Procurement and Debt[1]

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December 29, 2010

Abstract: This paper studies dynamic procurement design and the effect of bankruptcy on this design. Firms differ in their ability to self-finance their presence in the market. I study the optimal financial contract for the firm in need of funding and the optimal procurement contract in a setting with both a self-financed and a cash-constrained firm. This paper identifies two reasons, the sampling and duality effect, for favoring the financially weak firm and and one reason, the predatory effect, for not doing so.

JEL Classification: D82, G30, H57.

Keywords: Procurement, Dynamic contracts, Debt, Dual Sourcing.

[1]I particularly thank David Martimort for his patience, help and suggestions. I have benefited from discussions with Bruno Biais, Mike Burkart, Guillaume Cheikbossian, Tore Ellingsen, Heiko Karle, Sebastian Kranz, Roger Myerson, Frank Rosar and Vanessa Valero. I would also like to thank seminar participants at the Stockholm School of Economics, the University of Oslo (Økonomisk Institutt), the University of Mannheim, Toulouse School of Economics and Université Libre de Bruxelles as well as conference participants at the Spring Meeting of Young Economists 2010, the 2010 Doctoral Meeting of Montpellier, the Econometric Society World Congress 2010 and the 59^{th} annual Congress of L'Association Française de Science Economique (AFSE). All errors are mine.

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1 Introduction

The focus of this paper is on public procurement with an endogenously drawn number of bidders and whether dual sourcing can be maintained and should be so when firms face severe financial constraints. In fact public procurement is an important part of most countries' economic activity. In 2002 the value of public procurement was estimated to be about 16% of GDP in the EU[1] and around 20% in the United States[2]. Consider the case of procurement by the public health authorities. With regular intervals, the public health authorities need to provide new services or new facilities for the promotion or provision of public health. In many tenders by the public health authorities the number of potential providers is fairly low. The potential effect of a change in the number of competitors over time may have a huge effect on outcome and efficiency. Furthermore, this type of procurement can in many cases result in dual sourcing.[3]

With the last couple of years' financial crisis, bankruptcy has been a major economic and political concern. In May 2010, the U.S. bankruptcy filings were at their second highest level since 2005[4] and, even as credit markets improve, we can still expect to see firms filing for bankruptcy[5]. Furthermore, financial distress is not limited to the private sector. Public sectors such as public health care are also affected.[6] Even before the financial crises, bankruptcy and financial distress was an issue in highly leveraged health care organizations[7] and the financial situation is not likely to improve the situation for financially distressed hospitals. For instance, between 2000 and 2006 42 U.S. acute care hospitals filed for bankruptcy protection and 67% of these hospitals eventually ceased operating.[8]. But financial distress is not limited to the health care sector or the financial crises. In other sectors, such as construction, bankruptcy was also an issue before the financial crisis. Calveras, Ganuza and Hauk (2004) points out that in the US during 1990-1997 more than 80,000 contractors went bankrupt leaving unfinished

[1] http://ec.europa.eu/internal_market/publicprocurement/index_en.htm

[2] Handbook of Procurement (Cambridge University Press).

[3] This is for instance the case for ambulance contracts and rehabilitations centers in Norway. However, dual sourcing is not only limited to the public health sector. Anton and Yao (1992) point out that in the public sector the application of this dual sourcing *runs the gamut from high-technology systems for telecommunications to their use in obtaining services such as refuse collection and street cleaning. In the private sector procurements of items such as customized computer chips and commercial aircraft have involved split awards.*

[4] http://www.bankruptcy-statistics.com/us-bankruptcy-filing-rate-near-5-year-high.html

[5] Bankruptcies: The Next Wave, Businessweek, July 29, 2009.

[6] See for instance Langabeer (2006) and Kim (2010)

[7] Moore, Coddington and Byrne (2009)

[8] Landry and Landry (2009)

private and public construction projects with liabilities exceeding $21 billion.

This paper combines the two observation of bankruptcy and a low number of bidders and studies the effect of this on dynamic procurement. In this paper there is a trade-off between the need to secure future competition and the optimal allocation of today's provision. The main result of this paper is that when the procurement agency and the investor behave non-cooperatively[9] and the financially constrained (highly leveraged) firm is not *efficient enough*, then the first-period optimal procurement contract should be biased. In other words one of the firms should be given a larger share of total production than in the case where both firms face the same constraints. But which firm to give an advantage is not clear and I identify three different effects; the *sampling* effect, the *predatory* effect and the *duality* effect. In fact there is a trade-off between more aggressive competition today[10] (predatory effect) and the benefit of potential future dual sourcing and competition (duality and sampling effect). On the one hand some firms will behave more aggressively today in order to get rid of its competitors in future periods. This allows the procurement agency to restrict its payment in the first period. But this has a negative effect on second-period surplus because the possibility for dual sourcing, and competition in general, decreases and second-period rents increase.

In this paper there are two types of firms. On the one hand, there are self-financed firms with a "deep-pocket", who have enough capital to fund their participation in the market in every period. On the other hand, there are cash-constrained firms with little or no proper funding ("shallow-pocket" firms). These latter firms rely on external investors to fund their market participation and their existence in future markets depends on their performance in today's market and their financial contract. One could think of a "deep-pocket" firm as being a big (multi-)national firm and a "shallow-pocket" firm as being a local firm or a start-up.[11]Both of these firms compete for the procurement of a good or service (which can potentially be split between producers).

The market or procurement setting in question is one where the procurement agency

[9]This is for instance the case in public procurement because strict rule for competition in these tenders forbid the procurement agency to finance only a selected group of market participants.

[10]Through the self-financed firm accepting a lower transfer for the first-period contract.

[11]An example of this would be the case of medical transport in Finnmark, Norway. In 2008, two out of three areas that were up for procurement went to Veolia, a strong player in the Norwegian transport sector, and the last area was contracted out to Loppa Legeskyssbåter, a firm with more local roots and which because of this contract can ensure further existence on the market. For more information see `http://www.altaposten.no/lokalt/nyheter/article203130.ece` and `http://www.helse-finnmark.no/pressemeldinger/ambulansekontrakt-til-loppa-legeskyssbaater-article60750-25745.html`

wishes to procure a good or a service in each period. However, the good or service in each period is not the same but similar enough so that the same firms have the competence and skill to provide both goods. In the case of treatments of drug abuse, this could be in-house (residential) treatment for abuse of a specific drug in one period and outpatient services in the second period. Alternatively, it could be different types of treatment programs in each period or going from one type of treatment program to a polydrug treatment program.[12] Formally, in each period a procurement agency decides how to split the provision of a good or service between the two firms (or organizations). The procurement agency can freely choose the optimal split of production for dual sourcing (including degenerate splits that would be equivalent to sole sourcing). However, since some firms do not have the ability to self-finance their presence in the market, they need to borrow from an investor before entering the market. It is assumed that there is a fixed cost of participation in each period.[13] The actual number of firms at the second-period procurement stage will be endogenous and depends on whether the financial contracts push to bankruptcy or not as well as the outcome of the first-period procurement. This paper studies the optimal financial contract for the firm in need of funding and the optimal procurement contract in a setting with self-financed and cash-constrained firms.

The financial contract in this paper extends the result of Faure-Grimaud (2000) to the case where the realization of profits is endogenously determined by an equilibrium procurement mechanism. Because the "probability of non-liquidation" is increasing in the efficiency of the firm and increasing in profits, the financial structure of the firms has implications on the design of the optimal procurement contract. If the financially constrained firm doesn't perform well enough in the first period, it risks being liquidated by the investor. This gives incentives to the other firm to engage in predation. Earlier literature on predation (Bolton and Scharfstein 1990, Snyder 1996 and Faure-Grimaud 2000) consider situations where profits are private information but their value is exogenous. The level of profits and their distribution is taken as given by all firms and either profits are privately observed by the firm or they are observable but not verifiable.[14] It should be noted however that this paper does not solely focus

[12]For instance, oploids abuse treatment program in the first period and alcohol abuse treatment program in the second period would require different approaches for treatment.

[13]This fixed cost could either be thought of as an investment in specialized production equipment for this specific good or as a cost related to the research and work needed for the firm to learn its cost for the specific project.

[14]By considering the financial contract as being a contract with a third party (and the procurement contract being the main contract) my paper also relates to the literature on games with side-contracts. For an introduction to this and related literature on games with third parties, see Gerratana and Kock-

on predation, but studies the allocation of production and distortions on this allocation due to asymmetric information and differences in firms' financial structure.

This paper analysis a situation where one of the firms need to contract with an outside investor to obtain the necessary funds for market participation. Brander and Lewis (1986), Faure-Grimaud (2000) and Spiegel & Spulber (1994, 1997) study the effect of strategic acquisition of debt. This paper abstracts completely from these issues and concentrate on situations where one firm is truly *cash-constrained* and the effect on the optimal procurement contract of this cash-constraint.

A recent part of the literature on contract theory builds on Baron and Besanko (1984) and looks at dynamic contracting. One part of this literature considers an environment with a dynamic population of agents whose private information remains fixed throughout time (Gershkov and Moldovanu, 2009, 2010, Pai and Vohra, 2009, Said, 2010a, 2010b). This paper is related to this literature but takes a more applied approach. It also focuses on exit only and instead of imposing an exogenous stochastic process for market exit, it uses the literature on financial contract and liquidation to explain why a firm might have to leave the market.

This paper is organized as follows. The model is presented in Section 2. Section 3 solves for the benchmark procurement contract where none of the firms need a financial contract. The non-cooperative solution is presented in Section 4. In this section the optimal financial contract as well as the optimal procurement contract and equilibrium conditions are derived. Section 5 derives comparative statics while extensions are discussed in Section 6. Section 7 briefly concludes.

2 The model

• **Technology, information and preferences:** There are four types of players in this model; the investor, the procurement agency, the cash-constrained firm and the self-financed firm.

There exists a competitive market for investors. Therefore a firm seeking funding has all the bargaining power when it comes to the details of the financial contract.[15]

In each of the two periods, the procurement agency wants to divide the production

ensen (2009).

[15] As pointed out in Faure-Grimaud (1997) when the investor has all the bargaining power the optimal contract has the same structure as in this case.

of an amount $\bar{q}$ of a certain good between the two firms. It enjoys a gross surplus $\bar{S}$ from the provision of such a service.[16][17] Denote by δ the discount factor which can also be interpreted as a measure of the importance of the second-period project.

The cash-constrained and the self-financed firms both have the ability to provide the goods to the procurement agency. There is a fixed cost D to be paid in each period for staying on the market and that the self-financed firm has got all the assets it needs to pay this fee whereas the cash-constrained firm has no such asset. D needs to be paid before each period by all firms who want to be present on the market. In other words, D is *not* a cost to enter the market, D is a fixed cost related to a project and needs to be paid for each new project.[18] Ex ante, the only difference between the firms is that the self-financed firm has a "deep pocket" and does not need external financing to stay on the market. The cash-constrained firm however, has a "shallow pocket" and need an investor to finance him in order to stay on the market.

In each period a firm's cost of procuring the required amount q of the good is $C(\theta, q) = \theta q + \mu \frac{q^2}{2}$ where $\mu \geq 0$. Here costs are convex. In general, convex costs gives an intrinsic efficiency reason for using dual sourcing. Since costs increase with the production level, splitting the production between two (or more) producers allows the procurement agency to obtain the good at a lower total cost than if he used only one provider. The specification of the cost function also includes the case of linear cost (when $\mu = 0$). The parameter μ is common to all firms[19] and is public knowledge. But θ is private information and independent across time and firms. Furthermore, costs are drawn from the same cumulative distribution function $F(\theta)$ with support $\Theta = [\underline{\theta}, \bar{\theta}]$. The associated density function is denoted $f(\theta)$. I assume that the inverse hazard rate $\frac{F}{f}$ is increasing.

For ease of notation, denote by C the cash-constrained firm and S the self-financed firm. Furthermore, define $\theta_1 \equiv (\theta_{1C}, \theta_{1S})$ and $\theta_2 \equiv (\theta_{2C}, \theta_{2S})$.

[16]In general, the per period total quantities $\bar{q}_1$ and $\bar{q}_2$ do not need to be the same. However, for simplicity, $\bar{q} = \bar{q}_1 = \bar{q}_2$.

[17] For instance, when the public health authorities decide to provide a treatment facility for drug abuse, it seems reasonable that the public health authorities already knows how much of this good it needs. This facility is needed to cope with a growing public health problem and will, in general, not be very sensitive to production costs. So in this case it seems reasonable to assume that the total quantity demanded is either fixed or varies very little. Another example where quantities are fixed is the auctioning of broadcast permits for big events (such as the UEFA Champions League and the Olympic Games). In general there is a fixed quantity of licenses and the decision concerns to whom allocate these broadcasting rights and not how many licenses to allocate.

[18]This could also be thought of as an administrative cost associated with starting on a new project.

[19]Industry specific

• **Contracts:** In this model there are two contracts, a procurement contract and a financial contract.

The procurement contract is a long-term contract which stipulates transfers to both firms and the associated quantities to be produced by the two firms in each period. First-period transfers and quantities are only contingent on first-period announcements θ_1. However, second-period announcement are contingent on first-period history (such as who dropped out of the market between periods and first-period announcements) as well as second-period announcement of types. Formally, the procurement contract can be written

$$\Big\{q_{1C}(\cdot), q_{1S}(\cdot), t_{1C}(\cdot), t_{1S}(\cdot), (\{q_{2C}(\cdot;n), q_{2S}(\cdot;n), t_{2C}(\cdot;n), t_{2S}(\cdot;n)\})_{n=1,2}\Big\},$$

where n represents the number of firms (one or two) left in the second period and is observable (and verifiable) between periods.

A cash-constrained firm will have to finance its fixed costs by entering into a financial contract with an investor. As in Bolton and Scharfstein (1990) and Faure-Grimaud (1997, 2000), a financial contract stipulates repayment schemes for the first period and a non-liquidation probability function contingent on the cash-constrained firm's realized profit in the first period as well as a repayment scheme for the second period[20]. Denoting by $R_i(\cdot)$ the repayment scheme in period i and by $\beta(\cdot)$ the non-liquidation probability function, a financial contract can be defined as follows.

$$\{(R_1(\cdot), R_2(\cdot), \beta(\cdot)\}$$

• **Timing:** The game therefore unfolds as follows:

1. First-period:

    - The cash-constrained firm negotiates a financial contract with the investor. This contract is not verifiable by outside players.
    - Firms pay the fixed cost D and privately learn their first-period cost θ.
    - The procurement agency offers a long-term procurement contract to the firms and firms privately announce their first-period type.
    - The outcome of the first-period procurement stage is realized and observed by both firms and the procurement agency (but not by the investor).

[20]For further explanation regarding the financial contract and its structure see Section 4.1.

- The cash-constrained firm announces its profits to the investor and makes its first-period repayment. Depending on its announcement of realized profits, this firm is forced by the investor to leave the market or not.

2. Second period:

    - Firms that are still active on the market pay the fixed cost D and privately learn their second-period cost.
    - The second-period procurement stage takes place and firms privately announce their type.
    - Second-period transfers and quantities are realized. These transfers and quantities are only observed by the firms and the procurement agency.
    - If the cash-constrained firm is still on the market, he makes the second-period repayment in accordance with the financial contract.

This timing is summarized in the following figure.

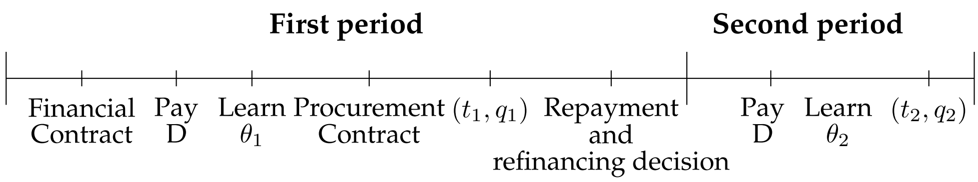

Figure: Timing

• **Comments:** Before presenting the benchmark and the results, some comments regarding the fixed payment, D, are in order. First, the cash-constrained firm cannot falsely claim zero profits and default on its repayment to the investor in order to use its profit to finance the second-period fixed cost itself. Although profits are unobservable, claiming to have made zero profits and then being able to pay D can be seen as a way of revealing that profits where positive and can therefore be prohibited by law and punished accordingly.

Finally, I focus on the case of a relatively small D. In fact, if D is large enough, a natural monopoly situation might be preferable. Here, I want to abstract from these issues and focus on situations where competition is beneficial (but fragile in the sense that some players are cash-constrained). Of course profits from the procurement stages will be endogenous, but I will in what follows assume that the (endogenous) expected value of participating in the market is higher than the up-front payment D.

• **First-Best Procurement:** Ignoring the difference in financial structure of the two firms and with symmetric information in both periods, the procurement agency's objective is to maximize the expected intertemporal social surplus subject to the firms accepting the deal in each period.[21]. Furthermore gros surplus is constant since the total quantity $\bar{q}$ to be procured in each period is fixed, so maximizing expected intertemporal social surplus amounts to minimizing expected total cost of procurement for both periods. With public information on the firms' production costs, the procurement agency will only pay the firms an amount equal to their costs. And finally, there is direct relation between q_{kC} and q_{kS} which means that for each unit of the good or service that is being provided the procurement agency will look at what firm can produce this additional unit at the lowest cost. In other words, if possible the procurement agency is going to choose to split the production so that marginal cost of each firm coincides. It is easy to see that if this is not the case, then the procurement agency can reduce its payment by transferring a small amount of the provision from the firm with a high marginal cost to the one with a low marginal cost. However, sometimes[22] such a split is not possible because the marginal cost of one firm is above the marginal cost of the other firm for all possible splits the production. In this case the procurement agency is going to ask the most efficient firm (the one with the lowest marginal cost) to provide the entire production.

Formally, in each period k and for an interior solution this yields the following condition for the optimal solution for the cash-constrained firm's production $q_{kC}(\theta_k)$.

$$\theta_{kC} + \mu q_{kC}(\theta_{kC}, \theta_{kS}) = \theta_{kS} + \mu(\bar{q} - q_{kC}(\theta_{kC}, \theta_{kS})). \tag{1}$$

If one firm is inherently more efficient than the other so that (1) does not have a solution in $[0, \bar{q}]$, then the optimal strategy for the procurement agency is to select sole sourcing from the most efficient firm. Assuming that there is an interior solution to (1)[23], this solution is such that at the optimal levels, $(q_{kC}(\theta_k), q_{kS}(\theta_k))$, both firms produce at the same marginal cost.

[21] I require interim participation constraints in both periods. In fact, any firm that finds the second-period part of the procurement contract *unfair* can choose to be inactive and not participate in the second-period provision of the good. This assumption is based on a ruling by the U.S. Supreme Court (and discussed in Spiegel and Spulber, 1994) stating that, a regulatory agency cannot force a firm to provide a good or a service at a loss. In the Texas Railroad Comm. v. Eastern Texas R.R. Co. case of 1924 the Supreme Court argued that "If at any time it develops with reasonable certainty that future operations must be at a loss, the company may discontinue operation and get what it can out of the property...To compel it to go on at a loss, or to give up the salvage value, would be to take its property without just compensation which is a part of due process of law." (264 U.S. 79, 85 (1924)).

[22] For instance in the special case of linear costs ($\mu = 0$).

[23] This is the case when $\forall(\theta_{kC}, \theta_{kS})$, $0 \leq \frac{\bar{q}}{2} + \frac{\theta_{kS} - \theta_{kC}}{2\mu} \leq \bar{q}$.

Firm S is obviously asked to produce the complimentary quantity $q_{kS}(\theta_k) = \bar{q} - q_{kC}(\theta_k)$. It is immediate to see that the only difference between the provision rule in the first and the second period is that each rule uses the type of the firms in the associated period. I can therefore simplyfy the notation and write $q_C(\cdot)$ for both $q_{1C}(\cdot)$ and $q_{2C}(\cdot)$. This first-best result generalizes the static result in Auriol and Laffont (1992).

In what follows, because total quantity in each period is fixed there can be no distortion of total quantity and I can therefore focus on the allocative role of asymmetric information and financial structure on the distribution of $q_{kC}(\cdot)$ and $q_{kS}(\cdot)$, $k = 1, 2$.

3 Benchmark: No Financial Contract

In the absence of financial constraints both firms will be around in both periods and the procurement contract is a long-term contract which stipulates for each firm, transfers and quantities for each period. First-period transfers and quantities are only contingent on first-period announcements of θ_1. However, second-period announcement are contingent on first-period history as well as second-period announcement of types. Formally a mechanism can be written as

$$\Big\{q_{1C}(\cdot), q_{1S}(\cdot), t_{1C}(\cdot), t_{1S}(\cdot), (\{q_{2C}(\cdot), q_{2S}(\cdot), t_{2C}(\cdot), t_{2S}(\cdot)\})\Big\}.$$

Since surplus is fixed, the only issue is how to optimally allocated the production between the firms. In other words, I will look for the split of production that minimizes procurement cost subject to participation and incentive constraints. To do so, I make use of the Revelation Principle (Myerson 1981).

At the beginning of the first period, the procurement agency chooses the menu of contracts that minimizes his intertemporal expected total cost subject to first- and second-period incentive compatibility and participation contraints.

The firms' IC constraints are

$$\begin{aligned}
U_{2i}(\theta_1, \theta_{2i}) &= \max_{\tilde{\theta}_{2i}} U_{2i}(\theta_1, \theta_{2i}, \tilde{\theta}_{2i}) = \max_{\tilde{\theta}_{2i}} E_{\theta_{2j}} \Big\{ t_{2i}(\theta_1, \tilde{\theta}_{2i}, \theta_{2j}) - C(\theta_{2i}, q_{2i}(\theta_1, \tilde{\theta}_{2i}, \theta_{2j})) \Big\}, \\
U_{1i}(\theta_{1i}) &= \max_{\tilde{\theta}_{1i}} U_{1i}(\tilde{\theta}_{1i}, \theta_{1i}) = \max_{\tilde{\theta}_{1i}} E_{\theta_{1j}} \{ t_{1i}(\tilde{\theta}_{1i}, \theta_{1j}) - C(\theta_{1i}, q_{1i}(\tilde{\theta}_{1i}, \theta_{1j})) \\
&\qquad + \delta \left(E_{\theta_{2i}} \left(U_{2i}(\tilde{\theta}_{1i}, \theta_{1j}, \theta_{2i}) \right) - D \right) \}, \\
&(i, j) \in \{C, S\}^2, i \neq j.
\end{aligned}$$

I immediately get:

Lemma 1

- *$U_{ki}(\cdot)$ is convex in θ_{ki} and absolutely continuous. Thus almost everywhere differentiable in θ_{ki} with at any differentiability point:*

$$\frac{\partial}{\partial\theta_{ki}}U_{ki}(\cdot) = -E_{\theta_{kj}}\left[q_{ki}(\theta_k)\right]. \tag{2}$$

- *Local second-order conditions for incentive compatibility can be written as:*

$$E_{\theta_{kj}}\frac{\partial q_{ki}}{\partial\theta_{ki}}(\cdot) \leq 0. \tag{3}$$

- *Local second-order conditions are sufficient for global optimality of the truthtelling strategy.*

Observe that (2) implies that $\frac{\partial}{\partial\theta_{ki}}U_{ki}(\cdot)$ is non-decreasing in θ_{ki}, so that the participation constraint is binding at $\bar{\theta}$ only. Integrating (2) with respect to θ_{ki} yields

$$\begin{aligned} U_{2i}(\theta_1,\theta_{2i}) &= E_{\theta_{2j}}\left\{\int_{\theta_{2i}}^{\bar{\theta}} q_{2i}(\theta_1,s,\theta_{2j})ds\right\} \\ U_{1i}(\theta_{1i}) &= E_{\theta_{1j}}\left\{\int_{\theta_{1i}}^{\bar{\theta}} q_{1i}(s,\theta_{1j})ds\right\},\ (i,j)\in\{C,S\}^2, i\neq j. \end{aligned}$$

The procurement agency will therefore minimize expected total virtual cost where virtual, as defined by Myerson (1984), refers to the actual cost plus the adjustment (or rent) required for the mechanism to be incentive compatible. The procurement agency's optimization problem can thus be written

$$\begin{aligned} \min_{(q_{kC}(\cdot),q_{kS}(\cdot))_{k=1,2}} E_{\theta_1,\theta_2}\Bigg[\sum_{i=S,C}\left(\theta_{1i}q_{1i}(\theta_1)+\frac{F}{f}(\theta_{1i})q_{1i}(\theta_1)+\mu\frac{q_{1i}(\theta_1)^2}{2}\right) \\ +\,\delta\sum_{i=S,C}\left(\theta_{2i}q_{2i}(\theta_1,\theta_2)+\mu\frac{q_{2i}(\theta_1,\theta_2)^2}{2}-D\right)\Bigg] \end{aligned}$$

subject to $q_{kS}(\cdot) = \bar{q} - q_{kC}(\cdot)$, $q_{ki}(\cdot) \geq 0$ and $q_{2i}(\cdot) \leq \bar{q}$ for $(i,j) \in \{C,S\}^2$, $i \neq j$ and $k = 1,2$.

The optimal quantity $q_{2C}(\theta_2)$ is independent of first-period types and, (unless it is a corner solution) is given by

$$\theta_{2C} + \mu q_{2C}(\theta_2) = \theta_{2S} + \mu(\bar{q} - q_{2C}(\theta_2)), \tag{4}$$

and the optimal quantity $q_{1C}(\theta_2)$ is given by (unless it is a corner solution)

$$\theta_{1C} + \frac{F}{f}(\theta_{1C}) + \mu q_{1C}(\theta_1) = \theta_{1S} + \frac{F}{f}(\theta_{1S}) + \mu(\bar{q} - q_{1C}(\theta_1)). \tag{5}$$

Assuming that there exists a $q_{kC}(\theta_2) \in (0, \bar{q})$ that solves the appropriate first-order condition, then the optimal solution for the procurement agency is to adopt dual sourcing with the cash-constrained firm producing $q_{1C}(\theta_1)$ given by (5) and $q_{2C}(\theta_2)$ given by (4). The self-financed firm produces the complementary quantity $q_{kS}(\cdot) = \bar{q} - q_{kC}(\theta_k)$.

Because there is symmetric information on second-period costs at the time of contracting, the first-best outcome can be implemented in the second period. In fact second-period transfers are chosen so as to ensure second-period incentive compatibility and first-period transfers are decreased by the expected amount of second-period transfers so that the regulatory agency can without giving up rents implement the first-best in this period.[24] It is also clear that this decision rule is independent of first-period types.

In the first-period, the solution to equation 5 is such that the virtual marginal cost of the cash-constrained firm for producing the second-period good equals the virtual marginal cost of the self-financed firm for producing the same good. In the case where both firm have the same efficiency ($\theta_{1C} = \theta_{1S}$), the production is split into two equal parts. If firms differ in their efficiency, the split is not equal.

If the left-hand side (LHS) of the above equation is always smaller than the right-hand side (RHS), then sole sourcing by the cash-constrained firm is optimal and $q_{1C}(\theta_1) = \bar{q}$. In this case, the virtual marginal cost of the cash-constrained firm is, for all admissible quantities[25], lower than the virtual marginal cost of the self-financed firm and it is therefore less costly to make the cash-constrained firm provide the entire quantity $\bar{q}$.

If the virtual marginal cost of the self-financed firm is smaller than the virtual marginal cost of the cash-constrained firm for all $q \in [0, \bar{q}]$ then sole sourcing by the self-financed firm is optimal and $q_{1S}(\theta_1) = \bar{q}$.

To sum up these findings, given the *types* of the firms, the procurement agency chooses to split the production in the most efficient way between the two firms. If one firm is inherently more efficient than the other then the procurement agency will opt for sole sourcing. However, in the opposite case, when firms differ less in their *types*,

[24]The rent it has to give out in the second period to ensure incentive compatibility has been "recovered" in the first period through decreased transfers.

[25]$\forall q_{1C} \in (0, \bar{q})$.

the procurement agency will opt for dual sourcing since it allows to enjoy lower costs. In this case the procurement agency will ask the firms to produce quantities such that their respective virtual marginal costs are equal because if the procurement agency transfers some of the production from one firm to the other, the overall production cost increases.

Rearranging (5) we get

$$q_{1C}(\theta_1) = \frac{\bar{q}}{2} + \frac{\theta_{1S} + \frac{F}{f}(\theta_{1S}) - \theta_{1C} - \frac{F}{f}(\theta_{1C})}{2\mu}. \tag{6}$$

Since the inverse hazard rate is increasing, it is straightforward to see that the most efficient firm produces more both under symmetric and asymmetric information. However, under asymmetric information the distortions are going to be such that the procurement agency will favor the more efficient firm even more in order to reduce the rent he has to pay for the firms to behave truthfully. In a standard principal-agent framework with a non-constant surplus function, quantities are reduced (distorted downward) to reduce the rent of the agents. Here total quantities are fixed, but by shifting some of the production from the less efficient firm to the most efficient one the procurement agency decreases the rent he has to pay to high types which again allows him to reduce the rent to more efficient types.[26] Therefore, when the cash-constrained firm is more efficient than the self-financed firm, $q_{1C}(\theta_1)$ is shifted upward compared to the first-best case. For the same reason, when the self-financed firm is more efficient than the cash-constrained firm, $q_{1C}(\theta_1)$ is shifted downward to allow more of the production of the efficient the self-financed firm.

The findings in the case where firms have no financial constraints are summarized in the following Proposition.

Proposition 1 *When none of the firms face financial constraints, the optimal solution is to implement the first-best quantities in the second period and first-period quantities are such that, if* $\forall(\theta_C, \theta_S)$, $\frac{\bar{q}}{2} + \frac{\theta_S - \frac{F}{f}(\theta_S) - \theta_C - \frac{F}{f}(\theta_C)}{2\mu} \in [0, \bar{q}]$, *then (non-degenerate) dual sourcing is optimal. Furthermore these production levels are such that each firm produces at the same virtual marginal cost.*

This result can be viewed as an extension of Baron and Myerson (1982) and Auriol

[26]The more efficient you are the more rent you will require. But if the inefficient types get very little rent, then an efficient type will be more willing to accept lower levels of rent since this is still better than deviating.

and Laffont (1992) to the dynamic contracting problem, and as a multi-agent version of Baron and Besanko (1984).

It can also be shown that the result in the previous Proposition extends to the case where the regulatory agency and the investor are the same entity.[27] A contract in this setting will be defined in the same way as the contract above except that it will also include a refinancing decision between periods and second-period quantities and transfers are also contingent on the number of firms around in this period. In other words, a dynamic (cooperative) contract can be written as

$$\left\{ q_{1C}(\cdot), q_{1S}(\cdot), t_{1C}(\cdot), t_{1S}(\cdot), \beta(\cdot), (\{q_{2C}(\cdot;n), q_{2S}(\cdot;n), t_{2C}(\cdot;n), t_{2S}(\cdot;n)\})_{n=1,2} \right\},$$

where $\beta(\cdot)$ is the probability of non-liquidation of the cash-constrained firm between periods.

The optimal contract in this setting is such that the cash-constrained firm is always refinanced and transfers and quantities are the same as in Proposition 1. This is summarized in the following Proposition.

Proposition 2 *When the procurement agency and the investor behave cooperatively,*

- *It is always optimal to refinance the financially constrained firm.*
- *The optimal quantities are the same as in Proposition 1.*

4 Non-Cooperative Solution

In this section, I study the optimal dynamic procurement problem when one firm is financially constrained. Here financially constrained refers to the fact that the firm in question does not have enough cash to finance the fixed cost D and therefore needs to contract with an outside investor. In many cases, especially in public procurement, the procurement agency does not offer financial support for potential service providers. Firms in lack of financial resources therefore have to contract separately with their investors.

In this setting I need to characterize two distinct contracts; the optimal long-term procurement contract and the optimal financial contract.

[27] Or behave cooperatively.

For future use, define the cash-constrained firm's first-period realized profit as $\Pi_1(\theta_1) = t_{1C}(\theta_1) - \theta_{1C} q_{1C}(\theta_1) - \mu \frac{q_{1C}(\theta_1)^2}{2}$.

Notice that because there is no communication among players between the offering of the two contracts, the timing of the financial contract and the first procurement contract is such that it is strategically equivalent to a simultaneous game. When two principals contract simultaneously with an agent, but each principal (investor and procurement agency) only controls part of the agent's activity, then the Revelation Principle does not necessarily hold.[28]

Looking at the non-cooperative solution to this problem I first derive best-replies for the procurement agency and investor. On the one hand, I characterize the investor's financial contract. On the other hand, I study the behavior of the procurement agency. Having obtained the best-reply contracts for each principal, I characterize a Nash equilibrium of this game.

This section starts by deriving the optimal financial contract for the cash-constrained firm. Then I present the optimal procurement contract and the equilibrium conditions for this game.

4.1 Optimal Financial Contract

Without asymmetric information, the financial contract between the investor and the cash-constrained firm would simply be a sharing rule of realized profits between the investor and the firm in exchange for the investor paying D. However, when profits are privately observed by firms we need a more sophisticated mechanism. Here, I make the assumption of private information on realized profits because costs are privately observed by firms and also the announcements in the procurement stage remain private.[29] Furthermore, for the investor, the outcome (quantities and dual/sole sourcing decision) in the procurement contract is non contractible. Once a firm has realized some profits, it needs to be induced to repay the investor rather than strategically defaulting by pretending not to have made any profits. Following Bolton and Scharfstein

[28] Martimort (1992) and Martimort and Stole (2002) shows that in situations where several principals control the agent's activity through non linear tariffs, the Revelation Principle does not apply, and needs to be replaced by a weaker concept; the Delegation Principle.

[29] Bolton and Scharfstein (1990) point out different reasons for contracts not being profit-contingent. First, the managers of the firm might be able to divert profits (cash-diversion argument). Second, the firm in question might be related to another firm and therefore has some flexibility in the (joint) allocation of costs and revenues between these firms. These reasons for non-contractibility carry over to this paper.

(1990) and Faure-Grimaud (2000), I study contracts with a repayment schedule and a "reward" function. If a firm claims to have made no profits in the first-period just in order not to pay back the investor, he can be punished by not being refinanced in the second period. This penalizes default deviations and helps the investor in monitoring the firm.

A financial contract consists of repayment schemes for each period and probabilities of the firm being refinanced in the second period. A priori, if the identity of the producers chosen to produce a non-negative quantity in the first-period is publicly announced, the financial contract could be contingent on this information. However, in the case of dual sourcing, knowing the type of one particular firm is not sufficient to deduce the profits earned by the cash-constrained firm. Indeed, the investor must also learn the type of the other firm (which the investor has no contact with). Or equivalently, the investor needs to know the quantity produced by the cash-constrained firm. In fact, if the investor knows both firms' type he can deduce the quantities and if he knows one firm's type and the quantity he can deduce the other firm's type in the case of dual sourcing. With sole sourcing the other firm's type becomes irrelevant for the investor. Notice however that the only reason for which the investor wants this information is so that he can deduce the profit level of the cash-constrained firm. Having this in mind, I will therefore take a different approach and assume that the financial contract is contingent on the (announced) profit level of the cash-constrained firm rather than on its type and quantities produced. To simplify the exposition I will assume that the investor cannot make the contract contingent on which and how many producers are chosen. The reason being either that this information is not public (or verifiable) or that the investor only cares about profits.

Formally, a financial contract is a menu

$$\big\{(R_1(\hat{\Pi}_1), (R_2(\hat{\Pi}_1, \hat{\Pi}_2)), \beta(\hat{\Pi}_1))\big\},$$

where $R_1(\hat{\Pi})$ is the repayment in period 1 for a firm announcing profit level $\hat{\Pi}_1$, $R_2(\hat{\Pi}_1, \hat{\Pi}_2)$ is the repayment in period 2 for a firm announcing profit level $\hat{\Pi}_1$ in the first period and $\hat{\Pi}_2$ in the second period, and, finally, $\beta(\hat{\Pi}_1)$ is the probability of being refinanced in the second period following this annoucement.

A financial contract is a menu of repayment and refinancing probabilities that satisfy three types of constraints: incentive compatibility (IC), limited liability (LL) and individual rationality (IR^I) constraints. The two first types of constraints are related to the firm whereas the latter is related to the investor (hence the superscript I). In fact, since there is perfect competition between investors, the financial contract will maximize the firm's expected inter-temporal profit subject to the participation constraint of the investor (and, of course, incentive compatibility).

For ease of notation I denote by Π^d the ex ante expected profit in the second-period procurement contract (given that both firms are still on the market). Both Π_1 and Π^d are endogenous and will be determined by the procurement stage. Define $G(\Pi_1) = Prob\{\theta_1, \Pi_1(\theta_1) \leq \Pi_1\}$ and $g(\Pi_1)$ the associated density function. The support of $\Pi_1(\theta_1)$ is $[\min \Pi_1(\theta_1), \max \Pi_1(\theta_1)]$ and, to simplify notations, I use $\bar{\Pi}_1 = \max \Pi(\theta_1)$ and $\underline{\Pi} = \min \Pi_1(\theta_1)$.

Before solving the above optimization problem, let us define what is meant by a debt contract.

Definition 1 *A debt contract is a financial contract $\{(R_1(\hat{\Pi}_1), \beta(\hat{\Pi}_1))\}$ with a fixed first-period repayment and where upon repayment the firm is always refinanced. If the firm cannot repay the required fixed amount, he has to give his entire profit to the investor and risks liquidation ($\beta(\hat{\Pi}_1) < 1$).*

Proposition 3 *(Faure-Grimaud, 2000) The optimal financial contract $\{(R_1(\hat{\Pi}_1), R_2(\hat{\Pi}_1), \beta(\hat{\Pi}_1))\}$ takes the form of a debt contract with all $R_2(.)$ equal to zero.*

- *If the firm makes high enough profits, it reimburses a fixed amount Π^* and is never liquidated.*
  $\forall \Pi_1 \geq \Pi_1^*$, $\beta(\Pi_1) = 1$ *and* $R_1(\Pi_1) = \Pi_1^*$.
- *If profits are not high enough, the firm has to repay all it has and the probability of refinancing is less than one.*
  $\forall \Pi_1 \leq \Pi_1^*$, $\beta(\Pi_1) = 1 - \frac{\Pi_1^* - \Pi_1}{\delta \Pi^d} < 1$ *and* $R_1(\Pi_1) = \Pi_1$.

The fixed repayment is given by the following equation

$$\int_{\underline{\Pi}_1}^{\bar{\Pi}_1} \left[R_1(\Pi_1) - \delta\beta(\Pi_1)D\right] dG(\Pi_1) = D$$

or equivalently

$$\Pi_1^* - \frac{\Pi^d - D}{\Pi^d} \int_{\underline{\Pi}_1}^{\Pi_1^*} G(\Pi_1) d\Pi_1 = (1+\delta)D \tag{7}$$

Notice that the financial contract is the same as in Faure-Grimaud (2000). If the cash-constrained firm is sufficiently efficient, it is always refinanced and does not pay its entire first-period gain. If the firm is less efficient, but it produces in the first-period, it is refinanced with a certain probability and has to pay its entire first-period gain from the production. The difference with Faure-Grimaud (2000) is that profits, Π_1, are endogenous and are determined by the procurement contracts. Recall that the financial contract is a best response to a given profit distribution. Proposition 3 characterize the optimal financial contract for a given distribution of profits, but this distribution will be determined by the contract offered by the procurement agency. After having characterized the optimal procurement agency, this paper determines the endogenous distribution of profits that occur at equilibrium.

The debt contract described in the above Proposition also resembles the "smoothed debt" contract presented in Hart and Moore (1998). In that model, the focus is slightly different from here and the authors focus on a situation with symmetric unverifiable information and renegotiation. In their setting they derive sufficient conditions for debt contracts to be optimal.

4.2 Optimal Procurement Contract

The optimal procurement contract is a menu of transfers and quantities for both firms contingent on the types in the corresponding period[30] and the number of active firms in this period. In previous sections, the two firms were active in both periods. Here, there is a possibility of only one firm being active in the second period.

• **Both firms are present in the second period:** If both firms are still present on the market and given the repayment scheme for the second period (which I have now shown is always equal to zero), the problem is the same as in Section 3.

• **Only the self-financed is present in the second period:** If only the self-financing firm is left on the market in the second period, it is immediat that it provides $\bar{q}$. In this case incentive compatibility implies that $t_{2S}^m = \bar{\theta}\bar{q} + \mu\frac{\bar{q}^2}{2}$ and therefore $U_{2S}^m(\theta_{2S}) = (\bar{\theta} - \theta_{2S})\bar{q}$.

Clearly the principal's surplus when there is only one firm left on the market in the second period is lower than when there are two firms. This is simply because the firm's rent when he is in a monopoly situation, Π^m, is higher than when there is competition,

[30]Because types are independent across time, second-period transfers and quantities which can be contingent on first-period types, will at the optimum be independent of first-period types and this contingency is therefore ignored. This is discussed and proved in Section 3.

Π^d.[31]

• First-period optimal procurement contract

Incentive compatibility of the first-period part of the contract requires that the procurement contract, $\{q_{1C}(\hat{\theta}_1), q_{1S}(\hat{\theta}_1), t_{1C}(\hat{\theta}_1), t_{1S}(\hat{\theta}_1)\}$, satisfies the following incentive constraints.

$$\theta_{1C} \in \arg\max_{\tilde{\theta}} E_{\theta_{1S}}\{t_{1C}(\tilde{\theta}, \theta_{1S}) - \theta_{1C} q_{1C}(\tilde{\theta}, \theta_{1S}) - \mu \frac{q_{1C}(\tilde{\theta}, \theta_{1S})^2}{2}$$
$$- R_1(\Pi_1(\theta_1, \tilde{\theta})) + \delta\beta(\Pi_1(\theta_1, \tilde{\theta}))\Pi^d\},$$

where $\Pi_1(\theta_1, \tilde{\theta}) = t_{1C}(\tilde{\theta}, \theta_{1S}) - \theta_{1C} q_{1C}(\tilde{\theta}, \theta_{1S}) - \mu \frac{q_{1C}(\tilde{\theta}, \theta_{1S})^2}{2}$.

From the incentive compatibility and Lemma 2, $-R_1(\Pi_1) + \delta\beta(\Pi_1)\Pi^d = C$.The cash-constrained firm completely internalizes the effect of the financial contract and only considers the effect of his first-period type announcement on first-period profits.

Incentive compatibility for the self-financed firm can be written as

$$\theta_{1S} \in \arg\max_{\tilde{\theta}} E_{\theta_{1C}}\{t_{1S}(\theta_{1C}, \tilde{\theta}) - \theta_{1S} q_{1S}(\theta_{1C}, \tilde{\theta}) - \mu \frac{q_{1S}(\theta_{1C}, \tilde{\theta})^2}{2} + \delta\Pi(\theta_{1C}, \tilde{\theta})\}$$

where

$$E_{\theta_{1C}}(\Pi(\theta_{1C}, \tilde{\theta})) = E_{\theta_{1C}}\{\beta(\Pi_1(\theta_{1C}, \tilde{\theta}, \theta_{1C}))\Pi^d + (1 - \beta(\Pi_1(\theta_{1C}, \tilde{\theta}, \theta_{1C}))\Pi^m\} - D$$

It is clear that the expected profit of the self-financed firm depends on the financial contract of the cash-constrained firm. If the self-financed firm, through its behavior in the first-period, can reduce the value of $\beta(\theta)$, it can increase its own expected profit from the second-period.

So even if the cash-constrained firm internalizes the effect of the financial contract, its competitor, the self-financed firm, does not and a predatory effect from the financial contract appears in the self-financed firm's first-period incentives. This will be of crucial importance in the next Proposition.

The optimal procurement contract also needs to satisfy the participation constraints. For the cash-constrained firm this writes

$$U_{1C}(\theta_{1C}) = E_{\theta_{1S}}\{t_{1C}(\theta_1) - \theta_{1C} q_{1C}(\theta_1) - \mu \frac{q_{1C}(\theta_1)^2}{2} + C\} \geq 0.$$

[31] Note that here the only remaining firm gets the entire production of $\bar{q}$. Because of the assumption that $\bar{q}$ is fixed, there is no room in this model for strategies such that when only one provider is available total provision is reduced. Here, it is as if demand is inelastic and the cost does not matter. For instance, it is not because only one institution is available to treat drug abusers that the public sector wants to reduce the availability of the treatment. Drug abuse is a serious problem, that needs to be dealt with, regardless of the competition for the provision of this service.

And the participation constraint of the self-financed firm:

$$U_{1S}(\theta_{1S}) = E_{\theta_{1C}}\{t_{1S}(\theta_1) - \theta_{1S}q_{1S}(\theta_1) - \mu\frac{q_{1S}(\theta_1)^2}{2} + \delta\Pi(\theta_1)\} \geq 0.$$

Lemma 1 still applies and the participation constraint will be binding for the least efficient type only and therefore

$$U_{1i}(\theta_{1i}) = E_{\theta_{1j}}\int_{\theta_{1i}}^{\bar{\theta}} q_{1i}(s)ds.$$

• **Optimization** Replacing transfers in the regulatory agency's minimization problem by the expression derived using incentive compatibility constraints and participation constraints yields the following relaxed minimization problem.

$$\begin{aligned}\min_{(q_{ki}(\cdot))_{i=C,S,k=1,2}} E_{\theta_1,\theta_2}\Bigg[\sum_{i=C,S}\left(\theta_{1i}q_{1i}(\theta_{1i}) + \frac{\mu}{2}q_{1i}^2(\theta_{1i}) + \frac{F}{f}(\theta_{1i})q_{1i}(\theta_{1i})\right) + R_1(\Pi_1(\theta_1))\\ + \delta\beta(\Pi_1(\theta_1))\sum_{i=C,S}\left(\theta_{2i}q_{2i}(\theta_{2i}) + \frac{\mu}{2}q_{2i}^2(\theta_{2i})\right)\\ + \delta(1-\beta(\Pi_1(\theta_1)))\left(\theta\bar{q} + \frac{\mu}{2}\bar{q}^2\right) - \delta D\Bigg]\end{aligned}$$

subject to

$$\bar{q} = q_{kC}(\cdot) + q_{kS}(\cdot)$$

It is straightforward to conclude that second-best production still follows the first-best provision rule as defined in equation 4. In other words, the second-period split of production is such that firms produce at the same marginal cost (unless there is a corner solution).

Denote the expected second-period transfer when there is only one firm left on the market, T_2^d, and when there are two firms around, T_2^d.

$$T_2^m = t_{2S}^m = \bar{\theta}\bar{q} + \frac{\mu}{2}\bar{q}^2$$

$$T_2^d = E_{\theta_2}[\sum_{i=C,S}\left(\theta_{2i}q_{2i}(\theta_2) + \frac{\mu}{2}q_{2i}^2(\theta_2) + \frac{F}{f}(\theta_{2i})q_{2i}(\theta_2)\right)]$$

And the expected profits

$$\Pi^m = E_{\theta_{2S}}\left[(\bar{\theta} - \theta)\bar{q}\right]$$

$$\Pi^d = E_{\theta_2}[\frac{F}{f}(\theta_{2i})q_{2i}(\theta_2)].$$

The condition $\Pi_1 \geq \Pi_1^*$ in Proposition 3 can be replaced by $\theta_{1C} \leq \theta^*(\theta_{1S})$ where $\theta^*(\theta_{1S})$ is such that $\Pi_1(\theta^*(\theta_{1S}), \theta_{1S}) = \Pi_1^*$. The expected probability of non-liquidation can therefore be rewritten in the following way.

$$E_{\theta_1}\beta(\theta_1) = \frac{1}{\delta\Pi^d} E_{\theta_1}\left[\int_{\theta^*}^{\bar{\theta}} \frac{F}{f}(\theta_{1C}) q_{1C}(\theta_1) dF(\theta_{1C})\right].$$

Piecewise optimization yields the following two conditions for interior solution. If $\theta_{1C} \leq \theta^*(\theta_{1S})$ then the optimal quantity $q_{1C}(\theta_1)$ satisfies:

$$\theta_{1C} + \frac{F}{f}(\theta_{1C}) + \mu q_{1C}(\theta_1) = \theta_{1S} + \frac{F}{f}(\theta_{1S}) + \mu(\bar{q} - q_{1C}(\theta_1)), \tag{8}$$

If $\theta_{1C} > \theta^*(\theta_{1S})$ then the optimal quantity $q_{1C}(\theta_1)$ satisfies:

$$\begin{aligned} \theta_{1C} + \frac{F}{f}(\theta_{1C}) + \mu q_{1C}(\theta_1) - \frac{1}{\Pi^d}\frac{F}{f}(\theta_{1C})\left(\Pi^d - \Pi^m - T^d + T^m\right) \\ = \theta_{1S} + \frac{F}{f}(\theta_{1S}) + \mu(\bar{q} - q_{1C}(\theta_1)) \end{aligned} \tag{9}$$

In the case where the cash-constrained firm is sufficiently efficient compared to the self-financed firm, the condition giving the optimal quantity is the same as when firms are symmetric in their financial structure.

However, when the cash-constrained firm is not sufficiently efficient compared to its competitor, the condition changes slightly. But it can be interpreted in the same way as the previous conditions for q_{2C}. If there exists a $q_{1C}(\theta_1) \in (0, \bar{q})$ that solves (9), then the optimal solution for the procurement agency is to adopt dual sourcing with the cash-constrained firm producing $q_{1C}(\theta_1)$ given by (9) and the self-financed firm producing $q_{1S}(\theta_1) = \bar{q} - q_{1C}(\theta_1)$. Furthermore, this means that the virtual marginal cost of the cash-constrained firm equals that of the self-financed firm plus an extra term. If the left-hand side of (9) is always smaller than the right-hand side then sole sourcing by the cash-constrained firm is optimal and $q_{1C}(\theta_1) = \bar{q}$. If the left-hand side of (9) is always bigger than the right-hand side, then sole sourcing by the self-financed firm is optimal and $q_{1S}(\theta_1) = \bar{q}$.

Define the cash-constrained firm's modified virtual marginal cost as its virtual marginal cost plus the bias term (which can be positive or negative). When $\theta_{1C} \leq \theta^*(\theta_{1S})$, the virtual marginal cost and the modified virtual marginal cost coincide.

Proposition 4 summarizes these findings.

Proposition 4 • *Suppose that one of the firms faces a financial constraint and that there is an interior solution to the procurement agency optimization problem. Then, in the first period, the procurement agency chooses $q_{1C}(\theta_1)$ according to (8) when the cash-constrained firm is relatively efficient. When the cash-constrained firm is not sufficiently efficient, it is optimal for the procurement agency to slightly bias the procurement rule following (9).*

The optimal $q_{1C}(\theta_1)$ and $q_{1S}(\theta_1)$ are such that the (possibly modified) virtual marginal cost of the cash-constrained firm equals the virtual marginal cost of the self-financed firm.

- *In the second-period, the first-best solution remains optimal.*

If the cash-constrained firm is sufficiently efficient, the split of production remains the same as in the benchmark case without financial contracts. This is because when the profits of the cash-constrained firm is high enough it is always refinanced. However, when the self-financed firm is not efficient enough, condition (9) takes into account the effect of the split of production on the financial contract (or more precisely on the probability of refinancing) and thus the effect on second-period surplus and profits. The sign of the bias term is ambiguous (even in the linear case). If the effect on net gain for the procurement agency in the second period is higher than the effect from the self-financed firm being more aggressive (requiring a lower transfer in the first period), then the cash-constrained firm will be asked to produce at a higher level than in the benchmark. This will increase its probability of being refinanced as well as the expected surplus of the procurer in the second period because the probability of more competition increases. In the opposite case, when the gain from the self-financed firm being more aggressive today outweighs the gains from increased competition tomorrow, then the interior solution for $q_{1C}(\theta_1)$ is such that the virtual cost of the self-financed firm is lower than the virtual cost of the cash-constrained firm and the cash-constrained firm will therefore be asked to produce at a lower level in the first procurement contract.

In fact there are three different effects that influence the trade-off between favoring the cash-constrained firm or the self-financed firm.

Define

$$P \equiv - \left\{ \Pi^d - \Pi^m - T^d + T^m \right\}$$

If P is positive, then the procurement contract will favor[32] the self-financed firm, but if

[32]Here favored means increased production by the favored firm.

P is negative the cash-constrained firm will be favored. To determine the effects in P, replace the expected transfers and profits by their value so that

$$P = \int_{\Theta^2} \left\{ \left(\theta_{2C} - \theta_{2S} + \frac{F}{f}(\theta_{2C}) \right) q_{2C}(\theta_2) + \mu q_{2C}(\theta_2)(q_{2C}(\theta_2) - \bar{q}) \right\} dF(\theta_{2C}) dF(\theta_{2S})$$

Denote $p(\theta_2)$ the integrand of this integral. If the solution for $q_{2C}(\theta_2)$ is interior, using (4), I get

$$p(\theta_2) = \frac{F}{f}(\theta_{2C}) q_{2C}(\theta_2) - \mu q_{2C}^2(\theta_2).$$

The first term in this expression is the *predatory* effect and the second term the *duality* effect. It is easy to see that the second term is negative. Thus the duality effect favors the cash-constrained firm. Since convex costs in itself is a reason to consider dual sourcing, favoring the cash-constrained firm increases the probability that both firms are active in the second period and thus increases the probability that the dual sourcing option is available in this period.

In the definition of P, the procurement agency's expected surplus and the self-financed firm's expected profits are taken into account. The cash-constrained firm's expected profit does not enter into this expression. Having both the procurement agency's expected surplus and the self-financed firm's expected profits in the equations means that the procurement agency is not preoccupied by the amount of rent given to this firm because this amount is recovered by the procurement agency through the self-financed firm's more aggressive bidding in the first period. This is not the case for the cash-constrained firm, and the principal takes into account that giving up rent to this firm cannot be recovered and is therefore costly. The predatory effect is therefore positive and favors the self-financed firm.

If the solution is such that $q_{2C}(\theta_2) = 0$, then $p(\theta_2) = 0$. However, if the solution is such that $q_{2C}(\theta_2) = \bar{q}$ then

$$p(\theta_2) = \frac{F}{f}(\theta_{2C})\bar{q} + (\theta_{2C} - \theta_{2S})\bar{q}.$$

Again there is a predatory effect, Here, it is actually stronger since $q_{2C}(\theta_2) = \bar{q}$. The opportunity cost of giving up (non-recoverable) rent to the cash-constrained firm is higher. The second effect is an *sampling* effect, which is negative because $q_{2C}(\theta_2)$ is equal to $\bar{q}$ only when the cash-constrained firm is more efficient than the self-financed firm. Therefore the effect favors the cash-constrained firm. In fact, to make sure that there is competition in the second period, which itself implies that the probability of drawing a low marginal cost for the industry is higher, the procurement agency is willing to bias the procurement in favor of the cash-constrained firm.

Proposition 5 *There are three effects that dictate what firm to bias the production in favor of.*

- *Sampling effect: Favor the cash-constrained firm to benefit from an increased probability of low production cost in the second-period.*
- *Duality effect: Favor the cash-constrained firm to benefit from the possibility of dual sourcing in the future.*
- *Predatory effect: Favor the self-financed firm to save on first-period transfer and reduce the opportunity cost of second-period rent.*

If the duality and sampling effect dominates the predatory effect, the procurement contract is biased in favor of the cash-constrained firm. Otherwise, it is biased in favor of the self-financed firm.

4.3 Equilibrium

In the presentation of the optimal procurement contract above I have already taken into account the interaction with the financial contract. This has been done by replacing variables and function related to the financial contract by their corresponding values in the financial contract and by using Lemma 2. As pointed out in Section 4.1, the optimal financial contract is characterized for a given distribution of realized profits. But these profits are endogenous and depend on the optimal procurement contract. In order to fully characterize the Nash equilibrium of this game, it remains to characterize this distribution function.

Section 4.2 pins down the optimal split of production in each period, but because incentive compatibility is required before the firm learns the type of its competitor there exists an infinity of solutions for the associated transfers. To be able to characterize an equilibrium I focus on transfers that are not only Bayesian incentive compatible but are also dominant strategy incentive compatible (Mookherjee and Reichelstein, 1992). Following U.S. Supreme Court rulings[33], this ensures a reasonable rate of return to the

[33] The Permian Basin Rate Case (390 U.S. 747, 1968), the Federal Power Commission v. Hope Natural Gas Co. (320 U.S. 591, 1944) and Texas Railroad Comm. v. Eastern Texas R.R. Co. (264 U.S. 79, 85 (1924)). For further explanation see Spulber (1989, chapter 10).

firm and ensures that the actual production and transfer do not lead to profit losses[34].

Using these transfers, the first-period realized profits of the cash-constrained firm can be written

$$\Pi_{1C}(\theta_1) = \int_{\theta_{1C}}^{\bar{\theta}} q_{1C}(s, \theta_{1S})ds - C$$

Since $\int_{\theta_{1C}}^{\bar{\theta}} q_{1C}(s, \theta_{1S})ds$ is a decreasing function of θ_{1C}, I can rewrite $G(\Pi)$ as

$$G(\Pi) = \int_{\underline{\theta}}^{\bar{\theta}} Prob\left\{\theta_{1C} \geq \hat{\theta}_{1C}(\theta_{1S}, \Pi)\right\} dF(\theta_{1S}).$$

Rearranging this expression yields

$$G(\Pi) = 1 - \int_{\underline{\theta}}^{\bar{\theta}} F(\hat{\theta}_{1C}(\theta_{1S}, \Pi))dF(\theta_{1S}). \tag{10}$$

An equilibrium of this game is thus characterized by the optimal contracts presented in previous sections and the distribution of realized first-period profits given by (10).

Proposition 6 *At equilibrium and with transfers that are dominant-strategy incentive compatible, the distribution of realized first-period profits is characterized by (10).*

Having pinned down all the equilibrium conditions, the optimal financial contract and the optimal procurement design in this environment is fully characterized. However, one might wonder whether, at equilibrium, the debt contract is degenerate in the sense that it stipulates a fixed amount that even the least efficient firm can repay. All types of firms are thereby refinanced. This again implies that the bias in procurement rule disappears. The following proposition answers this question.

Proposition 7 *The equilibrium debt contract is not such that the cash-constrained firm always repays the fixed amount. For inefficient enough types of the cash-constrained firm, there will always be a non-zero risk of liquidation.*

This result implies that the bias in the first-period procurement is non-zero when the cash-constrained firm is relatively inefficient and thus the financial structure of firms does matter for the design of the procurement contract.

[34]Dominant strategy transfers would ensure that the firm does not abandon the project between the announcement of its type and the provision because of losses incurred due to a *strange* transfer rule. As pointed out in the ruling in Texas Railroad Comm. v. Eastern Texas R.R. Co. a regulatory commission can do very little to prevent liquidation because public utilities are not obligated to provide services at a loss.

5 Comparative Statics

5.1 An Example

Section 4.2 identifies the different effects in the biased procurement rule. However, it is not straightforward to determine what effect dominates the others or in what case one effect is likely to be important.

In the case where types are uniformly distributed on $[\underline{\theta}, \bar{\theta}]$ and when the parameters of the model are such that there is always an interior solution[35], then $P = \frac{\bar{q}}{4}(\bar{\theta} - 3\underline{\theta}) - \frac{(\bar{\theta}-\underline{\theta})^2}{6\mu}$.

It can easily be shown that if the support of the types increases, then P increases. In other words, if the variation in possible costs increases, then the bias in favor of the cash-constrained firm decreases, or the bias in favor of the self-financed firm increases. In fact, if $\Delta\theta \equiv \bar{\theta} - \underline{\theta}$ is important, recovering second-period rents is more important and it is therefore less likely to favor the cash-constrained firm. Similarly, if μ is important, then the predatory effect is likely to be more important since the change in second-period profit is important when going from a duopoly to a monopoly situation.

When the cost of the provision is high in the sense that $\underline{\theta}$ is high. Then it is more likely to favor the cash-constrained firm. This is done in order to increase the probability of a low production cost in the second period.

If $\bar{q}$ increases, the effect on the bias P is ambiguous. If $\Delta\theta > 2\underline{\theta}$ then an increase in $\bar{q}$ will increase P. In other words, the importance of the rent that can be recovered through the predatory effect makes it more likely to favor the self-financed firm. If $\Delta\theta \leq 2\underline{\theta}$, then the gain from not giving up rent is less important so that the likelihood of favoring the cash-constrained firm increases.

5.2 The Importance of the Second Period

In the model presented in Section 4, the optimal contracts and equilibrium conditions were derived for a given discount factor. In this section, I investigate what happens when the discount factor varies. The discount factor can be interpreted as a measure of the importance of the second period and is therefore a measure of the importance of the second-period project.

[35] This is the case when $2(\bar{\theta} - \underline{\theta}) < \mu\bar{q}$.

From (9) it is straightforward to see that the level of the bias is in fact independent of the discount factor. However the cut-off type above which you apply the bias changes. Recall the definition of the cut-off type $\theta_1^*(\theta_{1S})$ such that $\Pi_1(\theta_1^*(\theta_{1S}), \theta_{1S}) = \Pi_1^*$. If the importance of the second period increases, then the cut-off decreases making the decision rule in the first-period procurement biased more often.

In fact, an increase in the discount factor decreases the probability of refinancing. To compensate for this increased risk of liquidation, the procurement agency decreases its threshold for biasing the first-period decision rule. This finding is summarized in the following Proposition.

Proposition 8 *A change in δ does not influence the first-period procurement rules (8) and (9), but it has a negative effect on the cut-off type above which the biased rule, (9), is applied.*

6 Extensions

6.1 Changes in the First-Period Participation Constraint

In this model, the argument for imposing second-period participation constraints has been based on rulings by the U.S. Supreme Court stating that the procurement agency cannot force a provider of public utilities to operate at a loss. By pushing this argument even further, one might argue that the appropriate first-period constraint is not the one presented above (expected total gain) but that it should be a constraint requiring first-period profits to be positive.

If in the model presented in Section 2, the first-period participation constraint is replaced by a constraint requiring first-period profits to be positive, the resulting optimal procurement contract remain relatively similar to previous results. However, for the highest values of θ_{1C} the bias will decrease compared to Proposition 4 reflecting the trade-off between the higher cost of procurement from always requiring first-period profits to be positive and the fact that predation is more likely to be successful in this cost region.

Formally, the first-period procurement rule can be divided into three zones:

- If the cash-constrained firm is relatively efficient compared to the self-financed firm, the rule implies a non-biased procurement decision where (for an interior solution) each firm produces at the same level of virtual marginal cost. In other words, for $\theta_{1C} \in [\underline{\theta}, \theta^*_{1C}(\theta_{1S})]$,

$$\theta_{1C} + \frac{F}{f}(\theta_{1C}) + \mu q_{1C}(\theta_1) = \theta_{1S} + \frac{F}{f}(\theta_{1S}) + \mu(\bar{q} - q_{1C}(\theta_1)).$$

- If the cash-constrained firm is slightly less efficient than in the above case, the procurement rule states that the cash-constrained firm should be given the same bias as in Proposition 4. For $\theta_{1C} \in [\theta^*_{1C}(\theta_{1S}), \theta^*_{1C}(\theta_{1S})]$, the optimal quantity $q_{1C}(\theta_1)$ satisfies:

$$\theta_{1C} + \frac{F}{f}(\theta_{1C}) + \mu q_{1C}(\theta_1) - \frac{1}{\Pi^d}\frac{F}{f}(\theta_{1C})(\Pi^d - \Pi^m - T^d + T^m) = \frac{F}{f}(\theta_{1S}) + \mu(\bar{q} - q_{1C}(\theta_1)).$$

- If the cash-constrained firm is sufficiently inefficient, the first-period procurement rule unambiguously favors the cash-constrained firm. For these values of the cash-constrained firm, predation is more likely to succeed (for all types of the self-financed firm). Therefore, it is not necessary for the procurement agency to take into account the predatory effect as in the previous procurement rule. The decision rule on this segment unambiguously favors the cash-constrained firm and states that for $\theta_{1C} \in [\theta^*_{1C}(\bar{\theta}), \bar{\theta}]$, the optimal quantity $q_{1C}(\theta_1)$ satisfies:

$$\theta_{1C} + \frac{F}{f}(\theta_{1C}) + \mu q_{1C}(\theta_1) + \frac{1}{\Pi^d}\frac{F}{f}(\theta_{1C})(T^d - T^m) = \frac{F}{f}(\theta_{1S}) + \mu(\bar{q} - q_{1C}(\theta_1)).$$

6.2 Different Cost Function

Instead of considering an industry specific capacity constraint as when μ is common to all firms and costs are linear in the type of the firm, costs could be convex in the firm-specific component. In the case where costs are given by θq^2, it can be shown that a slightly modified version of Proposition 4 still holds. With this new cost function, the equation giving first-period provision level for the cash-constrained firm is If $\theta_{1C} > \theta^*(\theta_{1S})$ then the optimal quantity $q_{1C}(\theta_1)$ satisfies[36]:

$$\left[\theta_{1C} + \frac{F}{f}(\theta_{1C}) - \frac{1}{\Pi^d}\frac{F}{f}(\theta_{1C})\left(\Pi^d - \Pi^m - T^d + T^m\right)\right] q_{1C}(\theta_1) = \left[\theta_{1S} + \frac{F}{f}(\theta_{1S})\right](\bar{q} - q_{1C}(\theta_1)) \quad (11)$$

However, the idea conveyed in Section 4.2 remains the same.

[36]Where cut-offs and expected second-period profits and transfers are adequately redefined to take into account the new cost structure.

7 Conclusion and Future Research

This paper studies dynamic procurement design and the effect of bankruptcy on this design. Firms differ in their ability to self-finance their presence in the market. I study the optimal financial contract for the firm in need of funding and the optimal procurement contract in a setting with both a self-financed and a cash-constrained firm. This paper has identified two reasons, the sampling and duality effect, for favoring the financially weak firm and and one reason, the predatory effect, for not doing so.

One possible extension would be to analyze whether the self-financed firm would prefer or not to acquire debt or other external funding in these kinds of environments.

It would also be interesting to extend this paper to a situation where firms have different weights in the welfare function. For political reasons a procurement agency or a government may be more favorable towards certain firms than other.

Appendix

• **Proof of Lemma 1:**

- *Envelope Theorem.* First, the Envelope Theorem yields that $U_{ki}(\cdot)$ is absolutely continuous in θ_{ki} and thus almost everywhere differentiable. Moreover at any differentiability point, I have:

$$\frac{\partial}{\partial \theta_{1i}} U_{1i}(\theta_{1i}) = -\, E_{\theta_{1j}} \left[q_{1i}(\theta_1) \right] \tag{12}$$

$$\frac{\partial}{\partial \theta_{2i}} U_{2i}(\theta_{1i}, \theta_{2i}) = -\, E_{\theta_{1j}, \theta_{2j}} \left[q_{2i}(\theta_1, \theta_2) \right] \tag{13}$$

- *Local conditions for incentive compatibility.* The local first-order conditions for incentive compatibility can be written as:

$$E_{\theta_{1j}, \theta_2} \left[\frac{\partial}{\partial \theta_{1i}} t_{1i}(\theta_1) - \theta_{1i} \frac{\partial}{\partial \theta_{1i}} q_{1i}(\theta_1) - \mu q_{1i}(\theta_1) \frac{\partial}{\partial \theta_{1i}} q_{1i}(\theta_1) + \delta \frac{\partial}{\partial \theta_{1i}} U_{2i}(\theta_{1i}, \theta_{2i}) \right] = 0. \tag{14}$$

and

$$E_{\theta_{2j}} \left[\frac{\partial}{\partial \theta_{2i}} t_{2i}(\theta_1, \theta_2) - \theta_{2i} \frac{\partial}{\partial \theta_{2i}} q_{2i}(\theta_1, \theta_2) - \mu q_{2i}(\theta_1, \theta_2) \frac{\partial}{\partial \theta_{2i}} q_{2i}(\theta_1, \theta_2) \right] = 0. \tag{15}$$

It can be shown that the local second-order necessary conditions become:

$$E_{\theta_{1j}, \theta_2} \left[\frac{\partial}{\partial \theta_{1i}} q_{1i}(\theta_1) \right] \leq 0. \tag{16}$$

and

$$E_{\theta_{1j},\theta_{2j}}\left[\frac{\partial}{\partial\theta_{2i}}q_{2i}(\theta_1,\theta_2)\right]\leq 0. \tag{17}$$

- *Global incentive compatibility.* For global incentive compatibility to hold I need to prove that $U_{1i}(\theta_{1i},\theta_{1i})\geq U_{1i}(\theta_{1i},\hat{\theta}_{1i})$ and $U_{2i}(\theta_{1i},\theta_{2i},\hat{\theta}_{2i})\geq U_{2i}(\theta_{1i},\theta_{2i},\hat{\theta}_{2i})$.

  For any $(\theta_{1i},\tilde{\theta}_{1i})\in\Theta^2$, using (14) yields

$$U_{1i}(\theta_{1i},\theta_{1i})-U_{1i}(\theta_{1i},\hat{\theta}_{1i})=$$
$$\int_{\hat{\theta}_{1i}}^{\theta_{1i}}E_{\theta_{1j},\theta_2}\left[\frac{\partial}{\partial\theta_{1i}}t_{1i}(x,\theta_{1j})-\theta_{1i}\frac{\partial}{\partial\theta_{1i}}q_{1i}(x,\theta_{1j})-\mu q_{1i}(x,\theta_{1j})\frac{\partial}{\partial\theta_{1i}}q_{1i}(x,\theta_{1j})+\delta\frac{\partial}{\partial\theta_{1i}}U_{2i}(x,\theta_{2i})\right]dx.$$

  Define

$$\begin{aligned}K(x,\theta_{1i})=&E_{\theta_{1j},\theta_2}\Bigg[\frac{\partial}{\partial\theta_{1i}}t_{1i}(x,\theta_{1j})-\theta_{1i}\frac{\partial}{\partial\theta_{1i}}q_{1i}(x,\theta_{1j})-\mu q_{1i}(x,\theta_{1j})\frac{\partial}{\partial\theta_{1i}}q_{1i}(x,\theta_{1j})\\&+\delta\frac{\partial}{\partial\theta_{1i}}U_{2i}(x,\theta_{2i})\Bigg].\end{aligned}$$

  Notice that from the local first-order condition, $K(\theta_{1i},\theta_{1i})=0$, so that

$$U(\theta_{1i},\theta_{1i})-U(\theta_{1i},\hat{\theta}_{1i})=\int_{\tilde{\theta}_{1i}}^{\theta_{1i}}\int_{\theta_{1i}}^{x}K_1(y,\theta)dydx$$

  where $K_1(x,\theta_{1i})=E_{\theta_{1j},\theta_{2j}}\left[\frac{\partial}{\partial\theta_{2i}}q_{2i}(\theta_1,\theta_2)\right]\leq 0$.

  Therefore $U_{1i}(\theta_{1i},\theta_{1i})\geq U_{1i}(\theta_{1i},\hat{\theta}_{1i})$ for all $(\theta_{1i},\tilde{\theta}_{1i})\in\Theta^2$ (since "x"$\leq\theta$ in the second integral).

  For any $(\theta_{2i},\tilde{\theta}_{2i})\in\Theta^2$, using (15) yields

$$U_{2i}(\theta_{1i},\theta_{2i},\theta_{2i})-U_{2i}(\theta_{1i},\theta_{2i},\hat{\theta}_{2i})=\int_{\tilde{\theta}_{2i}}^{\theta_{2i}}E_{\theta_{1j},\theta_{2j}}\left[\frac{\partial}{\partial\theta_{2i}}q_{2i}(\theta_1,x,\theta_{2j})\right]dx.$$

  Define

$$L(\theta_{1i},x,\theta_{2i})=E_{\theta_{2j}}\left[\frac{\partial}{\partial\theta_{2i}}t_{2i}(\theta_1,x,\theta_{2j})-\theta_{2i}\frac{\partial}{\partial\theta_{2i}}q_{2i}(\theta_1,x,\theta_{2j})-\mu q_{2i}(\theta_1,x,\theta_{2j})\frac{\partial}{\partial\theta_{2i}}q_{2i}(\theta_1,x,\theta_{2j})\right].$$

  Notice that from the local first-order condition, $L(\theta_{1i}\theta_{2i},\theta_{2i})=0$, so that

$$U_{2i}(\theta_{1i},\theta_{2i},\theta_{2i})-U_{2i}(\theta_{1i},\theta_{2i},\hat{\theta}_{2i})=\int_{\tilde{\theta}_{2i}}^{\theta_{2i}}\int_{\theta_{2i}}^{x}L_2(\theta_{1i},y,\theta_{2i})dydx$$

  where $L_2(\theta_{1i},x,\theta_{2i})=E_{\theta_{1j},\theta_{2j}}\left[\frac{\partial}{\partial\theta_{2i}}q_{2i}(\theta_1,\theta_2)\right]\leq 0$.

  And I can conclude that $U_{2i}(\theta_{1i},\theta_{2i},\theta_{2i})\geq U_{2i}(\theta_{1i},\theta_{2i},\hat{\theta}_{2i})$ for all $(\theta_{2i},\tilde{\theta}_{2i})\in\Theta^2$.

Proof of Proposition 2:

Note that here there is no need to specify two different payments, one transfer for the provision of the good and another to compensate the principal for funding D, between the principal and the cash-constrained firm. Instead of considering separately the repayment in the financial contract and the transfer for providing the good, I can (w.l.o.g) focus on the transfer (including both the provision payment and the repayment in the financial contract).

Compared to the situation studied in Section 3, nothing changes with regard to the second-period incentive compatibility. Since the funding of the cash-constrained firm (if it takes place) is sunk before this stage is played, the previous result remain valid. Notice however, that if there is only one firm around in the second period, its expected profits are bigger than when there are two firms around. Define the expected duopoly profit as $\Pi^d = E_{\theta_2}\left(\frac{F}{f}(\theta_{2i})q_{2i}(\theta_2)\right)$, where $q_{2i}(\cdot)$ remains to be determined, and the expected monopoly profit $\Pi^m = \int_{\underline{\theta}}^{\bar{\theta}}\int_{\theta}^{\bar{\theta}} \bar{q}dsdF(\theta) = \bar{q}\left(\bar{\theta} - E_{\theta_{1i}}(\theta_{1i})\right)$.

For future use, let us also define the procurement agency's expected total second-period payment when there is one firm (resp. two firms) left on the market as $T_2^m = \bar{\theta}\bar{q} + \mu\frac{\bar{q}^2}{2}$ (resp. $T_2^d = E_{\theta_2}[t_{2C}(\theta_2) + t_{2S}(\theta_2)]$).

In the first period, the expected payment required by a firm depends on its underlying financial situation through the refinancing variable $\beta(\cdot)$. For the cash-constrained firm I have[37],

$$E_{\theta_{1S}}\left(t_{1C}(\theta_1)\right) = E_{\theta_{1S}}\left\{\theta_{1C}q_{1C}(\theta_1) + \mu\frac{q_{1C}(\theta_1)^2}{2} + \int_{\theta_{1C}}^{\bar{\theta}} q(s,\theta_{1S})ds - \delta\beta(\theta_1)\Pi^d\right\},$$

whereas for the self-financed firm the expected transfer is

$$\begin{aligned}E_{\theta_{1C}}\left(t_{1S}(\theta_1)\right) = E_{\theta_{1C}}\Big\{&\theta_{1S}q_{1S}(\theta_1) + \mu\frac{q_{1S}(\theta_1)^2}{2} + \int_{\theta_{1S}}^{\bar{\theta}} q_{1S}(s,\theta_{1C})ds \\ &-\delta\beta(\theta_1)\Pi^d - \delta(1-\beta(\theta_1))\Pi^m - \delta D\Big\}.\end{aligned}$$

In fact, the refinancing variable $\beta(\cdot)$ modifies the continuation valuation for the firms in different ways and thus it also modifies the transfer they will require to satisfy participation (and incentive) constraints in the initial procurement stage.

[37] Assuming that the merged principal pays the sunk cost for this firm in each period, if, of course, this firm is allowed to continue.

Lemma 1 still hold and when taking into account the value of the respective transfers, the principal's relaxed problem becomes

$$\begin{aligned}\min_{q_C(\cdot),\beta(\cdot)} E_{\theta_1}\Big\{&\theta_{1C}q_{1C}(\theta_1)+\frac{F}{f}(\theta_{1C})q_{1C}(\theta_1)+\mu\frac{q_{1C}(\theta_1)^2}{2}-\delta\beta(\theta_1)\Pi^d\\&+\theta_{1S}(\bar{q}-q_{1C}(\theta_1))+\frac{F}{f}(\theta_{1S})(\bar{q}-q_{1C}(\theta_1))+\mu\frac{(\bar{q}-q_{1C}(\theta_1))^2}{2}\\&-\delta\beta(\theta_1)\Pi^d-\delta(1-\beta(\theta_1))\Pi^m+\delta D+\delta\beta(\theta_1)(T^d+D)\\&+\delta(1-\beta(\theta_1))T^m\Big\}\end{aligned}$$

Replacing expected second-period transfers and profits and looking at the first-order condition with respect to q_{2C} allows to conclude that second-period quantities remain at the first-best level.

From the minimization problem above, it is straightforward to observe that there are no terms with both $\beta(\cdot)$ and $q_{1C}(\cdot)$. The problem is therefore separable in $\beta(\cdot)$ and $q_{1C}(\cdot)$.

Looking first at the first-order condition with respect to $q_{1C}(\cdot)$, it is immediate that the optimal quantity $q_{1C}(.)$ is still given by Equation 5. It follows that when both firms are around in each period the decision rule does not change (but is of course dependent on the relevant period's private information).

To completely solve the optimization problem, the optimal value of $\beta(\cdot)$ remains to be determined. Denote by $V(q_{1C},\beta)$ the expression that the procurement agency is minimizing.

$$\frac{1}{\delta}\frac{\partial V(q_{1C},\beta)}{\partial\beta}=-2\Pi^d+T^d+D+\Pi^m-T^m. \tag{18}$$

Define $V^m=S(\bar{q})-T^m$ (respectively $V^d=S(\bar{q})-T^d$) and the previous expression can be written as $V^m+\Pi^m-(V^d-2\Pi^d-D)$. This is the difference in expected total surplus with only one firm in the second period and the equivalent surplus with two firms left in the second period. Replacing these expressions by their value and rearranging term yield

$$\frac{1}{\delta}\frac{\partial V(q_{1C},\beta)}{\partial\beta}=E_{\theta_2}\left\{q_{2C}(\theta_2)(\theta_{2C}-\theta_{2S})+\mu q_{2C}(\theta_2)(q_{2C}(\theta_2)-\bar{q})+D\right\}. \tag{19}$$

The second-term in this expression is negative. Note that $q_{2C}(\cdot)$ increases when the cash-constrained firm is relatively more efficient than the self-financed firm. In other words, the coefficient $q_{2C}(\cdot)$ is more important when $\theta_{2C}<\theta_{2S}$. The second term is therefore also negative. To finish the discussion on the optimal choice of $\beta(\cdot)$, note that if D is small enough, $\frac{\partial V(q_{1C},\beta)}{\partial \beta} \leq 0$. The optimal choice of β is thus one.

This implies that it is always optimal to keep the cash-constrained firm in the second period. Keeping the cash-constrained firm not only allows a more efficient split of the second-period production, but it also reduces the rent that the principal has to pay for second-period incentive compatibility. It is therefore optimal for the merged principals to always refinance the cash-constrained firm, and the solution obtained is the same as the one presented without financial contracts.

Proof of Proposition 3:
It is assumed that firms are protected by limited liability. In other words, none of the repayments stipulated in the financial contract can exceed the gains of the firm. This implies that, given that the true type and hence the profits of the firm are unobservable, a second-period repayment can never be bigger than the first-period profits of the firm net of its first-period repayment. This is because the firm can always strategically default in the second period and avoid any excessive payment. In fact, there is no possibility for the investor to screen second-period profits.

The remaining limited liability (LL) constraints are therefore, for all Π_1,

$$R_1(\Pi_1) \leq \Pi_1$$
$$\delta R_2(\Pi_1, \Pi_2) \leq \Pi_1 - R_1(\Pi_1) + \delta \underline{\Pi}_2$$

Notice that any potential repayment in the second period must come from the first-period profit since there is no incentives for the firm to report truthfully its second-period profits if its repayment is contingent on this. This is because there is no possibility of using a threat of non-refinancing the firm in future periods to act as a incentive device to truthfully report profits in the current period. In the second period, a firm will always strategically default on its repayment. And so any repayment in the second period can by redefining repayment variables be included in the corresponding first-period repayment and $R_2(\cdot) = 0$.

Taking Π_1 and Π^d as given, incentive compatibility (IC) requires, for all Π_1[38],

$$\Pi_1 \in \arg\max_{\hat{\Pi}_1 \leq \Pi_1} \Pi_1 - R_1(\hat{\Pi}_1) + \delta\beta(\hat{\Pi}_1)\Pi^d\big].$$

This incentive-compatibility constraint can be replaced by its first-order condition.

Lemma 2 *(Faure-Grimaud 1997, 2000) The incentive compatibility constraints is binding and* $\forall \Pi_1$,

$$-R_1(\Pi_1) + \delta\beta(\Pi_1)\Pi^d = C$$

The individual rationality of the investor, (IR^I), can be written as

$$\int_{\underline{\Pi}_1}^{\bar{\Pi}_1} \big[R_1(\Pi_1) + \delta\beta(\Pi_1)\big(R_2(\Pi_1) - D\big)\big] dG(\Pi_1) \geq D$$

Perfect competition on the financial market implies that this constraint will be binding.

The firm's maximization problem can be written as

$$\max_{\{R_1(),R_2(),\beta()\}} \int_{\underline{\Pi}_1}^{\bar{\Pi}_1} \Big[\Pi_1 - R_1(\Pi_1) + \delta\beta(\Pi_1)[\Pi^d - R_2(\Pi_1)]\Big] dG(\Pi_1)$$

subject to (IC), (LL) and (IR^I).

Using Lemma 2, I can define the Lagrangian as

$$\begin{aligned} L(C, \beta(\Pi_1)) =& [\Pi_1 + C + \alpha(-C + \delta\beta(\Pi_1))(\Pi^d - D)\big] g(\Pi_1) \\ & - \lambda_0\big[\beta(\Pi_1) - \frac{\Pi_1 + C}{\delta\Pi^d}\big] - \lambda_1\big[\beta(\Pi_1) - 1\big] + \lambda_2\beta(\Pi_1) \end{aligned}$$

where α is the multiplier associated with the individual rationality constraint. λ_0 is the multiplier associated with the limited liability constraint. λ_1 (respectively λ_2) is the multiplier associated with the requirement $\beta(\Pi_1) \leq 1$ (respectively $\beta(\Pi_1) \geq 0$).

The first-order conditions are

$$\begin{aligned} \frac{\partial L}{\partial C} =& (1 - \alpha)g(\Pi_1) + \frac{\lambda_0}{\delta\Pi^d} = 0 \\ \frac{\partial L}{\partial\beta(\Pi_1)} =& \alpha\delta(\Pi^d - D)g(\Pi_1) - \lambda_0 - \lambda_1 + \lambda_2 = 0 \end{aligned}$$

[38]This model follows Faure-Grimaud (2000) and only allows the firm to announce below its realized profits. The justification for this restriction on announcements is simply that announcing a certain level of profit consists of showing this amount to the principal. Other models such as Gale and Hellwig (1984) and Townsend (1979) also only allow announcements below the realized level of profits whereas Bolton and Scharfstein does not make this restriction.

Notice first that $\alpha \neq 0$. In fact, $\alpha = 0$ would imply that $\frac{\partial L}{\partial C} > 0$ which is impossible.

Furthermore, it is not possible to have $\lambda_0 = \lambda_1 = 0$ because that would imply that $\frac{\partial L}{\partial \beta(\Pi_1)} > 0$ (since $\alpha \neq 0$). So either $\beta(\Pi_1) = 1$ or $\beta(\Pi_1) = \frac{\Pi_1 + C}{\delta \Pi^d}$.

Define Π_1^* to be such that $\Pi_1^* = \delta \Pi^d - C$.

$\forall\ \Pi_1 < \Pi_1^*$, it is impossible to have $\beta(\Pi_1) = 1$ since it would violate limited liability. In fact, limited liability implies $\Pi_1^* - \Pi_1 \leq 0$. However, since $\Pi_1 < \Pi_1^*$ this gives a contradiction. So $\beta(\Pi_1) = \frac{\Pi_1 - C}{\delta \Pi^d}$. By definition of Π_1^*, I finally get $\beta(\Pi_1) = 1 - \frac{\Pi_1^* - \Pi_1}{\delta \Pi^d} < 1$.

$\forall\ \Pi_1 > \Pi_1^*$, limited liability is always satisfied for the same reason that it did not hold for $\Pi_1 < \Pi_1^*$. So $\lambda_0 = 0$ which implies that $\beta(\Pi_1) = 1$ (since λ_0 and λ_1 cannot simultaneously be equal to zero). From the definition of Π_1^* I get $R_1(\Pi_1) = \Pi_1^*$.

To complete the proof, I need to find C as big as possible or, equivalently, $\hat{\Pi}_1$ as big as possible, such that individual rationality is satisfied.

Formally, Π_1^* is such that

$$\int_{\underline{\Pi}_1}^{\bar{\Pi}_1} \big[R_1(\Pi_1) - \delta\beta(\Pi_1)D\big] dG(\Pi_1) = D,$$

or, equivalently

$$\Pi_1^* - \frac{\Pi^d - D}{\Pi^d} \int_{\underline{\Pi}_1}^{\Pi_1^*} G(\Pi_1) d\Pi_1 = (1+\delta)D.$$

Proof of Proposition 4:
Most of the proof of Proposition 4 follows straightforwardly from the discussion in the text. However, it remains to verify that second-order conditions hold.

For the self-financed firm, $\forall \theta_1$, $\frac{\partial q_{1S}}{\partial \theta_{1S}}(\theta_1) = \frac{-1}{2\mu}\left(1 + \frac{\partial \frac{F}{f}(\theta_{1S})}{\partial \theta_{1S}}\right) < 0$. And therefore, the second-order condition $E_{\theta_{1C}}(q_{1S}(\theta_1))$ holds.

For the cash-constrained firm,

$$E_{\theta_{1S}}\left[q_{1C}(\theta_{1C})\right] = E_{\theta_{1S}}\left(\frac{\bar{q}}{2} + \frac{\theta_{1S} + \frac{F}{f}(\theta_{1S}) - \theta_{1C} - \frac{F}{f}(\theta_{1C})}{2\mu}\right) - \frac{P}{2\Pi^d \mu}\frac{F}{f}(\theta_{1C})\left(1 - F(\tilde{\theta}_{1S}(\theta_{1C}))\right),$$

where $\tilde{\theta}_{1S}(\theta_{1C})$ is such that $\Pi(\theta_{1C}, \tilde{\theta}_{1S}(\theta_{1C})) = \Pi_1^*$ or equivalently $\int_{\theta_{1C}}^{\bar{\theta}} q_{1C}(s, \tilde{\theta}_{1S}(\theta_{1C}))ds = \Pi_1^* + C$.

Furthermore,

$$\frac{\partial}{\partial\theta_{1C}} E_{\theta_{1S}}\left[q_{1C}(\theta_{1C})\right] = -\frac{1}{2\mu}\Big(1 + \frac{\partial\frac{F}{f}}{\partial\theta_{1C}}(\theta_{1C}) + \frac{\partial\frac{F}{f}(\theta_{1C})}{\partial\theta_{1C}}\frac{P}{\Pi^d}(1 - F(\tilde{\theta}_{1S}(\theta_{1C}))) \\ - \frac{\partial\tilde{\theta}_{1S}(\theta_{1C})}{\partial\theta_{1C}}\frac{P}{\Pi^d}\frac{F}{f}(\theta_{1C})f(\tilde{\theta}_{1S}(\theta_{1C}))\Big), \tag{20}$$

and

$$\frac{\partial\tilde{\theta}_{1S}}{\partial\theta_{1C}} = \frac{q_{1C}(\theta_{1C}, \tilde{\theta}_{1S})}{\frac{1+\frac{d\frac{F}{f}}{d\theta_{1S}}(\tilde{\theta}_{1S})}{2\mu}(\bar{\theta} - \theta_{1C})} > 0.$$

So, finally, the second-order condition for the cash-constrained firm holds if and only if the two last term in (20) are not too positive.

Proof of Proposition 7:
The proof of Proposition 7 is by contradiction. Suppose that the optimal financial debt contract is degenerate. In other words, $R_1(\cdot) = \Pi_1^*$ and $\beta(\cdot) = 1$ regardless of the realized profits of the firm. Furthermore, Π_1^* has to be less or equal to $\Pi_1(\bar{\theta}, \theta_{1S})$, for all θ_{1S}.

Recall that $U(\bar{\theta}) = 0$. With transfers and quantities in dominant strategy, this can be rewritten for any θ_{1S},

$$\Pi_1(\bar{\theta}, \theta_{1S}) + \delta\beta(\Pi_1(\bar{\theta}, \theta_{1S}))\Pi^d = R_1(\Pi(\bar{\theta}, \theta_{1S}))$$

Replace $R_1(\cdot)$ and $\beta(\cdot)$ by their (degenerate) values. This yields

$$\Pi_1(\bar{\theta}, \theta_{1S}) + \delta\Pi^d = \Pi_1^*$$

Since $\delta\Pi^d > 0$, $\Pi_1(\bar{\theta}, \theta_{1S}) < \Pi_1^*$. A contradiction. The optimal financial contract is therefore not degenerate.

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