Level-k Thinking in Average Bid Procurement Auctions[*]

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Abstract

Contributions to economic literature have provided different explanations to the frequently observed deviations from equilibrium bidding in auctions. An approach to address the issue consists of nonequilibrium models where bidders are characterized by different levels of ability - i.e. *level-k thinking* - in performing an iterated process of strategic playing. We use this approach to investigate average bid auctions. Using a dataset on regional public procurement auctions in Italy and exploiting an exogenous change in the average bid format, we show that observed behavior of bidders in average bid auctions can be predicted by their level of *sophistication*, which is related to their past experience in analogous auctions and cognitive ability. We show that our empirical evidence is consistent with a level-k model of bidders' behavior.

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1 Introduction

Contributions to economic literature have provided different explanations to the frequently observed deviations from equilibrium bidding in auctions. Most of these explanations preserve the equilibrium approach, but hypothesize that actual bidders' payoff functions differ from the standard ones. For example, observed bids higher than the risk neutral Nash equilibrium in private-value auctions, a typical experimental anomaly, has been explained with *risk aversion* (Cox, Smith and Walker, 1983; 1988), *joy of winning* (Cox, Smith and Walker, 1992; Holt and Sherman, 1994) and *anticipated regret* (Filiz-Ozbay and Ozbay, 2007). An alternative approach to address the issue consists of nonequilibrium models in which bidders beliefs are not mutually consistent: an example of this approach is the *level-k thinking* model according to which bidders are characterized by different levels of ability in performing an iterated process of strategic playing so that a bidder fail to consider the possibility that others may be doing as much or more steps of thinking. Crawford and Iriberri (2007) have shown that a level-k thinking model may indeed explain nonequilibrium bidding in auction experiments, like winner's curse in common value and overbidding in private value auctions.[1]

In this paper we study nonequilibrium bidding behavior in average bid auctions - auctions where the bid closest to some endogenously defined threshold (average) wins.[2] A large empirical evidence - as well as ours - shows that, in these auctions, offers do systematically differ from equilibrium predictions.[3] To explain the observed deviation from the predicted equilibria, we borrow the intuition behind the level-k literature and hypothesize that bidders (firms) are heterogeneous' in their cognitive abilities (which we call "sophistication") and that their behavior is related to their level of sophistication.

We empirically investigate this question in Section 2. To do so, we define a measure of bidders' sophistication and test whether our intuition is supported by real data on average bid auctions taken from a dataset of procurement auctions in the Italian region of Valle d'Aosta.

[1]The *level-k thinking* approach (also known as cognitive hierarchy) has been initially developed by Stahl and Wilson (1994, 1995) and Nagel (1995), and further applied - among others - by Ho et al. (1998), Costa-Gomes et al. (2001), Bosch-Domenech et al. (2002), Crawford (2003), Camerer et al. (2004), Costa-Gomes and Crawford (2006).

[2]For extended investigations about average bid formats adopted in several national contexts, see Decarolis (2010); Decarolis and Klein (2011).

[3]This is well documented by empirical investigations on Italian public procurement average bid auctions (see, Bucciol, et al. 2013; Coviello and Gagliarducci 2010; Coviello and Mariniello 2010; Coviello, et al. 2012; Coviello, et al. 2012; Decarolis 2009; Decarolis 2012; Moretti and Valbonesi 2012).

Addressing a bidder's sophistication is not an easy task and - empirically - its measure can be related to unrevealed bidder's characteristics. We define an indicator that jointly takes into account: *(i)* bidder's past experience in that auction format and in that specific market; *(ii)* bidder's past ability to offer bids close to the actual (and auction-specific) *focal point* (that will be defined below). We are aware that this measure could suffer of problems of measurement error. In fact, the average bid is the typical format of auction used to award public works contracts in Italy and, thus, bidder's experience could have been gained in regions or periods that we do not observe. However, our dataset is particulary interesting because it records an exogenous change in the average bid format which allows us to control for problems of measurement error and to run a natural experiment.

In fact, until 2006, the region Valle d'Aosta - as well as many other Italian regions - used to award procurement contracts using the following average bid auction format (which we abbreviate in AB): given the distribution of all bids received for an auction, a first average ($A1$) is computed by averaging all bids except those located in the first and last deciles; then, a second average ($A2$) is computed by averaging all bids above $A1$ (again excluding those bids located in the last decile). The winning bid is the one immediately below $A2$ (see Figure A in Appendix). In case all bids are equal, the winner is chosen randomly. Given the rules of this auction, the value of $A2$ is crucial: to increase the probability of winning a bidder has to correctly predict the value $A2$ and place a bid close to it. It is then natural to define $A2$ as the focal point for the AB auction.

Since 2006, and *only* in the region we observe (Valle d'Aosta), a different and new average bid format has been introduced that includes an aleatory element (we call this format AB+lottery, or simply ABL). This format works as follows: given the threshold $A2$ computed as above, a random number (R) is extracted from the set of the nine equidistant numbers between the lowest bid above the first decile and the bid just below $A2$. Averaging R with $A2$, the winning threshold W is obtained and the winning bid is the one closest from above to W (see Figure B in Appendix). If no bid satisfies this criterion, the winning bid is the one which is equal or, in absence, closest from below to W. Notice that W will necessarily fall within an interval whose endpoints are the midpoint between the first decile and $A2$ (call this midpoint $A3$) and $A2$ itself. Clearly, given $A3$ and $A2$, any bid has the same probability of winning, since the exact position of W in this interval is the outcome of a lottery. However, given $A3$ and $A2$, a bid closer to $A3$ is for sure better, since it guarantees a higher payoff in case of winning. Thus, a bidder must correctly predict the values of $A3$ and $A2$ and, given these predictions, should locate his bid close to $A3$. It is then natural to define $A3$ as the focal point for the ABL auction.

The change in the average bid format from AB to ABL occurred in 2006 allows us to control whether the supposed relationship between a bidder's level of sophistication and his bid is affected by problems of measurement error since, given that the policy change is circumscribed to Valle d'Aosta, we can be confident that our indicator of bidder's sophistication computed after 2006 is not influenced by bidder's experience gained elsewhere or in periods we do not observe.

Moreover, this exogenous change gives us the opportunity to provide further evidence in a difference-in-difference spirit. In fact, we test whether more sophisticated bidders better adapt their bids to the new format of auction (ABL) with respect to less sophisticated bidders and to the previous format of AB.

Our estimation results show that, in both formats of average bid auctions we analyze, more sophisticated bidders offer bids that are closer to the predicted focal points. Moreover, this relationship is even stronger when the new format of average bid auction (ABL) is exogenously introduced.

In Section 3, we show that these empirical results are in line with the predictions from a level-k model of bidder's cognitive ability, in which level-0 bidders bid randomly according to some frequency distribution, while level-k bidders, $k \geq 1$, make optimal bids against level-0 to level-$k-1$ bidders. In fact, as the simulations we perform show, the model predicts that level-1 and level-2 bids will be concentrated around a specific value, which is close to the focal point, while level-0 bids will be dispersed and, on average, further from the focal points. Thus, level-0 bidders' predicted behavior seem to track the observed behavior of those bidders' whose value of sophistication is low, while level-1 and level-2 bidders' predicted behavior seem to track the observed behavior of those bidders' whose value of sophistication is high.

Section 4 briefly concludes.

2 Empirical evidence

2.1 Data

To investigate bidders' behavior in average bid auctions, we use data for the procurement of public works awarded from 2000 to 2009 by an Italian local contracting authority (CA) -the Regional Government of Valle d'Aosta-[4] by means of open auctions. The general rule governing these auctions is relatively simple: firms participate to the auction by offering a price consisting in a percentage discount - a rebate - on the reserve price set by the CA, and once the CA has verified the bidders's legal, fiscal, economic, financial and technical requirements, the winning firm is chosen according to AB or AB+lottery mechanism. Particularly, we have detailed information for 267 auctions, each corresponding to a tendered project of which we know some of characteristics (such as the reserve price, the tasks of the tendered project or the estimated duration of the works). For each auction we also have information about all bidding firms' names and offered rebates (the dataset we use is from Moretti and Valbonesi 2012).

Our dataset covered 267 auctions for public contracts, for which a total of 12,522 bids offered. The average reserve price (i.e., the price set by the CAs on the contracts awarded) was approximately 1.1 million euros (ranging from a 155,000 to 5.2 million euros), on average 74 bidders participated to an auction (ranging form 1 to 155 bidders), and they offered an average percentage rebate of 17.1%. In terms of tasks, these contracts refer mainly to road works (44.2%), river and hydraulic works (37.6%), and special structural works (9.3%). The subsample of the AB auctions includes: 10,555 bids offered by 626 different bidders; an average reserve price of about 1.1 million euros; on average 69 participants, and an average rebate of 17.9. While the subsample if AB+lottery auctions includes: 1,967 bids by 385 bidders; an average reserve price of 1.1 million euros; on average 103 participants which offered an average rebate of 13.5% (see Table 1 for summary statistics about the used sample of auctions/projects by type of awarding mechanism).

[4] Valle d'Aosta is a small mountainous region (3,263 sq. km, 951 MSL) with a population of 129,000 on Italy's north-western borders with France and Switzerland.

Table 1. Auctions' summary statistics

Type of acution/Variable	Obs.	Mean	St.Dev.	Min	Max
AB auctions (n=238):					
Rebate (%)	10555	17.852	4.726	0.001	42.060
log(Reserve price)	238	13.626	0.773	11.955	15.477
log (Participants)	238	3.760	0.719	0	4.883
log (Expected duration)	238	5.638	0.508	4.369	7.272
Road works	238	0.382	0.487	0	1
River and hydraulic works	238	0.298	0.458	0	1
Buildings	238	0.071	0.258	0	1
ABL auctions (n=29):					
Rebate (%)	1967	13.530	4.039	1.56	31.732
log(Reserve price)	29	13.760	0.545	12.900	14.980
log (Participants)	29	4.237	0.706	2.079	5.043
log (Expected duration)	29	5.899	0.471	4.860	6.733
Road works	29	0.310	0.471	0	1
River and hydraulic works	29	0.310	0.471	0	1
Buildings	29	0.138	0.351	0	1

Measurement of bidders' level of sophistication. One of the main challenge is to measure level of sophistication of a bidder. In this work we make the hypothesis that the bidder's joint knowledge of the market and focal points ($A2$ in AB and $A3$ in AB+lottery) represents a good proxy to reveal the bidder's sophistication.

In particular, we construct our indicator (*BidderSoph*) as bidder's past experience in auctions issued by a single CA (Regional Government of Valle d'Aosta) weighted by the distance of its bid from the auction-specific focal point. The higher the number of a bidder's past bids and the closer they are to the (auction-specific) focal point, the higher its level of sophistication. In case a bidder had intensely participated to past auctions for public works issued by the Regional Government of Valle d'Aosta but its bids were not close to the predicted focal point, or in case it did not have intensely participated but its bids were close to the focal point, we do not consider its joint knowledge of market and focal points particularly sophisticated. More precisely, a bidder's past participation takes value 1 if its bids was exactly equal to $A2$ in an AB auction, or to $A3$ in an ABL auction; it takes value 0 if its bids was the most distant from the focal point, among all submitted bids; it takes intermediate values according to the relative distance of its bid from the focal point. We sum up this measure by firm chronological order of participation and by format of auction (AB and ABL), so that the weighted experience of the firm in AB auctions does not count in ABL

auctions.

Since the AB format is commonly used in the awarding of contracts for public works in Italy, our indicator of past experience might suffer of problems related to measurement error as we are not able to take into account the firm's participation to AB auctions issued by any other Italian CA but the Government of Valle d'Aosta. We believe equally important the joint knowledge of the auction format and the market, so the experience gained by the bidder through the participation to auctions issued by other CAs, which govern markets with different characteristics, can be slightly downgraded. Furthermore, we are also not able to observe the firm's past experience in auctions issued by the Government of Valle d'Aosta before year 2000. For all these reasons, we consider particularly important to tackle this potential measurement error.

However, Valle d'Aosta was the only Italian region to replace in 2006 AB format with ABL format and this allows us to obtain for the latter format a measure of weighted past experience and performance which is not influenced by the firms' participation to similar auction format in other regions or in the past. In other words, looking at the ABL reform in Valle d'Aosta allows us not only to test the presence of bidders' hierarchical sophistication in two different average bid formats, but also to address the problems associated with the measurement error of our indicator of bidders' sophistication and to better capture the relationship between the bidders' characteristics and the bidding behavior (see Table 2 for summary statistics about our sophistication index and bidders' participation).

Table 2. Bidders' summary statistics

Type of auction: variable	Obs.	Mean	St.Dev.	Min	Max
AB: BidderSoph	10555	20.001	21.152	0	133.441
ABL: BidderSoph	1967	3.189	3.493	0	18.398
AB: Participation (626 bidders)	626	16.861	24.173	1	162
ABL: Participation (385 bidders)	1967	5.109	4.627	1	24

2.2 Distributional evidence

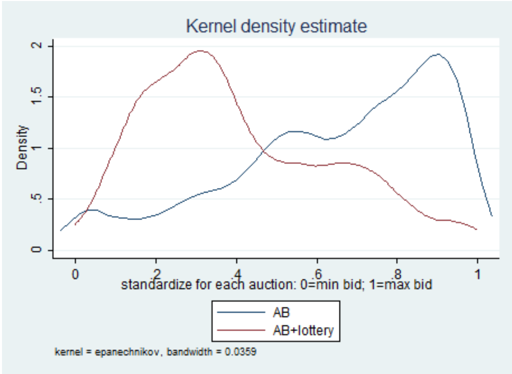
Before providing parametrical evidence of our predictions, we show some non-parametric kernel density estimation of the distributions of bids with the two different format of average bids auctions. Graph 1 shows the estimated distribution of the bids for the AB (blue line) and ABL (red line) formats. For each auction, the bids have been re-scaled using a min-max normalization (the lowest rebate in a auction takes value 0, while the highest takes value 1).

Graph 1 shows that there is more than one pick in the density of the distribution. In particular, for AB auction there are at least two picks: one around the value .9, which usually represents the area around $A2$, and one around the value .5 which usually represent the area around the average bids. Similarly, in the estimated distributions of bidding rebates for the ABL auctions we observe a concentration of bids around $A3$ (about the value 0.4) and around the value .8.

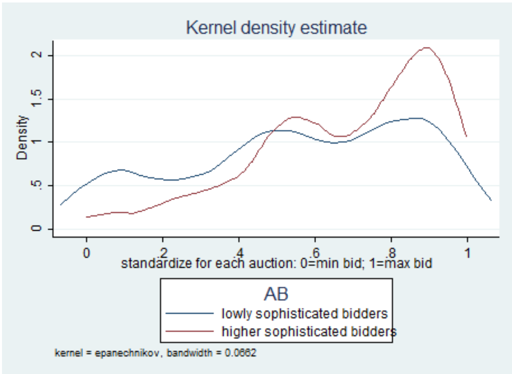
This descriptive evidence tells us three different facts: i) bids are not uniformly distributed; ii) there is a concentration of bids around $A2$ and $A3$ for both AB and ABL formats, respectively; iii) bids are concentrated also around values different than the focal points.

The comparison in the distribution of bids between highly sophisticated bidders and lowly sophisticates bidders (ie. those bidders with a value of the measure of sophistication above 90 percentile vs. those with a value below 10 percentile) in AB (Graph 2) and ABL (Graph 3) give us additional and more precise support to our predictions. In particular, Graph 2 and 3 show that lowly sophisticated bidders tend to offer bids less concentrated on particular values than highly sophisticated bidders. Furthermore, the graphs tell us that the highly sophisticated bidders are those that tend to concentrate their offered rebates around $A2$ in AB and $A3$ in an ABL environment.

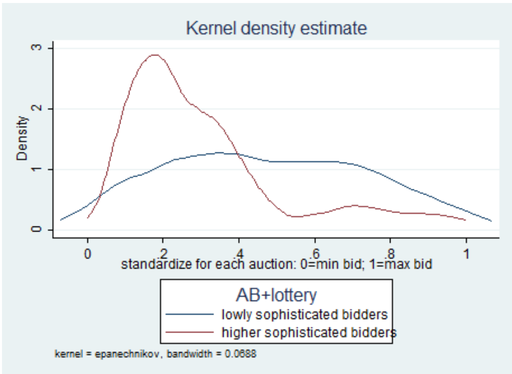
Graph 1: Kernel estimation density of bids within AB and ABL auctions



Graph 2: Kernel estimation density of bids within AB auctions



Graph 3: Kernel estimation density of bids within AB+lottery auctions



2.3 Model specification and estimation

Model specification. To better explore the previous evidence and the prediction about a relationship between bidder's level of sophistication and its bidding behavior, we introduce and estimate a reduced form model, which takes into account the projects' characteristics and bidders' characteristics.

The model specification for bidding rebates looks as follows:

$$log|Distance|_{ij} = \alpha + \beta log(BidderSophij) + \gamma Q_j + \theta X_i + \epsilon_{ij}. \tag{1}$$

where the dependent variable $Distance$ is the log of the absolute difference between the bidder's i offered rebate and the auction-specific focal point ($A2$ if auction's i has an AB format or $A3$ is auction's i has an ABL format). $BidderSoph$ is variable which measure the level of sophistication of the bidder. Q_j is a set of variables to control for the characteristics of the project and auctions: i.e., proxies for characteristics of the project such as its dimension or complexity and the type of work involved, and proxies for the auction's characteristics, such as the level of competitive pressure, the format (AB or ABL) of the auction, and year dummy variables to adjust for temporal shocks that might have affected both the time-related trends of the firm bidding behavior and the contracts chosen by the CA. In some specification we include auction fixed effects to reduce omitted variable problem and to exploit within auction variation of the indicator of bidders' sophistication. X_i represents a set of characteristics of the firm: again, to reduce the omitted variable problems, in some specifications, we also included firm's fixed effects to adjust for firm-specific characteristics (e.g., size, productivity, financial position, and location); this enabled us to focus on the within firm variation in the sophistication status. ϵ_{ij} is the error component.

Regression results. In Table 3, we present our first estimation results, which show that higher level of bidder's sophistication is associated with a smaller distance between the offered rebate and the (auction-specific) focal points.[5] This means that higher sophisticated bidders are more likely to offer bids close to the focal points than less sophisticated ones and confirm our predictions. This result is robust to different model specifications: a) when we include auction's covariates (columns 1 and 3) or when we included auction's fixed effects (columns 2 and 4); b) when we use

[5] Note that the number of observation is reduced to 11,472 bids because of the presence of firm's fixed-effects we need to exclude bidders that participated only one time - most of those are consortia.

our continuous variable representing the degree of sophistication (*BidderSoph*; columns 1 and 2) or we use categorial dummy variables for medium or high level bidder's sophistication (*MediumSoph* and *HighSoph* - low level of sophistication is the excluded category, columns 3 and 4).

Table 3: Baseline regression results

Dependent variable:	log |Distance|			
	1	2	3	4
(log)BidderSoph	-.197***	-.089***		
	(.026)	(.029)		
MediumSoph			-.260***	-.144***
			(.053)	(.054)
HighSoph			-.526***	-.227***
			(.071)	(.080)
Bidder fixed effects	YES	YES	YES	YES
Auction fixed effects	NO	YES	NO	YES
Auction controls	YES	NO	YES	NO
Observations	11472	11472	11472	11472
Adj. R-squared	0.223	0.393	0.221	0.393

OLS regression with robust SE clustered at firm-level;
significance levels: *** p<0.01, ** p<0.05, * p<0.1.

As previously discussed, to better exploit our dataset and to reduce the measurement error associated with our measure of bidder's sophistication, we look at the differential behavior of bidders with different level of sophistication in ABL and AB format of auction. The ABL format introduced only in Valle d'Aosta since 2006 allows us to gauge more accurate difference between highly and lowly sophisticated bidders as their experience in bidding for this particular format of average bid auction is not influenced by past or somewhere else gained experience.

We expect that if more sophisticated bidders' offered a rebate closer to the auction's focal point, this would be true also in ABL (i.e. the estimated coefficient of the interaction $BidderSoph{*}AB + lottery$ would be negative). Furthermore, given the fact that the ABL is a totally new format to all bidders, we expect that highly sophisticated bidders have an advantage over to lowly sophisticated ones in understanding the new mechanism, thus the estimated difference between highly and lowly sophisticated bidders would be higher in this format than in the AB format.

Estimation results reported in Table 4 confirm our hypothesis that i) in both format of auction highly sophisticated bidders offer rebates closer to the auction's focal point respect to less sophisticated ones, ii) this is particularly true in a newly introduced average bid format of auction (i.e. the *ABL* format in the context of procurement auctions in Italy). These results are hold a) when split the sample between *AB* and *ABL* formats (columns 1 and 2) or when we interact our measure of sophistication with the *ABL* format (columns 3 and 4); b) when we control for bidders fixed effects and auctions' characteristics (column 3) or when we also introduce auctions' fixed effects (column 4); c) hen we use our continuous variable representing the degree of sophistication (*BidderSoph*; columns 3 and 4) or we use categorial dummy variables for medium or high level bidder's sophistication (*MediumSoph* and *HighSoph* - low level of sophistication is the excluded category, columns 5 and 6).

Table 4: Regression results using the exogenous change in average bid format

Dependent variable:	log |Distance|					
	1	2	3	4	5	6
(log)BidderSoph	-.130***	-.440***	-.150***	-.059***		
	(.032)	(.066)	(.026)	(.028)		
(log)BidderSoph*(AB+lottery)			-.281***	-.215***		
			(.048)	(.057)		
MediumSoph					-.187***	-.109*
					(.057)	(.057)
HighSoph					-.385***	-.159*
					(.079)	(.083)
MediumSoph*(AB+lottery)					-.498***	-.313**
					(.126)	(.145)
HighSoph*(AB+lottery)					-.727***	-.466***
					(.147)	(.171)
Bidder fixed effects	YES	YES	YES	YES	YES	YES
Auction fixed effects	NO	NO	NO	YES	NO	YES
Auction controls	YES	YES	YES	NO	YES	NO
Sample	AB	AB+lottery	Both	Both	Both	Both
Observations	9896	1576	11472	11472	11472	11472
Adj. R-squared	0.223	0.341	0.225	0.394	0.223	0.393

OLS regression with robust SE clustered at firm-level; significance levels: *** p<0.01, ** p<0.05, * p<0.1.

2.4 Selection bias issues

The analysis of bidders' offers may suffer from problems relating to a selection bias because different levels of sophistication could also reflect structural and technological differences that influence the firms' decision to participate in an auction. In other words, if an high level of bidder's sophistication reflect firm's characteristics which reduce costs of project's execution, those firms' expected profits increase, so their probability of participation.

In our sample, potential bidders are all Italian firms qualified to operate in the public works market, but from our data we cannot estimate the probability of firms participating in auctions because we do not have data for all Italian firms' qualifications. Having included fixed effects allows us to focus on the with-in firm variability in the level of sophistication, which is by construction different for each auction the firm participates in (as the indicators is a weighted average of firms' past participations).

However, in this section we propose the application of a two-step Heckman model to explicitly take into account the selection bias problems, which might influence our main results. Given the problem of unavailability of data for all Italian firms' qualifications, we define the sample of potential participants in a conservative way, following Gil and Marion (2012) and Moretti and Valbonesi (2013). We consider as potential participants those firms that in a given year have participated at least to one auction in the Region of Valle d'Aosta for a given type of project (i.e. for projects with the same main category of work), since firms have to be qualified for the main category of work of the project to participate to an auction.

The other element we need to define in order to estimate the probability of participation is an exogenous instrument that is related to the probability of firms' participation but have an influence only on the costs of participation. We use the length of time between the date project is advertised and when the bid letting occurs (this instrument is also used by Gil and Marion (2012) and Moretti and Valbonesi (2013). The hypothesis is that longer is the time between the beginning of project's publicity and the deadline for bid's submission, longer is the time for firms to evaluate the project and the relative bid to submit, and lower is the cost associated with entry. Our data show that there is variability in terms of auctions' advertise lead time, with an average of 34.1 days (and a standard deviation of 33.6 days).

In Table 5, we report estimation results based on auctions for projects having road works as main category of work, since this is the most numerous category of work we observe in our data (46.5% of the auctions). In columns 1 and 2 of Table 5, we report OLS estimations of the probability of firm's participation and offered rebate, respectively, using our benchmark model specification (eq.

1). Estimation results show that the firm's higher level of sophistication is associated with higher probability of participation, and with offered rebates closer to the focal points (confirming that our main estimation results hold for the subsample of road works).

In columns 3 and 4 of Table 5, we report the first and second stage of an two-step Heckman selection model. Estimation results show that our instrument is positively (and statistically significant) associated with higher probability of participation as we expected, and that in the second stage of the model the estimated (statistically significant) coefficient of the firms' level of sophistication indicates that firms' with higher level of sophistication tend to bid closer to the focal points.

Table 5: Two step Heckman selction model: Road works

	OLS		Heckman	
Model:	Participation eq.	log |Distance|	Participation eq.	log |Distance|
Mean outcome:	0.28	-0.477	0.28	16.32
	1	2	3	4
(log)BidderSoph	0.071***	-0.190***	0.255***	-0.168**
	(0.006)	(0.026)	(0.031)	(0.022)
Auction advertise lead time			0.176***	
			(0.036)	
Res.price/Exp.dur.	YES	YES	YES	
Type of auction dummy	YES	YES	YES	
Firm size dummy	YES	YES	YES	
Year dummy	YES	YES	YES	
Observations	16,462	4,610	16,462	
Censored obs.			11,852	
Uncensored obs.			4,610	

Bootstrapped standard errors clustered at firm-level in parentheses.
*** $p<0.01$, ** $p<0.05$, * $p<0.1$.

3 Theory: equilibrium and a level-k model

A single contract is auctioned off. There are n firms participating in the auction. Each firm i has a cost c_i of completing the job. This cost is private information to the firm, but it is commonly known that costs are independent and identically distributed according to a strictly increasing cumulative distribution function $F(\cdot)$ over the interval $[\underline{c}, \bar{c}]$. Firms submit sealed bids formulated in terms of percentage discounts over the reserve price R. Let $d_i \in [0, 1]$ denote firm i's bid (discount). The firm submitting the winning bid d^* obtains the contract and it is paid $(1 - d^*)R$. Firm i's expected payoff is thus:

$$\pi_i(d_i; c_i) = [(1 - d_i)R - c_i]\,\mathrm{Pwin}(d_i),$$

where $\mathrm{Pwin}(d_i)$ is the probability that firm i wins when she bids d_i. Clearly, $\mathrm{Pwin}(d_i)$ (and thus π_i) depends on the bidding functions adopted by other firms and on the auction format.

In the AB auction, the winning bid is the bid closest *from below* to $A2$. If all bidders make the same bid, the contract is assigned randomly.

In the ABL auction, the winning bid is the bid closest *from above* to W, provided that this bid does not exceed $A2$. If no bid satisfies this requirement, the winning bid will be the one equal, if there is one, or closest from below to W.

We now characterize the properties of symmetric equilibria of these auctions.

Lemma 1. *Let $\delta_K(c)$, $K = AB, ABL$ denote a symmetric equilibrium of either type of auctions and assume it is almost everywhere continuous. The following three properties hold for both auction formats.*

(i) In equilibrium, the probability of winning the auction is strictly positive for all types $c \in [\underline{c}, \bar{c})$.

(ii) Equilibrium bids are weakly decreasing.

(iii) Equilibrium bids are flat at the bottom: there exists $\underline{c} < \hat{c} \leq \bar{c}$ such that $\delta(c) = \bar{d}$, for all $c \in [\underline{c}, \hat{c}]$. Notice that, because of (i), $\bar{d}$ is the highest discount offered.

Proof.

(i) Consider an interval (a, b) in which $\delta(\cdot)$ is continuous and weakly monotone. Then, necessarily $\mathrm{Pwin}(\delta(c)) > 0$ for all $c \in (a, b)$. This is obvious if $\delta(\cdot)$ is constant at c (in this case there will be a strictly positive probability that all bidders make the same bid, so that each bidder will have a $1/n$ probability of winning the auction). If instead $\delta(\cdot)$ is strictly monotone at c, then it is always possible to take two subintervals (a, a_1) and (b_1, b), with $a_1 < c < b_1$, such that, if m bidders' types belong to the first interval and $n - m - 1$ bidders' types belong to the

second, the winning bid is $\delta(c)$. Clearly, If the probability of winning is strictly positive for all $c \in (a,b)$, it must be strictly positive also for all types $c \leq a$ (by incentive compatibility, the probability of winning must be weakly decreasing in c). Now, because $\delta(\cdot)$ is continuous a.e. for all $\varepsilon > 0$, there will always be a subinterval of $[\bar{c}-\varepsilon, \bar{c}]$ in which $\delta(\cdot)$ is continuous and weakly monotone. Thus, we can always find a $c < \bar{c}$ such that $\text{Pwin}(\delta(c)) > 0$.

(ii) Suppose, by contradiction, that $\delta(\cdot)$ is not weakly decreasing. Then, there must exist types c_1 and c_2, with $c_1 > c_2$ such that $\delta(c_1) > \delta(c_2)$. Now, since $\delta(\cdot)$ is an equilibrium, it must hold that $[(1-\delta(c_1))R - c_1]\text{Pwin}(\delta(c_1)) \geq [(1-\delta(c_2))R - c_1]\text{Pwin}(\delta(c_2))$. Therefore, because of point *(i)*, it must be $\text{Pwin}(\delta(c_1)) > \text{Pwin}(\delta(c_2))$. But this contradicts the fact that, in equilibrium, $\text{Pwin}(\delta(c_1)) \leq \text{Pwin}(\delta(c_2))$.

(iii) Suppose, to the contrary, that $\delta(\cdot)$ is strictly decreasing at the bottom, i.e. that $(\delta(\underline{c})) > (\delta(c))$, for all $c \in (\underline{c}, \bar{c}]$. This implies that $\text{Pwin}(\delta(\underline{c})) = 0$, which cannot hold (see point *(i)*).

From the properties above, we can now derive more precise predictions on the Bayesian equilibria in the two formats.

Lemma 2.

(i) In the AB auction, there is a unique symmetric equilibrium in which bidders make a 0*-discount irrespective of their costs. The contract is assigned randomly.*

(ii) In the ABL auction, it is a symmetric equilibrium for bidders to make a constant discount d *irrespective of their costs, provided that* $d \in [0, 1-\bar{c}/R]$.

(iii) In the ABL auction, in equilibrium, the set of bidders' types who make the highest discount $\bar{d}$ *must be sufficiently large (i.e.* $\hat{c}$ *must be sufficiently high).*

Proof.

(i) By lemma 1, we know that, in equilibrium, types $c \leq \hat{c}$ make the same bid $\bar{d}$. Now, consider a bidder i of this type: he will win the AB auction if and only if all other bidders bid $\bar{d}$, in which case every bidder will have a $1/n$ chance of winning. If, instead, bidder i decreases his bid below $\bar{d}$, in case all other bidders bid $\bar{d}$ he will be the sole winner (moreover, with

a smaller discount), which is clearly profitable. The only situation in which no profitable deviation exists is when it is not possible to decrease bids, i.e. when $\bar{d} = 0$.

(ii) In ABL, If all bidders' types make the same bid $d \in [0, 1 - \bar{c}/R]$, every bidder will have a $1/n$ chance of winning. If bidder i increases is bid, then $A2$ will necessarily be equal to d and bidder i will not win as his bid exceeds $A2$. If bidder i decreases is bid, then W will necessarily be equal to d and the winner will be one of the other bidders. Clearly, all bidders' types making the same bid $d > \bar{c}/R$ cannot be an equilibrium, as in this case the expected payoff of a type $\bar{c}$ bidder will be strictly negative.

(iii) We give only an intuition of the proof. Suppose there exists an equilibrium of the ABL auction in which all types below $\hat{c} < \bar{c}$ bid $\bar{d}$ and all other types make strictly lower bids. Consider a bidder i of type $c < \hat{c}$. Let m the number of bidders who bid $\bar{d}$ (including bidder i) and $n - m$ the number of bidders bidding below $\bar{d}$ and denote by $\tilde{n}$ the lowest integer greater than or equal to $n/10$. The winning bid will be $\bar{d}$ if: (a) $n - m \leq \tilde{n} + 1$; or (b) $n - m > tilden + 1$ and W turns out to be greater than the highest bid of the $n - m$ bidders bidding less than $\bar{d}$. In all these cases, bidder i will have (only) a $1/m$ probability of winning the auction. Now suppose bidder i slightly decreases his bid below $\bar{d}$. Now, if $n - m \leq \tilde{n}$, again the winning bid will be $\bar{d}$ and bidder i will not win the auction. This is the cost of lowering his bid below $\bar{d}$. However, if $n - m > \tilde{n}$ then, whenever W turns out to be greater than the highest bid of the $n - m$ bidders bidding less than $\bar{d}$, bidder i will (essentially) be the sole winner. This is the benefit of lowering his bid below $\bar{d}$. Thus, a necessary condition to have an equilibrium of this kind is that the cost of deviating for a bidder like i must exceed the benefit. Hence, the probability that $n - m \leq \tilde{n}$ has to be sufficiently low. But this probability is nothing but $[1 - F(\hat{c})]^{\tilde{n}}$; this means that $\hat{c}$ must be sufficiently high.

Interestingly, what drives the difference between AB and ABL in terms of equilibria is not the fact that in ABL the winning threshold is computed differently and adding an element of casuality, but rather the fact that the winning bid is the one closest to the relevant threshold from *below* in AB, but from *above* in ABL.

However, there is a feature which is common to all equilibria of the two auctions: most or all bidders' types submit the same discount. All types making a 0-discount is an equilibrium in both auctions (in AB, it is the unique). In ABL, we have a continuum of flat equilibria in which all types make the same discount, provided that this discount guarantees a positive payoff in case of winning to all types. In ABL, in addition, there can be other equilibria, but all these must display a sufficiently large degree of pooling: all types below a sufficiently high threshold $\hat{c}$ make the same discount, which, because of the monotonicity of bidding functions, is the highest submitted discount.

A quick look at the data reveals immediately that actual bids are very different from what equilibrium analysis predicts: in both auctions bids are somewhat dispersed and for sure are far from 0. In the ABL auction, it is true that we observe a concentration of bids around a certain value; however this value is relatively low, while equilibrium would predict concentration of bids on the highest discount.

To offer a possible explanation of these deviations from equilibrium behavior we consider the cognitive hierarchy model introduced by Stahl and Wilson (1994, 1995) and Nagel (1995) and developed by Camerer, Ho and Chong (2004). Crawford (2007) has shown that this model is able to explain frequently documented phenomena like winner's curse and overbidding in first- and second-price auctions.

According to this model, players in a game differ by their level of "sophistication", i.e. their ability of performing an iterated process of strategic thinking. The proportion of each level in the population is given by a frequency distribution $P(k)$, where $k = 0, 1, 2, \ldots$ is the level of sophistication. Level-0 players are completely unsophisticated and simply play according to some probability distribution; a level-k player ($k = 1, 2, 3, ..$) believes that his opponents are distributed, according to a normalized version of $P(k)$, from level-0 to level-$(k-1)$ and choose their optimal strategy given these beliefs. For example, level-1 players believe their opponents are all of level-0; level-2 players believe their opponents are a mixture of level-0 and level-1 players, where the proportion of level-0 players is $P(0)/(P(0) + P(1))$; and so on. Essentially, while level-0 players are fully naive, level-k players, $k \geq 1$, are rational, but differ in their beliefs: a level-k player anticipates that he is facing players of levels 0 to $k-1$, but ignores the possibility that some players may be doing as much or more steps of thinking. Therefore, the higher is the level of a player, the more accurate are his beliefs on other players' behavior.[6]

[6]Thus, this model exhibits "increasing rational expectations". Crawford and Iriberri (2007) adopt a slightly

To assess what the implication of this model of behavior are in our context, we run a series of simulations. The model leaves us two degrees of freedom: the choice of the frequency distribution of the different levels $P(k)$ and the specification of level-0 players' (in our context, bidders') behavior. We will follow Camerer et al. (2004) and assume that the distribution of levels is Poisson with parameter λ.[7] As far as the specification of level-0 bidders' behavior is concerned, we will assume that they draw their bids d from a uniform distribution over a specific interval $[\underline{d}, \bar{d}]$.[8] This is the most common specification in the level-k literature and also the most neutral one.

In our simulations, we will confine our analysis only to level-1 and level-2 bidders because, as it is shown below, optimal bids by level-1 and level-2 bidders turn out to be rather close, especially when the number of bidders is large; thus, considering higher levels will not give additional insights, at least qualitatively. Moreover, experimental evidence has shown that the majority of subjects performs no more than 2 levels of iteration and that an average of 1.5 levels fits data from many games. Thus, even if higher levels can be present, their proportion should be relatively low.

The simulations were run using the following parameters:

- reserve price (R): 100;
- costs uniformly distributed on the interval $[\underline{c} = 50, \bar{c} = 70]$, with increments of 0.2;
- level-0 bidders bid according to a uniform distribution over the interval $[\underline{d} = 0, \bar{d} = 0.3]$;
- number of bidders (n): 10, 50;
- expected number of levels (λ): 0.5, 1, 1.5.

For each of the two auction formats, we run 6 simulations, one simulation for each possible combination of n and λ. This will allow us to assess whether and how the outcome is affected by the number of bidders and by the proportion of different levels. The other parameters are chosen to avoid that level-0 bidders end up having negative payoffs (a level-0 bidder with maximum cost (70) who makes the maximum discount (0.3) will get, in case he wins the auction, a payoff equal to zero).

different specification in which level-k players best respond only to level-$k-1$ players.

[7] In a Poisson distribution, $P(k) = e^{-\lambda}\frac{\lambda^k}{k!}$, $k = 0, 1, 2, \ldots$ The parameter $\lambda > 0$ is both the expected value and the variance of the distribution.

[8] The endpoint of this interval could for example be inferred from the distribution of bids in past auctions of the same kind.

The following table reports optimal discounts for level-1 and level-2 bidders in the two auctions. Discounts are normalized: discount of 10% in the table means an actual discount of 3%.

		AB		ABL	
n	λ	level 1	level 2	level 1	level 2
10	0.5	0.63 if $c \leq 55.6$ 0.62 if $55.6 < c$	0.64	0.57	0.54
	1	”	0.58	”	0.56 if $c \leq 60$ 0.54 if $60 < c$
	1.5	”	0.60 if $c \leq 55.6$ 0.59 if $55.6 < c$	”	0.51
50	0.5	0.69 if $c \leq 61.4$ 0.68 if $61.4 < c$	0.70	0.51	0.50
	1	”	0.70	”	0.50
	1.5	”	0.68	”	0.50

The observation of these numbers suggest the following considerations:

1. In both auctions, bids are essentially insensitive to costs.

2. Level-1 and level-2 bids are very close, especially when n is large.

   The intuition is straightforward: level-2 bidders optimize against the optimal bids of level-1 bidders. Clearly, the presence of level-1 bidders and the fact that they all essentially make the same bid will cause the winning threshold to be close to level-1 bidders' bid.

3. Bids are always lower in ABL than in AB.

   This is obvious as, for given bids, the winning threshold in ABL is necessarily lower than in AB. Thus, given that the behavior of level-0 bidders is assumed identical in the two auctions, level-1 bidders' optimal bids are lower in ABL than in AB; thus, also level-2 bidders' bids, which best-respond to level-0 and level-1, will be lower in ABL.

4. As n increases, optimal bids tend to increase in AB and to decrease in ABL. Thus, the difference between bids in the two auctions gets larger.

   This is true for both level-1 and level-2 bids. The reason is simple: take AB and consider a level-1 bidder i. For low n, the variance of the winning threshold is high. Placing a bid very close to the expected value of the winning threshold is thus risky because there is a high probability that the winning threshold will fall below bidder i's bid, in which case bidder i would not win. It is then optimal to reduce the bid. Moreover, since n is low, the probability that a level-0 bidder will place his bid between bidder i's bid and the winning threshold is low. Instead, when n is large, the probability that the actual winning threshold will be lower than expected is low, while the probability that the a level-0 bidder will place his bid between bidder i's bid and the winning threshold is high. Essentially, as n increases, bidder i has an incentive to bid closer (from below) to the expected value of the winning threshold. The same is true for level-2 bidders. An identical argument applies to ABL, but here, since the rules of the auction state that the winning bid is the one closer from above to the threshold, bids tend to decrease as n increases.

5. The effect of a change in λ on optimal bids is not clear.

These predictions seem to mimic quite well what we observe in our field data:

- As predicted by the model, in both auctions we actually do observe a concentration of bids in a narrow interval plus a number of dispersed bids.

- As predicted by the model, actual bids in AB are concentrated on significantly higher values than in ABL.

- The fact that our indicator of sophistication is a good predictor of the distance between a bidder's bid and the actual winning threshold is consistent with our model: our intuition is that bidders with a high value of sophistication should be more likely to belong to level-1 and level-2 and thus should concentrate their bids close to the expected winning threshold.

- Finally, although the proportion of different levels in the population (the value of λ) has no clear effect on optimal bids, by definition a higher value λ means a higher proportion of level-1 and level-2 bidders; hence, as λ increases, we should expect higher proportion of concentrated bids with respect to dispersed bids. Indeed we observe more concentration of bids over time, which is consistent with the prediction of the model as the average indicator of sophistication is necessarily increasing over time.

4 Conclusion

In this paper we study, empirically and theoretically, two different versions of the average bid auctions. In particular, starting from the observation that observed behaviors of bidders in this auctions display systematic deviations from the theoretical Nash equilibria, we propose a non-cooperative non-equilibrium explanation of such behavior. In this respect, our paper represent a novelty in the literature on average bid auctions, where most papers have focused on cooperative (i.e. collusive) arguments.

The explanation we offer borrows its intuition from the level-k thinking literature: bidders differ in their level of sophistication, with unsophisticated bidders bid randomly, while more sophisticated best respond to the behavior of less sophisticated bidders. For both auction formats we consider, the prediction of this model are consistent with the results of our analysis of actual bidders' behavior from a dataset taken from average bid auctions in the Italian region of Valle d'Aosta.

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Appendix A

Figure A: Average price auction - AB format

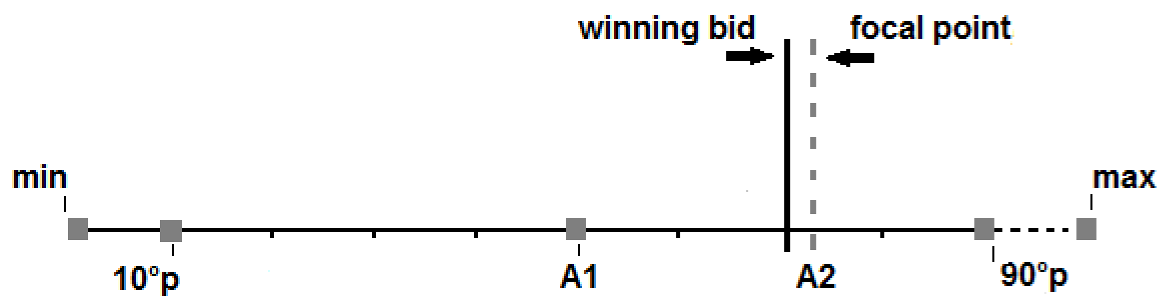

Figure B: Average price auction with uncertainty - AB+lottery format