds for $j = h$, this can be made by adopting the sharing rule (z_h, T_h), with z_h larger than q_l^* and duration longer in the good than in the bad state.

Proposition 1 *Suppose that* (5) *is violated, whereas* (11) *is satisfied for* $j = h$ *and* $T_l > T_h$. *Then, under full commitment, the payoff* $\mathbb{E}_i[W_i^*]$ *is attained with profits* (9a) *and* (9b), *together with the triplet* (M, C, t_0) *such that* (1) *holds, if and only if* $z_h \in (q_l^*, q_h^{\max}]$ *and* T_h *and* T_l *are set such that* $T_h \in [\underline{T}(z_h), T_l)$.

Proof. See Appendix A.2. ■

The proposition conveys a neat message. The sharing rule (z_h, T_h) can be conveniently used to construct an appropriate compensation-and-duration scheme that allows G to reach the largest attainable payoff in full-commitment frameworks in which the moral-hazard problem is especially harsh.

4 Limited commitment

Consider now the following framework, characterized by both *non-commitment* and *limited enforcement.* G and F sign the optimal contract. Accordingly, F and L sign the credit contract. However, neither G nor F commits to contractual obligations. Under this circumstance, the contract between G and F is hardly executed till the termination date agreed upon. Indeed, once the true value of θ becomes commonly known, parties may be dissatisfied with the realized payoffs. On top of that, F may stop reimbursing L. This involves that F would be unable to take a loan in the first place. The optimal contract yields an efficient allocation, for it would be

[8] Looking at one or the other change allows for a more immediate comparison with the fixed-term regime and a more intuitive presentation of the associated implications. This is why we find it useful to take this approach here and elsewhere in the text.

desirable to have it enforced. Besides, insofar as a loan is hardly obtained, it may be difficult to undertake the project at all, for budgetary reasons. One thus needs to understand how can the contract be made self-enforcing so that, even under limited commitment, the efficient allocation is implemented for the whole stipulated duration and, if opportune, external financiers can be involved in the project.

The analysis developed in the sequel of this section applies whether duration is fixed or state-dependent. Thus, while presentation is restricted to the latter scenario to avoid redundancy, it should be kept in mind that, *mutatis mutandis*, things carry over when $T_l = T_h \equiv T$ and the concerned contract is Ψ (rather than Ψ_j).

4.1 Conditional guarantees

To secure financiers' participation, debt guarantees can be provided. We allow for this assuming that, when contracts are signed, G guarantees the amount of funds that F is instructed to borrow, taking into account that, at some point, F may stop making payments to L. To make guarantees credible, G can rely on some authoritative third party, such as an Investment Insurance Agency, the World Bank or a multilateral development bank. One can think of G as depositing resources onto one such institution, whose task would then be to release money directly to L, should F suspend transfers. The exact amount that F should be instructed to borrow under the governmental guarantee depends upon the state and date at which the latter would take effect. In principle, an entire profile of debt (and guarantee) levels could result, one for each state-date pair at which F could stop honouring debt obligations. Importantly, guarantees are *conditional i.e.*, they are supposed to come into force only if the relationship with F goes on, whether under the initial or a revised deal.

4.2 The renegotiation game

To identify the conditions under which parties have no interest in reneging on Ψ_j, it is necessary to take into account what would happen, should that scenario materialize. Obvious aim of the reneging party would be to enhance its *ex-post* payoff by means of a new negotiation. Thus, once at some date $\tau \in (0, T_i)$, in some commonly-known state $i \in \{l, h\}$, either F or G reneges on Ψ_j, they come back to the contracting table. If renegotiation fails, F is replaced with another firm F'. If renegotiation succeeds, the relationship between F and G persists under a revised contract.

4.2.1 Break-up of the relationship and replacement of F

When renegotiation fails, the partnership between F and G breaks down. As a consequence, all actors bear a cost. F is relieved of the activity and no longer receives any compensation. This involves foregoing the part of the monetary and non-monetary contribution that F no longer recovers between date τ and date T_i. F has no reason to make further payments to L and the guarantee does not come into force. Hence, in turn, L foregoes the part of the loan that remains unpaid between date τ and date T_i. In other words, both F and L incur an *expropriation cost*, which is larger the earlier the termination. G appropriates the resources of F and L that are locked in the activity, from which it still benefits thanks to the production performed by F'. Nonetheless, symmetrically, it bears a reputation and/or credibility loss, hereafter referred to as the *replacement cost* and denoted R_{δ_i}, with $\delta_i \equiv T_i - \tau$. This cost is, in turn, positively related to the length of the residual contractual period (δ_i). Formally, taking R_{δ_i} to be continuously differentiable on $(0, T_i)$, $R'_{\delta_i} \equiv (dR/d\delta_i) > 0 \; \forall \delta_i \in (0, T_i)$. The cost is positive even in the event that the contract is stopped just before the date originally stipulated *i.e.*, $R_{\delta_i} > 0 \; \forall \delta_i \in (0, T_i)$, with $\lim_{\delta_i \to 0} R_{\delta_i} = \varepsilon > 0$. It only vanishes when $\tau = T_i$ so that $R_0 = 0$. Appending the superscript rp to denote the *rep*lacement scenario, the payoffs of F and G are given by

$$\Pi_{i,\tau}^{rp} = 0 \tag{12a}$$

$$V_{i,\tau}^{rp} = w_i^* \frac{1 - e^{-r\delta_i}}{r} - R_{\delta_i}. \tag{12b}$$

4.2.2 Renegotiation

Following Hart and Moore [13], we assume that, with probability $\alpha \in [0, 1]$, G makes a take-it-or-leave-it offer to F; with probability $1 - \alpha$, F makes a take-it-or-leave-it offer to G. The party that takes the initiative optimally makes the offer that leaves the partner just indifferent between renegotiation and the alternative regime (replacement). F ceases to abide by the reimbursement plan stipulated in the credit contract and lets the guarantee take effect. The payoffs that parties attain are determined accordingly. While F gets the same payoff as under replacement if G makes the offer, it extracts the resources that G would lose in the replacement scenario, net of the guarantee, if it makes the offer. In turn, whoever makes the offer, G obtains the largest gross payoff from consumption of the good. However, this is diminished by the social cost of the surplus that is given up to F when it makes the offer, plus the debt guaranteed to L. Appending the superscript rn to indicate the *ren*egotiation regime, the payoffs of F and G are given by

$$\Pi_{i,\tau}^{rn} = (1-\alpha)\left(\frac{R_{\delta_i}}{1+\lambda} - D_{i,\tau}^{rn}\right) \quad (13a)$$

$$V_{i,\tau}^{rn} = w_i^* \frac{1-e^{-r\delta_i}}{r} - (1+\lambda)\left[(1-\alpha)\left(\frac{R_{\delta_i}}{1+\lambda} - D_{i,\tau}^{rn}\right) + D_{i,\tau}^{rn}\right], \quad (13b)$$

where, to avoid confusion with the notation previously used for debt, $D_{i,\tau}^{rn} = \int_{\tau}^{T_i} d_{i,\tau}^{rn} e^{-r(x-\tau)} dx$ indicates the value at date τ of the debt guaranteed at the contracting stage in the event that the contract is renegotiated in state $i \in \{l, h\}$ at that date and then lasts till date T_i.

Payoffs (13a) and (13b) reflect the implicit assumption that the contract renegotiated at date τ remains in place till date T_i. However, *a priori*, one cannot rule out the possibility of parties negotiating again after date τ. Danau and Vinella [4] show how the profile of debt guarantees should be set to make repeated renegotiation unattractive. This point is not core to the present work, for we do not insist on it and rather privilege aspects that are here more salient.

4.3 The incentives of F and G to renege

Inspection of the payoffs in the renegotiation game highlights the incentives that parties display to come back to the contracting table.

F is aware that break-up of the relationship and replacement with F' would be costly to G. After discovering the true state i, F may attempt to raise its payoff by threatening G to abandon the project and let it bear the replacement cost, unless the initial deal is favorably revised. The temptation to renege is naturally stronger in state h, in which the contractual compensation is lower. By quitting the activity, F would in turn be expropriated (a part of) the contribution made up-front. Even so, the threat of default may still be effective, provided that replacement is costly enough to G and the initial contribution of F relatively modest.

Symmetrically, G is aware that it would be able to (partially) appropriate the initial contribution of F and L if the partnership were to break down. After learning θ_i, G may attempt to raise its payoff by threatening F to stop any compensation, hence prevent investment recovery, and continue the project with a new firm (to which no surplus is to be conceded), unless the contract is conveniently revised. The incentive to renege is stronger in state l, in which G owes a higher return to F and, yet, rewarding F is no longer optimal once information has been released. The change of partner would not come for free to G. The threat is nonetheless credible, provided that sufficient private capital is involved.

5 Contract enforcement under limited commitment

Whether duration is fixed or state-dependent, in the optimal contract, quantities and profits are set at the efficient levels. Hence, establishing conditions under which parties are motivated to abide by their obligations, so that the contract is executed and L repaid, boils down to identifying an appropriate mix of funds and termination date(s) for this to occur, still ensuring that both the financing condition and the moral-hazard restriction on duration are met.

As far as private funds are concerned, one can show that implementation of the contract involves weaker requirements when replacement, rather than renegotiation, is to be prevented. To facilitate enforcement, one should thus find a way to warrant that, were some party to renege, the relationship would be interrupted, eventually. At this aim, it is useful to observe that renegotiation fails or succeeds depending upon the magnitude of $D_{i,\tau}^{rn}$, suggesting that setting properly guarantees makes the job. Specifically, replacement is, indeed, the expected outcome of an ideal renegotiation process if and only if, for each relevant (i, τ) −pair:

$$D_{i,\tau}^{rn} \geq \frac{R_{\delta_i}}{1+\lambda}, \tag{14}$$

with $\delta_i \equiv \delta$ for all $i \in \{l, h\}$ if duration is fixed. When guaranteed debt is set to satisfy (14), any benefit that F and G could obtain by renegotiating in the state $i \in \{l, h\}$ that was correctly announced at the outset of the operation phase is washed out. It is thus natural that they both (weakly) prefer replacement in that same state. Importantly, raising guarantees is not an issue, provided they are only meant to facilitate contract enforcement and, in definitive, to never actually matter at equilibrium. Once (14) is met, one can focus on how F and G could be refrained from breaking up the partnership *i.e.*, on the seek of the *weakest* conditions under which the optimal contract is enforceable.

Identifying those conditions requires making a preliminary step of analysis. This consists in exploring the circumstances under which the two following events occur. First, conditional on F truthtelling on θ_i at the outset of the operation phase, both F and G (weakly) prefer honouring the contract rather than betraying the partnership at some date τ during the operation phase. Second, F (weakly) prefers truthtelling on θ_i at the outset of the operation phase rather than mimicking $\theta_{i'}$ in the perspective that the contract will be reneged at some date τ.

5.1 Fixed duration

As far as the fixed-term contract is concerned, the analysis just described is fully developed in Danau and Vinella [4]. The next proposition is stated to recall the set of weakest conditions, that is thereby derived, under which Ψ is enforceable.

Proposition 2 *Ψ is enforceable if and only if $\exists z \in Z$, $T \in [\underline{T}(z), +\infty)$ such that*

$$R_\delta \geq (1+\lambda)\,\Delta\theta z \frac{1-e^{-r\delta}}{r}, \ \forall \delta \in (0,T), \tag{15}$$

and, additionally,

$$E \geq \nu_1 \frac{\psi}{\Delta\nu} \tag{16}$$
$$C > 0. \tag{17}$$

First, for the contract to be effected, replacement of F must be so onerous that G does not find it attractive. Specifically, for all possible residual periods till the termination date T, the replacement cost must not fall below the value, at date τ, of the profit wedge under the contract (as inflated by the shadow cost of public funds). The latter measures the additional return that G owes to F in the good, relative to the bad state, if it sticks to Ψ at date τ. Second, it is necessary that both own funds of the firm and outside financing be available to run the project. On the one hand, involving own funds reinforces the willingness of the firm to preserve the relationship in order to escape expropriation. Reasonably enough, F should be able to contribute at least the amount of resources for which it is willing to remain in the contract, even in the bad state, for the shortest admissible time length $\underline{T}(z)$. Of course, F is to be wealthier than that if a longer duration is stipulated, in which case more important a contribution is to be called for to make the contract viable on the firm's side. On the other hand, recommending the firm to take a loan creates the opportunity to provide guarantees in favour of the lender. Conditional guarantees lower the benefit that the government could obtain and, symmetrically, the surplus that the firm could extract in the event of renegotiation. Once any perspective of profitable renegotiation is deliberately removed, there is no longer any incentive to come back to the contracting table. Ψ is then enforceable.

The possibility of enforcing Ψ is evidently related to the choice of the termination date through the properties of the replacement-cost function, on one side, and the magnitude of the firm's endowment, on the other. On top of that, the choice of the termination date is functional to address moral-hazard concerns, as previously seen. Thus, ultimately, the range of admissible durations results from how these three determinants combine.

Corollary 1 *Take* (16) *and* (17) *to hold.* (i) *Suppose that*

$$R'_\delta \geq (1+\lambda)\,\Delta\theta z e^{-r\delta}, \ \forall \delta \in (0,T), \ \forall T \in [\underline{T}(z), \infty), \ z \in Z. \tag{18}$$

Then, there exist values of T *for which* Ψ *is enforceable:*

$$\begin{aligned} T &\in \left[\underline{T}(z), \widetilde{T}(z,E)\right] \quad \textit{when } E \in \left[\nu_1 \frac{\psi}{\Delta\nu}, \nu_1 \frac{\Delta\theta z}{r} - \psi\right) \\ T &\in \left[\underline{T}(z), \infty\right) \quad \textit{when } E \geq \nu_1 \frac{\Delta\theta z}{r} - \psi, \end{aligned}$$

where

$$\widetilde{T}(z,E) \equiv \frac{1}{r} \ln \frac{\nu_1 \Delta\theta z}{\nu_1 \Delta\theta z - r(E+\psi)} < \infty. \tag{19}$$

(ii) *Suppose that*

$$R'_\delta < (1+\lambda)\Delta\theta z e^{-r\delta}, \; \forall\delta \in (0,T), \; \forall T \in [\underline{T}(z), \infty), \; z \in Z. \tag{20}$$

Then, there exist values of T *for which* Ψ *is enforceable if and only if* $R_{\overline{T}(z)} \geq (1+\lambda)\psi/\Delta\nu$:

$$\begin{aligned} T &\in \left[\underline{T}(z), \min\left\{\widetilde{T}(z,E); \overline{T}(z)\right\}\right] \quad \textit{when } E \in \left[\nu_1 \frac{\psi}{\Delta\nu}, \nu_1 \frac{\Delta\theta z}{r} - \psi\right) \\ T &\in \left[\underline{T}(z), \overline{T}(z)\right] \quad \textit{when } E \geq \nu_1 \frac{\Delta\theta z}{r} - \psi, \end{aligned}$$

where

$$\overline{T}(z) \equiv \frac{1}{r} \ln \frac{(1+\lambda)\Delta\theta z}{(1+\lambda)\Delta\theta z - rR_{\overline{T}(z)}} < \infty. \tag{21}$$

Two scenarios are possible. Which one arises depends upon how sharply the cost of replacing F increases, relative to the (inflated) cost of sticking to Ψ, which is measured by the profit wedge, as the duration of the contract is extended, for any given τ. In the first scenario (part (i) of the corollary), the replacement cost increases more sharply than the profit wedge. Then, (15) is satisfied no matter how T is picked within the interval $[\underline{T}(z), \infty)$, given the sharing rule $z \in Z$. Under this circumstance, the sole possible restriction on contract duration, other than (7), is related to how deep the firm's pocket is. When F is little wealthy, the contract cannot last too long but there still is room to match (7). In the second scenario (part (ii) of the corollary), on the opposite, the replacement cost increases less sharply than the profit wedge. Then, depending upon how T is picked, (15) may no longer be satisfied. It means that there is potentially less freedom at choosing duration. Implementation of Ψ requires the replacement cost being "sufficiently large" *i.e.*, $R_{\overline{T}(z)}$, with $\overline{T}(z)$ as defined by (21), not falling below the minimum profit wedge that is required to prevent shirking in construction. When this is not so, it is impossible to pick a duration such that the *ex-post* incentive of G to renege is removed together with the *ex-ante* incentive of F to shirk. The admissible lapse of time shrinks to the point that effort provision would be foregone in the first place. Ψ is then unenforceable.

The next proposition states how private funds should be mixed and quantified for Ψ to be effected under limited commitment, provided that the termination date does comply with the requirements of Corollary 1. Once M and C are set, t_0 is also determined, according to (1).

Corollary 2 *Suppose that* (15), (16) *and* (17) *hold. Then,* Ψ *is enforced by choosing* M *and* C *such that*

$$\nu_1 \Delta\theta z \frac{1-e^{-rT}}{r} - \psi \leq M \leq \left(\frac{R_\delta}{1+\lambda} \frac{r}{1-e^{-r\delta}} - (1-\nu_1) \Delta\theta z \right) \frac{1-e^{-rT}}{r} - \psi, \ \forall \delta \in (0, T), \tag{22}$$

together with

$$C \leq \frac{R_T}{1+\lambda} - (M+\psi). \tag{23}$$

F should be required to invest neither too little nor too much. With M too small, F would prefer not to produce; with M too large, G would like to appropriate the partner's investment. F should also be instructed to take a loan, though not encouraged to rely on external financing massively. While the presence of debt paves the way to a convenient use of conditional guarantees, too large C would trigger expropriation, in turn. The higher the amount of own funds picked in compliance with (22), the lower the admissible amount of borrowed funds. Expropriation is escaped provided that the maximum grab that break-up would secure to G (*i.e.*, $M+C+\psi$) does not exceed the largest cost that it could yield (*i.e.*, $R_T/(1+\lambda)$), which is attained if F is replaced as soon as it starts operating.

5.2 State-dependent duration

We now come back to situations where the duration of the contract is made contingent on the realized cost. In Appendix B.1, we derive the conditions under which, provided that F releases information as soon as the facility is in place, both F and G (weakly) prefer honouring the contract rather than breaking up the partnership beforehand. In Appendix B.2, we further derive the conditions under which F has no reason to lie as soon as it discovers θ_i, expecting parties to come back to the contracting table at a later stage. Here, we emphasize the benefits that making duration state-dependent yields in terms of contract enforcement.

At this aim, it is useful to observe that, when some sharing rule (z_j, T_j), $j \in \{l, h\}$, is adopted, the requirement on the replacement cost becomes

$$R_{\delta_l} \geq (1+\lambda) \Delta\theta z_j \frac{1-e^{-rT_j}}{r} \frac{1-e^{-r\delta_l}}{1-e^{-rT_l}}, \ \forall \delta_l \in (0, T_l). \tag{24}$$

This condition mirrors the circumstance that, in terms of enforceability of the contract, what matters is the replacement cost that G faces in the *good* state. The reason is that this is the state in which G is more hardly retained in the partnership. As long as duration is fixed, this aspect is little apparent because G faces an equal replacement cost in the two states. By contrast, it becomes evident when duration is differentiated and replacement costs diverge. As with a fixed duration, (24) requires comparing the cost of replacing F in state l (the left-hand side) with the (social) cost of proceeding with the contract in that same state, which is measured by the value that the profit wedge takes at the time when replacement would occur (the right-hand side).

From now on, for the sake of shortness, we refer to $(24)_h$ and $(24)_l$ when, respectively, $j = h$ and $j = l$.

5.2.1 The possibility of a double dividend with sharing rule (z_h, T_h)

We previously saw that more ambitious outcomes can be attained, under full commitment, if the fixed-term contract is replaced with a flexible-term contract in which the sharing rule (z_h, T_h) is adopted, setting z_h and $T_l > T_h$ according to Proposition 1. Specifically, the wedge between the date-0 profits is widened and more risk imposed on the firm. Consequently, the motivation to shirk at date 0 is lessened even when the disutility of effort is particularly big. While it is clear that this strategy is useful under full commitment, it is less evident whether the same is true under limited commitment. We now investigate this aspect.

From Proposition 2 we learn that, in situations where parties do not commit, the optimal fixed-term contract is enforced only if F is not too poor, on top of being able to borrow money, and replacement of F is sufficiently onerous to G. The former requirement follows from the firm's lack of commitment, the latter from the government's. From Corollary 1 we further learn that, the poorer F and the less costly replacement, the shorter a viable contract. To know whether adopting the sharing rule (z_h, T_h) and setting T_l above T_h facilitates enforcement, one needs to establish the impact that this has on the wealth constraint of F and the cost that replacement would yield to G relative to the continuation of the contract.

Let us begin with the wealth constraint of F. As we explained, requiring the firm to contribute up-front is meant to warrant that, subsequently, it will have an interest in preserving the contract even in the bad state. The higher T_h, the more money F should be invited to contribute, the larger the endowment it needs to hold. When, starting from $T_l = T_h \equiv T$, duration is raised in state l, there is no implication on how tight the wealth constraint is. Hence, this strategy does not make Ψ_h harder to enforce, on the firm's side, as compared to Ψ.[9]

Let us next turn to the replacement and continuation cost. In Appendix B.3, we show that, for any given τ, raising T_l above T_h yields an increase not only in the replacement cost but also

[9] Of course, while raising T_l, T_h could be simultaneously decreased, which would *relax* the wealth constraint of F.

in the cost of sticking to the contract in state l. The relative magnitude of these two increments dictates whether $(24)_h$ is more or less stringent than (15), hence whether Ψ_h is more or less difficult to enforce, on the government's side, as compared to Ψ.

First consider the case in which the cost of replacing F increases much more than the cost of proceeding with the contract in state l *i.e.*, for all $\delta_l, \tau \in (0, T_l)$,

$$R'_{\delta_l} \geq \left(R_{\delta_l} \frac{r\left(e^{-r\tau}-1\right)}{1-e^{-r\delta_l}} + (1+\lambda)\,\Delta\theta z_h \frac{1-e^{-rT_h}}{1-e^{-rT_l}} \right) \frac{e^{-rT_l}}{1-e^{-rT_l}}. \tag{25}$$

Then, the sharing rule (z_h, T_h) yields a double dividend. Not only more serious moral-hazard problems are tackled, still ensuring that the full-commitment contract attains efficiency (Proposition 1). Also, that contract is more easily sustained under limited commitment. This additional benefit is formalized hereafter.

Proposition 3 *Suppose that* (15) *does not hold for* $z = q_l^*$ *and* $T \geq \underline{T}(q_l^*)$. *Then,* $\exists z_h \in (q_l^*, q_h^{\max}]$ *for which* Ψ_h *is enforceable if and only if* $T_l > T_h$ *and* (16) *and* (17) *are satisfied together with* $(24)_h$ *and* (25).

Proof. See Appendix B.3. ■

Next turn to the case in which the cost of replacing F increases less or, at best, slightly more than the cost of honouring the contract in state l (*i.e.*, (25) is violated). This scenario is less favourable as the wish to prevent the firm from shirking *ex ante* exacerbates the commitment problem on the government's side. Then, insisting on the sharing rule (z_h, T_h) is a good idea only if moral hazard is harsh relative to non-commitment. Otherwise, it is convenient to resort to the alternative sharing rule (z_l, T_l). Although this latter rule does not deliver any particular benefit in the full-commitment setting, it comes out to be useful in the limited-commitment framework. This is illustrated hereafter.

5.2.2 Facilitating contract enforcement with sharing rule (z_l, T_l)

Under Ψ_l, the profit wedge at date 0 is narrowed. It means that, even putting aside any commitment issue, the sharing rule (z_l, T_l) yields the best outcome only if the disutility of effort is so small that the compensation scheme in (9a) and (9b), with $j = l$, does allow G to prevent shirking in construction, despite imposing little risk on F. In other words, (5) must hold. Take this to be the case, for it does make sense to investigate the benefit of signing Ψ_l, in the place of Ψ, in a limited-commitment framework.

To begin with, recall that part (ii) of Corollary 1 requires the replacement cost being "sufficiently large" and that, even so, the contract is not viable unless duration is shortened enough, all the more when the firm is little wealthy. Yet, this strategy contrasts with the moral-hazard requirement of letting the contract last so long that F can enjoy the return from effort. When the two conflicting necessities cannot be reconciled, there is no room for implementing Ψ. It is precisely in that case that it can pay to switch to the flexible-term contract Ψ_l.

Proposition 4 *Suppose that* (15) *does not hold for* $z = q_h^*$ *and* $T \geq \underline{T}(q_h^*)$. *Then,* $\exists z_l \in \left[q_l^{\min}, q_h^*\right)$ *for which* Ψ_l *is enforceable if and only if* $T_h < T_l$ *and* (16) *and* (17) *are satisfied together with* $(24)_l$.

Proof. See Appendix B.3. ■

To understand this result, start from the fixed duration T and decrease T_h below $T_l \equiv T$. Following to this change, it becomes easier to retain F in the contract when the cost is high. That is, the firm can be required to contribute less, for the wealth constraint is relaxed. Thus, as a first positive consequence, the limited-enforcement problem is mitigated. On top of that, while reducing T_h has no impact on the replacement cost in state l, which is relevant in terms of contract enforceability, it does narrow the profit wedge at date 0, as we said. The value that this narrower wedge takes at the time when replacement would occur depends upon T_l. With T_l unchanged, decreasing T_h reduces the date$-\tau$ value of the profit wedge, overall, meaning that $(24)_l$ is more relaxed than (15). Because honouring the contract in state l becomes less costly, G is obviously more prone to that.[10] The second positive consequence is, thus, that the non-commitment problem is alleviated. In definitive, the sharing rule (z_l, T_l) attenuates the commitment problem on both the firm's and the government's side. Therefore, as compared to Ψ, Ψ_l is more easily sustained.

In Example 1 below, we take a specific replacement-cost function to further illustrate how making duration state-contingent and targeting the sharing rule (z_l, T_l), with $T_h < T_l$, facilitates enforcement.

Example 1 *First suppose that duration is fixed to* T *and that* $R_\delta = \left(1 - ae^{-r\delta}\right)/r$, *where*

$$a < (1 + \lambda)\,\Delta\theta q_h^*. \tag{26}$$

As $\lim_{\delta \to 0} \left(1 - ae^{-r\delta}\right) = 1 - a > 0$, R_δ *is such that* $\lim_{\delta \to 0} R_\delta > 0$, *consistently with our assumptions. Take* $z = q_h^*$. *Under* (26), *part* (*ii*) *of Corollary* 1 *applies.* Ψ *is not enforceable unless the interval* $\left[\underline{T}(q_h^*), \overline{T}(q_h^*)\right]$ *exists. This occurs as long as*

$$a \leq \Delta\theta q_h^* \frac{\Delta\nu - r(1 + \lambda)\psi}{\Delta\nu\Delta\theta q_h^* - r\psi}. \tag{27}$$

[10] While decreasing T_h, T_l could be simultaneously raised. Albeit this would increase the date$-\tau$ value of the profit wedge, it would also make replacement more costly. Thus, with a proper adjustment of T_h and T_l, $(24)_l$ would still be more relaxed than (15).

Next allow for $T_l \neq T_h$. Suppose that the sharing rule (z_l, T_l) is picked, with T_l such that $T_l > T_h$ and $T_l \geq \underline{T}(z_l)$, and with $q_l^{\min} \leq z_l$ for $z_l = a/(1+\lambda)\Delta\theta$. Enforcement of Ψ_l does not require (27) *being satisfied.*

5.2.3 Capital structure

We identified conditions under which Ψ_j, $j \in \{l, h\}$, is sustainable. Provided that they are satisfied, actual enforcement of the contract still requires calibrating private funds M and C (and, consequently, t_0) in a proper manner. The next corollary concludes the analysis, stating how exactly this should be made.

Corollary 3 *Suppose that $\exists (z_j, T_j)$, $j \in \{l, h\}$, with $z_j \in Z_j$ and T_j satisfying* (10), *for which* (24) *holds, together with* (16) *and* (17). *Then, Ψ_j is enforced by setting M and C such that*

$$\nu_1 \Delta\theta z_j \frac{1-e^{-rT_j}}{r} - \psi \quad \leq \quad M \leq \min \left\{ \frac{R_{\delta_h}}{1+\lambda} \frac{1-e^{-rT_h}}{1-e^{-r\delta_h}} + \nu_1 \Delta\theta z_j \frac{1-e^{-rT_j}}{r}; \right. \tag{28}$$
$$\left. \frac{R_{\delta_l}}{1+\lambda} \frac{1-e^{-rT_l}}{1-e^{-r\delta_l}} - (1-\nu_1)\Delta\theta z_j \frac{1-e^{-rT_j}}{r} \right\} - \psi, \ \forall \delta_i \in (0, T_i),$$

together with

$$C \leq \frac{\mathbb{E}_i [R_{T_i}]}{1+\lambda} - (M + \psi). \tag{29}$$

Proof. See Appendix B.4. ■

This corollary can be interpreted, *mutatis mutandis*, along the same line as Corollary 2, for we do not insist on the general message that it conveys. We only make one final observation with regards to the restrictions on the choice of M.

According to (28), the maximum amount of own funds that F should be required to invest depends upon how large the replacement cost is in the good relative to the bad state, given the respective durations. To see this, first suppose that, for some given $j \in \{l, h\}$, the cost is large enough to satisfy

$$R_{\delta_l} \geq \left(R_{\delta_h} \frac{1-e^{-rT_h}}{1-e^{-r\delta_h}} + (1+\lambda)\Delta\theta z_j \frac{1-e^{-rT_j}}{r} \right) \frac{1-e^{-r\delta_l}}{1-e^{-rT_l}}, \ \forall \delta_i \in (0, T_i), \ i \in \{l, h\}. \tag{30}$$

This is clearly tighter in comparison with (24). It means that, under (30), the commitment problem is so weak on the government's side that F can be instructed to contribute a bigger amount of own funds without triggering the temptation of G to grab that investment. Then, the pertinent upper bound on M is $\left(\frac{R_{\delta_h}}{1+\lambda} \frac{1-e^{-rT_h}}{1-e^{-r\delta_h}} + \nu_1 \Delta\theta z_j \frac{1-e^{-rT_j}}{r} - \psi \right)$.

Next suppose that, for some given $j \in \{l, h\}$, (30) is violated, whereas (24) holds. The non-commitment problem is now more serious. Expropriation is not prevented unless the contribution of F is contained within a maximum of $\left(\frac{R_{\delta_l}}{1+\lambda}\frac{1-e^{-rT_l}}{1-e^{-r\delta_l}}-(1-\nu_1)\,\Delta\theta z_j\frac{1-e^{-rT_j}}{r}-\psi\right)$.

The reader may have noticed that, while the former bound is dictated by the replacement cost in state h, the latter is dictated by that in state l. This discrepancy, which might look weird at a first glance, is easily explained, in fact. Recall that the propensity of G to abide by its obligations depends upon how much it owes to F under the contract and how costly replacing F would be. How much exactly G owes to F also depends upon how much money F contributed up-front. All else equal, in either state, the higher M, the larger the profit of F, the less appealing the execution of Ψ_j to G. As long as replacement is so costly in state l that (30) holds, raising M would not be a problem if that state is realized eventually. However, it could be a problem if state h is realized, instead. This is more easily viewed by reformulating (30) as

$$R_{\delta_l}-R_{\delta_h}\frac{1-e^{-rT_h}}{1-e^{-r\delta_h}}\frac{1-e^{-r\delta_l}}{1-e^{-rT_l}}\geq(1+\lambda)\,\Delta\theta z_j\frac{1-e^{-rT_j}}{r}\frac{1-e^{-r\delta_l}}{1-e^{-rT_l}},\ \forall\delta_i\in(0,T_i)\,,\ i\in\{l,h\}\,.$$

The left-hand side is a measure, at date τ, of the cost that G would incur if it were to interrupt the partnership in state l, in which duration is longer, net of the cost that it would rather save by *not* doing that in state h, in which duration is shorter. As previously seen, the right-hand side is a measure, at date τ, of the additional cost that G bears if it continues to execute Ψ_j in state l, relative to state h. When the former cost is at least as large as the latter for all $\delta_i\in(0,T_i)$, $i\in\{l,h\}$, raising M does not make break-up more appealing to G than continuation of the contract in state l, but it might lead to that outcome in state h. This explains why R_{δ_h} dictates the upper bound on own funds. By contrast, when (30) is violated, raising M would be a problem already in state l, for R_{δ_l} determines the upper bound on own funds in that situation. Once M is downsized accordingly, there is no longer any worry of G interrupting the relationship if the high cost is rather realized.

6 Conclusion

In public-private contracting, reliance on a flexible-term agreement, that lasts longer when operating conditions come out to be favourable, is useful both in an "ideal" full-commitment world, in which only information issues are present, and in more realistic and complex frameworks, in which information issues coexist with enforcement difficulties on both the government's and the firm's side. However, a contract with that characteristics allows for two distinct options, associated with different levels of dispersion in the distribution of the firm's profits. This raises the necessity, in each relevant context, to individuate the option that is more appropriate to attain a desirable outcome.

Under full commitment, things are clear-cut. As imposing more risk facilitates the task of motivating the firm to exert effort at the construction stage, it is preferable to induce more dispersion in the profit distribution.

Under limited commitment, additional features of the two contractual options come to matter and should be considered, in turn, for a sound choice to be made. That is, the less dispersed compensation scheme is more easily sustained during operation. For the other, this occurs only under very specific circumstances. When the latter do arise, the decision is still as immediate as it would be under full commitment. The more dispersed scheme does represent the one-for-all recipe: not only it boosts effort, it is also more handily enforced. In all other cases, establishing which of the two options is more suitable, given the respective merits and limits, requires appraising how serious the moral-hazard problem is relative to the enforcement problem.

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A Full commitment

A.1 The profit wedge with state-dependent duration

Using (9a) and (9b), we can write

$$\Pi^*_{l,0}(z_j, T_j) - \Pi^*_{h,0}(z_j, T_j) = \Delta\theta \int_0^{T_j} z_j e^{-rx} dx, \ \forall j \in \{l, h\}, \tag{31}$$

from which we get the following relationship between z_l and z_h :

$$z_l = z_h \frac{1 - e^{-rT_h}}{1 - e^{-rT_l}}.$$

The lowest feasible value of z_j is $z_l = q^*_h \frac{1-e^{-rT_h}}{1-e^{-rT_l}}$, attained when $z_h = q^*_h$. The highest feasible value of z_j is $z_h = q^*_l \frac{1-e^{-rT_l}}{1-e^{-rT_h}}$, attained when $z_l = q^*_l$. Hence, $z_l \in Z_l \equiv \left[q^*_h \frac{1-e^{-rT_h}}{1-e^{-rT_l}}, q^*_l\right]$ and $z_h \in Z_h \equiv \left[q^*_h, q^*_l \frac{1-e^{-rT_l}}{1-e^{-rT_h}}\right]$.

A.1.1 The profit wedge is enlarged by raising T_l

Start from $T_l = T_h \equiv T$, as it is in Ψ. The profit wedge is equal to $\Delta\theta z \frac{1-e^{-rT}}{r}$. We can write

$$\Delta\theta z \frac{1 - e^{-rT}}{r} = \Delta\theta z \frac{1 - e^{-rT_l}}{1 - e^{-rT_h}} \frac{1 - e^{-rT_h}}{r}.$$

Now raise T_l above T_h so that $T_l > T_h \equiv T$. Also let $z_h \equiv z \frac{1-e^{-rT_l}}{1-e^{-rT_h}}$ to further write

$$\Delta\theta z \frac{1 - e^{-rT_l}}{1 - e^{-rT_h}} \frac{1 - e^{-rT_h}}{r} = \Delta\theta z_h \frac{1 - e^{-rT_h}}{r} > \Delta\theta z \frac{1 - e^{-rT_h}}{r},$$

where the inequality follows from $\frac{1-e^{-rT_l}}{1-e^{-rT_h}} > 1$. It means that, by raising T_l, the profit wedge is *enlarged*. The wedge is maximized by picking the sharing rule (z_h, T_h), with $z_h = q^*_l \frac{1-e^{-rT_l}}{1-e^{-rT_h}}$.

Then, the weakest restriction of duration applies:

$$T_h \geq \underline{T}\left(q_l^* \frac{1-e^{-rT_l}}{1-e^{-rT_h}}\right), \text{ with } \underline{T}\left(q_l^* \frac{1-e^{-rT_l}}{1-e^{-rT_h}}\right) < \underline{T}\left(q_l^*\right).$$

A.1.2 The profit wedge is narrowed by decreasing T_h

Start from $T_l = T_h \equiv T$, as it is in Ψ. The profit wedge is equal to $\Delta\theta z \frac{1-e^{-rT}}{r}$. We can write

$$\Delta\theta z \frac{1-e^{-rT}}{r} = \Delta\theta z \frac{1-e^{-rT_h}}{1-e^{-rT_l}} \frac{1-e^{-rT_l}}{r}.$$

Now decrease T_h below T_l so that $T_h < T_l \equiv T$. Also let $z_l \equiv z \frac{1-e^{-rT_h}}{1-e^{-rT_l}}$ to further write

$$\Delta\theta z \frac{1-e^{-rT_h}}{1-e^{-rT_l}} \frac{1-e^{-rT_l}}{r} = \Delta\theta z_l \frac{1-e^{-rT_l}}{r} < \Delta\theta z \frac{1-e^{-rT_l}}{r},$$

where the inequality follows from $\frac{1-e^{-rT_h}}{1-e^{-rT_l}} < 1$. It means that, by decreasing T_h, the profit wedge is *narrowed*. The wedge is minimized by picking the sharing rule (z_l, T_l), with $z_l = q_h^* \frac{1-e^{-rT_h}}{1-e^{-rT_l}}$. Then, the tightest restriction on duration applies:

$$T_l \geq \underline{T}\left(q_h^* \frac{1-e^{-rT_h}}{1-e^{-rT_l}}\right) > \underline{T}\left(q_h^*\right).$$

A.2 Proof of Proposition 1

Using the sharing rule (z_h, T_h), (4b) is written as

$$z_h\left(1-e^{-rT_h}\right) \geq \frac{r\psi}{\Delta\nu\Delta\theta}$$

so that the restriction on T_h is

$$T_h \geq \underline{T}\left(z_h\right) \equiv \frac{1}{r} \ln \frac{\Delta\nu\Delta\theta z_h}{\Delta\nu\Delta\theta z_h - r\psi}. \tag{32}$$

The logarithm is not defined unless (11) holds. The largest feasible value of z_h is $q_l^* \frac{1-e^{-rT_l}}{1-e^{-rT_h}}$. This is bigger than q_l^* as long as $T_l > T_h$. Hence, when this is the case, (32) is weaker than (7).

B Enforcement under limited commitment

B.1 Removing incentives to break up the partnership

We identify conditions under which F and G prefer honouring the contract rather than breaking up the partnership. We take duration as given and (14) to hold.

B.1.1 Removing the incentives of F

For F to be willing to honour Ψ_j once θ_i is observed, it must be the case that, conditional on truthtelling at the outset of the operation phase, F is at least as well off in Ψ_j as it would be if the relationship were to stop at some date $\tau \in (0, T_i)$:

$$\Pi_{i,\tau}^{*}(z_j, T_j) \geq \Pi_{i,\tau}^{rp} = 0. \tag{33}$$

Under Ψ_j, the discounted profits at date τ are given by

$$\Pi_{l,\tau}^{*}(z_j, T_j) \equiv \left(M + \psi + (1 - \nu_1)\Delta\theta z_j \frac{1 - e^{-rT_j}}{r}\right) \frac{1 - e^{-r\delta_l}}{1 - e^{-rT_l}} \tag{34a}$$

$$\Pi_{h,\tau}^{*}(z_j, T_j) = \left(M + \psi - \nu_1 \Delta\theta z_j \frac{1 - e^{-rT_j}}{r}\right) \frac{1 - e^{-r\delta_h}}{1 - e^{-rT_h}}, \tag{34b}$$

with $\delta_i \equiv T_i - \tau$, $\forall i \in \{l, h\}$. Thus, when $i = l$, (33) is clearly satisfied. When $i = h$, it is if and only if

$$M \geq \nu_1 \Delta\theta z_j \frac{1 - e^{-rT_j}}{r} - \psi. \tag{35}$$

B.1.2 Removing the incentives of G

For G to be willing to honour Ψ_j once θ_i is revealed, it must be the case that, conditional on F truthtelling at the outset of the operation phase, G is at least as well off by remaining in Ψ_j as it would be by stopping the relationship at date $\tau \in (0, T_i)$:

$$V_{i,\tau}^{*}(z_j, T_j) \geq V_{i,\tau}^{rp}. \tag{36}$$

Under Ψ_j, the discounted benefits of G from private management are written as

$$V_{l,\tau}^{*}(z_j, T_j) = w_l^{*} \frac{1 - e^{-r\delta_l}}{r} - (1 + \lambda)\left[\left(M + \psi + (1 - \nu_1)\Delta\theta z_j \frac{1 - e^{-rT_j}}{r}\right) \frac{1 - e^{-r\delta_l}}{1 - e^{-rT_l}} + D_{l,\tau}\right]$$

$$V_{h,\tau}^{*}(z_j, T_j) = w_h^{*} \frac{1 - e^{-r\delta_h}}{r} - (1 + \lambda)\left[\left(M + \psi - \nu_1 \Delta\theta z_j \frac{1 - e^{-rT_j}}{r}\right) \frac{1 - e^{-r\delta_h}}{1 - e^{-rT_h}} + D_{h,\tau}\right].$$

First take $i = l$. Then, (36) is satisfied if and only if

$$M \leq \frac{R_{\delta_l}}{1 + \lambda} \frac{1 - e^{-rT_l}}{1 - e^{-r\delta_l}} - (1 - \nu_1)\Delta\theta z_j \frac{1 - e^{-rT_j}}{r} - \psi. \tag{38a}$$

$$D_{l,\tau} \leq \frac{R_{\delta_l}}{1 + \lambda} - \left(M + \psi + (1 - \nu_1)\Delta\theta z_j \frac{1 - e^{-rT_j}}{r}\right) \frac{1 - e^{-r\delta_l}}{1 - e^{-rT_l}} \tag{38b}$$

Next take $i = h$. Then, (36) is satisfied if and only if

$$M \leq \frac{R_{\delta_h}}{1+\lambda}\frac{1-e^{-rT_h}}{1-e^{-r\delta_h}} + \nu_1 \Delta\theta z_j \frac{1-e^{-rT_j}}{r} - \psi. \tag{39a}$$

$$D_{h,\tau} \leq \frac{R_{\delta_h}}{1+\lambda} - \left(M + \psi - \nu_1 \Delta\theta z_j \frac{1-e^{-rT_j}}{r}\right)\frac{1-e^{-r\delta_h}}{1-e^{-rT_h}} \tag{39b}$$

As long as

$$R_{\delta_l} \geq \left(R_{\delta_h}\frac{1-e^{-rT_h}}{1-e^{-r\delta_h}} + (1+\lambda)\,\Delta\theta z_j \frac{1-e^{-rT_j}}{r}\right)\frac{1-e^{-r\delta_l}}{1-e^{-rT_l}}, \tag{40}$$

the relevant condition on M is (39a); otherwise, it is (38a).

B.2 Removing the incentives of F to cheat anticipating renege

In addition to making sure that parties are willing to honour the contract, conditional on F truthtelling as soon as the facility is in place, one should make sure that F has no incentive to lie on θ_i anticipating that some party will renege at some date τ during the operation phase.

Let $\Pi^{RN}_{i,\tau}$ denote the stream of profits that F would obtain in state i, discounted at time τ, if it were to cheat at the outset of the operation phase and Ψ_j were reneged at $\tau \in (0, T_i)$. In state l and h, F has no incentive to lie, anticipating that some party will renege on Ψ_j at some date τ, if and only if, respectively:

$$\Pi^*_{l,0}(z_j, T_j) \geq \int_0^\tau \left(\pi^*_{h,x} + \Delta\theta q^*_h\right) e^{-rx}dx + \max\left\{0; \Pi^{RN}_{l,\tau}\right\} \tag{41a}$$

$$\Pi^*_{h,0}(z_j, T_j) \geq \int_0^\tau \left(\pi^*_{l,x} - \Delta\theta q^*_l\right) e^{-rx}dx + \max\left\{0; \Pi^{RN}_{h,\tau}\right\}. \tag{41b}$$

Let us show that (41a) holds. When, in state l, F reports h at time 0 and then the contract is renegotiated at some instant $\tau \in (0, T_h)$, its instantaneous profit is given by

$$\pi^{RN}_{l,\tau} = t^{rn}_h + p\left(q^*_h\right) q^*_h - \left(\theta_l q^*_h + K\right) - d^{rn}_{h,\tau}, \tag{42}$$

where t^{rn}_h denotes the expected transfer that results from renegotiating at τ, given the report h. In the renegotiation game, when G makes the offer after being announced h, the instantaneous transfer t^G_h that it proposes to F, together with the quantity q^G_h, is given by

$$t^G_h = \theta_h q^G_h + K - p(q^G_h)q^G_h + d^{rn}_{h,\tau}.$$

When F makes the offer after announcing h, the instantaneous transfer t^F_h that it proposes to G, together with the quantity q^F_h, is given by

$$t^F_h \equiv \frac{1}{1+\lambda}\left(S(q^F_h) - p\left(q^F_h\right) q^F_h - w^*_h + \frac{rR_{\delta_h}}{1-e^{-r\delta_h}}\right).$$

We can thus write

$$\begin{aligned}
t_h^{rn} &= \alpha t_h^G + (1-\alpha)\, t_h^F \\
&= \alpha(\theta_h q_h^* + K + d_{h,\tau}^{rn}) + \frac{1-\alpha}{1+\lambda}\left(S\left(q_h^*\right) - w_h^* + \frac{rR_{\delta_h}}{1-e^{-r\delta_h}}\right) - \frac{1+\alpha\lambda}{1+\lambda} p\left(q_h^*\right) q_h^*.
\end{aligned}$$

Replacing into (42), we obtain

$$\pi_{l,\tau}^{RN} = (1-\alpha)\left(\frac{R_{\delta_h}}{1+\lambda}\frac{r}{1-e^{-r\delta_h}} - d_{h,\tau}^{rn}\right) + \Delta\theta_h q_h^*.$$

In discounted terms:

$$\begin{aligned}
\Pi_{l,\tau}^{RN} &= \int_\tau^{T_h}\left((1-\alpha)\left(\frac{R_{\delta_h}}{1+\lambda}\frac{r}{1-e^{-r\delta_h}} - d_{h,\tau}^{rn}\right) + \Delta\theta q_h^*\right) e^{-r(x-\tau)} d\tau \\
&= \Pi_{h,\tau}^{rn} + \int_\tau^{T_h} \Delta\theta q_h^* e^{-r(x-\tau)} d\tau.
\end{aligned}$$

Replacing this, (41b) becomes

$$\Pi_{l,0}^*\left(z_j, T_j\right) \geq \int_0^\tau (\pi_{h,x}^* + \Delta\theta q_h^*) e^{-rx} dx + e^{-r\tau} \max\left\{0; \Pi_{h,\tau}^{rn} + \int_\tau^{T_h} \Delta\theta q_h^* e^{-r(x-\tau)} dx\right\},$$

which is further equivalent to

$$\begin{aligned}
\Pi_{l,0}^*\left(z_j, T_j\right) \geq\ & \Pi_{h,0}^*\left(z_j, T_j\right) + \int_0^{T_h} \Delta\theta q_h^* e^{-rx} dx \\
& + e^{-r\tau}\left(\max\left\{0; \Pi_{h,\tau}^{rn} + \int_\tau^{T_h} \Delta\theta q_h^* e^{-r(x-\tau)} dx\right\} - \left(\Pi_{h,\tau}^* + \int_\tau^{T_h} \Delta\theta q_h^* e^{-r(x-\tau)} dx\right)\right).
\end{aligned} \tag{43}$$

When $\max\left\{0; \Pi_{h,\tau}^{rn} + \int_\tau^{T_h} \Delta\theta q_h^* e^{-r(x-\tau)} dx\right\} = 0$, (43) reduces to

$$\Pi_{l,0}^*\left(z_j, T_j\right) \geq \Pi_{h,0}^*\left(z_j, T_j\right) + \int_0^{T_h} \Delta\theta q_h^* e^{-rx} dx - e^{-r\tau}\left(\Pi_{h,\tau}^* + \int_\tau^{T_h} \Delta\theta q_h^* e^{-r(x-\tau)} dx\right).$$

This is implied by (8), hence it is satisfied. When $\max\left\{0; \Pi_{h,\tau}^{rn} + \int_\tau^{T_h} \Delta\theta q_h^* e^{-r(x-\tau)} dx\right\} = \Pi_{h,\tau}^{rn} + \int_\tau^{T_h} \Delta\theta q_h^* e^{-r(x-\tau)} dx$, (43) becomes

$$\Pi_{l,0}^*\left(z_j, T_j\right) \geq \Pi_{h,0}^*\left(z_j, T_j\right) + \int_0^{T_h} \Delta\theta q_h^* e^{-rx} dx - e^{-r\tau}\left(\Pi_{h,\tau}^* - \Pi_{h,\tau}^{rn}\right). \tag{44}$$

As far as (14) holds, we have $\Pi_{h,\tau}^{rp} = 0 \geq \Pi_{h,\tau}^{rn}$. Thus, (33) implies that $\Pi_{h,\tau}^* - \Pi_{h,\tau}^{rn} \geq 0$. Consequently, (8) implies (44), which holds true.

Proceeding analogously, one can prove that (8) and (33) imply (41b), which is thus satisfied.

B.3 Necessary conditions for contract enforcement

B.3.1 Deriving (16)

Using $T_j \geq \underline{T}(z_j)$, $E \geq M$ and (35) altogether, (16) follows.

B.3.2 Deriving (17)

In the state-dependent framework, (17) is derived just as under fixed duration, for formal details are here omitted.

B.3.3 Deriving $(24)_h$ and (25)

First suppose that (40) is satisfied so that (39a) must hold. Provided $R_{\delta_h} \geq 0$, (39a) is compatible with (35). That is, there always exists a range of feasible values of M. Next suppose that (40) is violated so that (38a) must hold. For (38a) to be compatible with (35), it must be the case that (24) holds. This is rewritten as

$$\frac{R_{\delta_l}}{1+\lambda}\frac{1-e^{-rT_l}}{1-e^{-r\delta_l}} \geq \Delta\theta z_j \frac{1-e^{-rT_j}}{r}. \tag{45}$$

Start from $T_l = T_h \equiv T$, as it is in Ψ, so that $\delta_l = \delta_h \equiv \delta$. The relevant condition on the replacement cost is (15), which is rewritten as

$$\frac{R_{\delta_l}}{1+\lambda}\frac{1-e^{-rT_l}}{1-e^{-r\delta_l}} \geq \Delta\theta z \frac{1-e^{-rT_l}}{1-e^{-rT_h}}\frac{1-e^{-rT_h}}{r}.$$

Now raise T_l above $T_h \equiv T$ so that $T_l > T_h \equiv T$. Also let $z_h \equiv z\frac{1-e^{-rT_l}}{1-e^{-rT_h}}$ to further write

$$\frac{R_{\delta_l}}{1+\lambda}\frac{1-e^{-rT_l}}{1-e^{-r\delta_l}} \geq \Delta\theta z_h \frac{1-e^{-rT_h}}{r}.$$

The right-hand side is now bigger because $\frac{1-e^{-rT_l}}{1-e^{-rT_h}} > 1$ so that $z_h > z$. To establish whether the change in T_l tightens or weakens (15), we need to check which variation it triggers in the term in the left-hand side. At this aim, we compute

$$\begin{aligned}
\frac{d}{dT_l}\left(R_{\delta_l}\frac{1-e^{-rT_l}}{1-e^{-r\delta_l}}\right) &= R'_{\delta_l}\frac{1-e^{-rT_l}}{1-e^{-r\delta_l}} + R_{\delta_l}\frac{re^{-rT_l}\left(1-e^{-r\delta_l}\right) - re^{-r\delta_l}\left(1-e^{-rT_l}\right)}{\left(1-e^{-r\delta_l}\right)^2} \\
&= \frac{1}{1-e^{-r\delta_l}}\left(R'_{\delta_l}\left(1-e^{-rT_l}\right) - rR_{\delta_l}\frac{e^{-r\delta_l}-e^{-rT_l}}{1-e^{-r\delta_l}}\right).
\end{aligned}$$

Because $R'_{\delta_l} > 0$ and $e^{-r\delta_l} > e^{-rT_l}$, we have

$$\operatorname{Si}gn\left\{\frac{d}{dT_l}\left(R_{\delta_l}\frac{1-e^{-rT_l}}{1-e^{-r\delta_l}}\right)\right\} = \operatorname{Si}gn\left\{R'_{\delta_l}\left(1-e^{-rT_l}\right) - rR_{\delta_l}\frac{e^{-r\delta_l}-e^{-rT_l}}{1-e^{-r\delta_l}}\right\}.$$

As long as $R'_{\delta_l} \leq R_{\delta_l}\frac{r\left(e^{-r\delta_l}-e^{-rT_l}\right)}{\left(1-e^{-rT_l}\right)\left(1-e^{-r\delta_l}\right)}$, $(24)_h$ is definitely tighter than (15) for all $\delta_l \in (0, T_l)$. Otherwise, the possibility of $(24)_h$ being weaker than (15) cannot be ruled out. Computing

$$\frac{d}{dT_l}\left(\Delta\theta z\frac{1-e^{-rT_l}}{1-e^{-rT_h}}\frac{1-e^{-rT_h}}{r}\right) = \Delta\theta z e^{-rT_l},$$

one finds that $(24)_h$ is weaker than (15) if and only if, for all τ and δ_l, (25) is satisfied.

B.3.4 Deriving $(24)_l$

Recall (45) and start from $T_l = T_h \equiv T$, as it is in Ψ, so that $\delta_l = \delta_h \equiv \delta$. The relevant condition on the replacement cost is (15), which we can rewrite as

$$\frac{R_{\delta_l}}{1+\lambda}\frac{1-e^{-rT_l}}{1-e^{-r\delta_l}} \geq \Delta\theta z\frac{1-e^{-rT_h}}{1-e^{-rT_l}}\frac{1-e^{-rT_l}}{r}.$$

Now decrease T_h below $T_l \equiv T$ so that $T_h < T_l \equiv T$. Also let $z_l \equiv z\frac{1-e^{-rT_h}}{1-e^{-rT_l}}$ to further write

$$\frac{R_{\delta_l}}{1+\lambda}\frac{1-e^{-rT_l}}{1-e^{-r\delta_l}} \geq \Delta\theta z_l\frac{1-e^{-rT_l}}{r}.$$

The term in the left-hand side is unchanged. The right-hand side is now smaller because $\frac{1-e^{-rT_h}}{1-e^{-rT_l}} < 1$ so that $z_l < z$. Therefore, $(24)_l$ is weaker than (15) for all $\delta_l \in (0, T_l)$.

B.4 Proof of Corollary 3

Condition (28) is obtained from (35), (38a) and (39a).
Using the definition of $\mathbb{E}_i\left[D_{i,\tau}\right]$ in (38b) and (39b), we get

$$\mathbb{E}_i\left[D_{i,\tau}\right] \leq \frac{\mathbb{E}_i\left[R_{\delta_i}\right]}{1+\lambda} - (M+\psi)\,\mathbb{E}_i\left[\frac{1-e^{-r\delta_i}}{1-e^{-rT_i}}\right].$$

Then, recalling that $\mathbb{E}_i[D_{i,0}] = C$, this condition together with $C > 0$ collapses onto (29).