Favoritism in Russian public procurement: does e-auction format matter

Maria Ostrovnaya and Elena Podkolzina

Abstract
Since the 2000s e-auctions have been actively used in public and private procurement in many countries. They are often organized by intermediaries – specialized e-platforms, which seem to raise price competition through increased transparency and lower participation costs. Meanwhile, it is not clear yet to what consequences e-auctions lead, if an e-platform is corrupt. In this paper we examine how a corrupt e-platform affects favoritism in public procurement. We have found that the impact of a corrupt e-platform on favoritism depends on characteristics of the preferred bidder. The model shows that if the preferred bidder carries out low costs, corruption of e-platform encourages favoritism. On the contrary, if the preferred bidder carries out high costs and meets the strict requirements of the public procurer, incentives for favoritism and the size of the bribe may decrease. Thus, corruption of e-platform replaces the corruption of the public procurer and the society can benefit from it.

Keywords: Favoritism, Public procurement, Auctions
JEL codes: C7, D44, L5

1. Introduction
Public procurement constitutes 10-25% of GDP and a significant share of domestic demand in most countries, and Russia is no exception. These huge financial resources can be used to achieve various purposes: provision of goods and services, fostering innovation and small business development, etc. Corruption can hinder this, so it is one of the key problems in public procurement. E-auctions are usually treated as transparent public procedures, which can solve this problem (e.g. Lengwiler & Wolfstetter; 2006). However, current procurement practice does not always support these conclusions. The purpose of this article is to show how a corrupt electronic platform, which is responsible for conducting e-auctions, affects favoritism, the effectiveness of the auction and social welfare. By favoritism we mean a special type of corruption when the procurer (hereinafter it) receives a bribe only from its preferred bidder

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5 http://www.gks.ru/metod/torg.html
(hereinafter he) and manipulates the requirements for purchased product or service in order to change his chances of winning the auction.

We consider Russian e-procurement reform as the starting point for this paper. In 2009-2010 the Russian government implemented the reform of public procurement, which was aimed at decreasing incentives for corruption, including favoritism. The reform presumed that e-auctions organized by specialized e-platforms would replace traditional (outrary) auctions organized by public entities (mostly, by various public procurers). It was implicitly assumed that e-platforms operated for the sake of public interests, but this situation is not always true. As far as corruption is wide-spread in Russia and employees working on the e-platform have incentives to take bribes, perhaps, some employees took bribes from bidders in exchange for assistance in winning e-auctions. According to the anecdotal evidence, bidders have made many complaints to the regulator about actions of e-platform that prevented them from bidding in e-auction. For instance, one company could not make bids in several auctions, because of some technical difficulties on e-platform that did not prevent the other bidder from making a bid and winning e-auction with a with slight decrease in price. This situation may be caused either by real technical difficulties, or by corrupt behavior of e-platform that is in the center of this paper.

We model traditional auction, which is organized by the public procurer, and e-auction, which is organized by a corrupt e-platform, and compare their results. We have found that the impact of a corrupt e-platform on favoritism depends on characteristics of the preferred bidder. If the preferred bidder has low production costs, the corrupt e-platform stimulates favoritism. If the preferred bidder has high production costs, meets the restrictive requirements of the public procurer, and e-platform requires a large bribe, incentives for favoritism and bribe may decrease. Thus, corruption of e-platform replaces the corruption of the public procurer.

This paper is organized as follows. Section 2 briefly analyzes the literature related to our paper. Section 3 describes public procurement in Russia and summarizes key differences between e-auctions and traditional auctions. Section 4 proposes a theoretical model of favoritism in public procurement and considers its preliminary results. Section 5 concludes and summarizes ideas for the future research.

2. Literature review

This paper is close to two strands of economic research. Firstly, since we compare the results of two types of auctions (traditional auction and e-auction), the empirical and theoretical literature that analyzes the various procurement mechanisms is closely related to our work. In

\[ \text{http://www.transparency.org/cpi2013/results} \]
\[ \text{http://forum.gov-zakupki.ru/topic16447.html} \]
\[ \text{We consider corrupt electronic platform as a single player. Drawing parallels with the reality separate employees can take bribes.} \]
this literature auctions are usually treated as objective competitive mechanisms, while the
negotiations, on the contrary, are an extreme case of the discretionary power of the auctioneer.
Bulow & Klemperer (1996) and Laffont & Tirole (1991a) are among the first researchers who
started the theoretical discussion. Bulow & Klemperer (1996) compare the auction and
negotiates and conclude that in most cases one additional bidder in the auction leads to a better
result for the auctioneer than negotiations with a smaller number of bidders. Laffont & Tirole
(1991a) examine in a theoretical model what rule the government should set to choose the
supplier. The authors identify four cases based on how the information about non-price
characteristics of the bidders is distributed and which bidder that can make a corrupt deal with
the procurer: any bidder or only preferred one. Subsequent empirical work has shown that the
choice between auction and negotiation depends on such factors, as the complexity of the
contract, the competence of the public procurer, the competition among bidders (Bajari et al.,
2007, 2009; Estache et al., 2009; Chong et al., 2010) and the level of political competition
(Chong et al., 2011; Moszoro & Spiller, 2012).

Since the beginning of the 2000s articles on e-auctions have been published. They address
various issues arising in electronic public procedures, for instance, new forms of rent-seeking
behavior (Kauffman & Wood, 2003) the redistribution of income between the seller and the
buyer (Chong et al., 2006) and unequal access of different companies to e-procurement (Albano
et al., 2008). Most studies state such features of e-auctions, as lower participation costs and the
anonymity of bidders. Conditions of Russian public procurement make more crucial another
feature of e-auction – the participation of the intermediary in the procurement process, on which
we focus in this paper.

Secondly, we rely on research on corruption. Søreide (2002), Boehm & Olaya (2006) show
that public procurers have ample opportunities to restrict competition before an auction starts.
Burguet & Che (2004), Compte et al. (2005) examine the relationship between corruption and
competition of procurers and bidders. A separate layer of papers is dedicated to the question of
how intermediaries affect corruption between officials and their clients (Bayar, 2005; Drugov et
al., 2014; Hasker & Okten, 2007). These articles assume that the client can give a bribe to
official directly or through an intermediary. Using the intermediary is costly, but reduces the risk
that the client will give a bribe to the honest official and suffer huge losses. In laboratory
experiments Drugov et al. (2014) have shown that the use of the intermediary reduces the moral
costs of officials and clients, therefore, encouraging corruption.

Our approach differs from previous papers in the following way. We assume that the e-
platform organizes e-auction and, as the public procurer, can take bribes from bidders. However
a corrupt deal between the public procurer and the bidder takes place without any participation of
e-platform. Collusion between the public procurer and the e-platform is beyond the scope of this paper. The idea of our study is to analyze how the corrupt behavior of e-platform affects incentives to favoritism in public procurement.

3. **Russian e-procurement**

In this section we will consider two procurement procedures: traditional auction and e-auction. Traditional auctions were wide-spread in the Russian public procurement in 2006-2010; in 2010 they were replaced by e-auctions. Procurement legislation and practices were gradually changing, and in this section we will describe the main features of the two types of auctions, when they were clearly defined.

Traditional auctions were run in three stages, namely, registration, (dis)approval of applications and the auction itself. At the first stage procurer placed information about the auction at the official web-site, and then bidders applied for participation in the auction. Their applications contained two parts that were put together: a bidder’s agreement to perform the contract (supply goods of certain quality) and private information about bidder (identity). At the second stage the special commission of the public procurer considered the applications of bidders. If the commission approved an application, a bidder could participate in traditional auction; if the commission rejected it, he could not. At the third stage the procurer organized an auction with approved bidders. The bidder who made the lowest bid won the auction, and procurer announced it on the regional web-site.

The process of e-auction is more complex because of the appearance of a new player – e-platform, which serves as an intermediary between the public procurer and bidders. E-platform registers bidders, organizes e-auction and exchanges information with the public procurer. E-auction consists of four stages and runs as follows. At the first stage procurer chooses any of five accredited e-platforms and provides her with the auction documentation. Then both procurer and e-platform put information about the auction at the official web-site and web-site of e-platform, respectively, and bidders apply for participation in e-auction. In order to guarantee anonymity, bidders submit their applications in two separate parts: a bidder’s agreement to perform the contract in one part, and private information about the bidder in another. At the second stage the auction commission considers the first parts of applications. At the third stage the e-platform holds e-auction, where only bidders whose applications have been approved can participate. When e-auction finishes, the platform ranks bids made by bidders in ascending order and provides the public procurer with the ranking and the second parts of applications. At the fourth stage public procurer considers applications starting from the application of the bidder that has made the lowest bid (Article 60 point 3, 44 Federal Law). At the fifth stage the public procurer
announces the winner of e-auction. The winner is the bidder, which made the lowest bid among the bidders, which applications were approved.

The Table 1 shows how the differences between traditional auction and e-auction influence their results. Most researchers and practitioners distinguish such differences, as the level of participation costs and the anonymity of the auction. Firstly, e-auction decreases the transaction costs of bidders [e.g., Garicano & Kaplan; 2001]. The lower the transaction costs are, the more companies will participate in auction and the more they will reduce the price. Secondly, e-auctions provide anonymity of bidders through technical tools [Li et al., 2011; Treisman, 2000]. So obtaining the information about costs or previous prices of other bidders is more costly than in traditional auction. Therefore, bidders will compete more aggressively in e-auction.

**Table 1. Differences between e-auctions and traditional auctions**

<table>
<thead>
<tr>
<th>Differences</th>
<th>Consequences</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. E-auctions decrease participation costs</td>
<td>More bidders =&gt; final prices decrease in e-auctions</td>
</tr>
<tr>
<td>2. E-auctions provide anonymity of bidders</td>
<td>Bidders less likely recognize costs or previous prices of their competitors =&gt; bidders bid more aggressively =&gt; final prices decrease in e-auctions</td>
</tr>
<tr>
<td>3. E-auctions are organized by intermediary (e-platform)</td>
<td>Corruption of e-platform =&gt; payoffs of the public procurer and its preferred bidder change =&gt; conditions of favoritism and the size of a bribe change in e-auction</td>
</tr>
</tbody>
</table>

We believe that these differences do not have significant impact on the results of Russian procurement auctions. Firstly, for a number of industries, such as construction, services, oil and gas industry, high transportation costs may exceed the effect of low participation costs. Even if e-auctions reduce participation costs to zero, effective companies located in the Kamchatka region will rarely participate in auctions for road construction in the Volga region, as transportation costs are very high. Therefore, reducing the participation costs in this case will not lead to a significant increase in the number of bidders. Secondly, there is probably no significant difference between costs of obtaining information about production costs of competitors in traditional auctions and e-auctions. There was s small number of bidders in both types of auctions. According to the Federal State Statistics Service, approximately three - four companies participated in both traditional and e-auctions on average in 2009-2010; slightly more than two companies participated in e-auctions on average in 2011-2012. In addition to it, procurement auctions are held periodically from year to year and all information about prices is published on the official website. Thereby, companies can roughly calculate bids that will make their competitors in both types of auctions.

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In our opinion, the key difference between traditional auction and e-auction is that e-auction is organized by the e-platform. The behavior of the e-platform changes expected payoffs of the public procurer and its preferred bidder, and hence, incentives to favoritism. On the one hand, the e-platform can fight favoritism: report violations to the regulator and block "suspicious" auctions. However, at this moment, the incentives for such behavior are practically absent, since payoffs of the employees working on e-platforms are not connected with any indicators of competition in e-auctions. On the other hand, these employees can behave opportunistically and take bribes from bidders. As far as the public procurer is also susceptible to corruption, there may be one corrupt deal in traditional auction and two corrupt deals - in e-auction.

In the next section we will construct a theoretical model of favoritism in public procurement. We will show how the restriction of competition before the auction affect the effectiveness of the auction and social welfare, as well as how corrupt behavior of e-platform affects favoritism and effectiveness of the auction.

4. Model of favoritism in public procurement

A. Basic assumptions and set-up

A public procurer receives funding size \( r \) for the purchase of one indivisible product via auction. We wonder which type of auction maximizes social welfare and leads to less favoritism: traditional auction organized by the public procurer or e-auction organized by e-platform. The format for both auctions is a reverse English auction, which we model as a “clock auction” following Ausubel (Ausubel, 2003; p. 137).

Two bidders participate in each auction: bidder 1 (the preferred bidder) and bidder 2. They have two characteristics \((c_i; s_i)\). Firstly, each bidder carries out production costs \(c_i, i = 1, 2\), which with equal probability can be high \((\bar{c})\) or low \((\underline{c}, r > \bar{c} > \underline{c} > 0)\). Secondly, each bidder with equal probability can execute the contract with restrictive conditions \((s_i = S)\) or cannot do it \((s_i = 0)\). These restrictive conditions may be any non-price characteristics of the public contract, which can be inherent not to all bidders. Let us assume that the values of characteristics \(c_i\) and \(s_i\) are not connected with each other. The procurer and bidders have information about probabilities with which each bidder can have a set of certain characteristics \((c_i; s_i)\) at the beginning of the game.

\(^{10}\) For example, the public procurer is going to buy gasoline through gas stations. Let us assume that there are two gas stations in the city, which are ready to perform the contract. Each gas station with a non-zero probability can be located close to the public procurer \((s_p = S)\) or far from it \((s_p = 0)\). Location of each gas station is not connected with production costs, but affects the utility of the public procurer. The procurer receives greater utility from the contract with the gas station, which is located near it, than from the contract with the gas station, which is located far from it, \(v(S_{WIN} = S) > v(S_{WIN} = 0)\).
Before the auction the procurer sets minimal requirements to the contract conditions $s_p$, which influences its expected utility $EU_{proc}$ in the following way:

$$EU_{proc} = Ev(s_{WIN} | s_p), v(S) > v(0) > 0$$  \hspace{1cm} (1)

where $v(s)$ – the utility of the public procurer depending on contract conditions $s$; $s_{WIN}$ – the type of the contract that the winner can execute: the contract with restrictive conditions ($s_{WIN} = S$) or without them ($s_{WIN} = 0$).

Bidder $i$ can participate in auction, if $s_i \geq s_p^{11}$. Hence, if the procurer does not set restrictive conditions ($s_p = 0$), all bidders can participate in auction. If the procurer sets restrictive condition ($s_p = S$), bidders that can execute only contract without restrictive conditions ($s = 0$) cannot participate in auction. Hence, with non-zero probability none of the bidders can execute the contract with restrictive conditions. In this case, the auction will not take place, and all players receive zero payoffs.

If only one bidder participates in auction, it wins auction at the reserve price $r > \bar{c}$. If two bidders participate in auction, the auctioneer (public procurer or e-platform) organizes reverse English auction, which starts from the reserve price $r$. Since we have assumed that production costs of the bidders are independent from each other, the bids in this auction will coincide with the bids in the similar second-price sealed-bid auction (Ausubel, 2003; p. 138). Auction runs as follows. Auctioneer consistently reduces the price; when the price becomes equal to the production costs of a bidder, he refuses to continue the auction. Another bidder wins the auction at this price.

Thus, if two bidders participate in auction, they get the following profits:

$$\pi_1 = \begin{cases} c_2 - c_1, & \text{if } c_1 < c_2; \\ 0, & \text{if } c_1 \geq c_2. \end{cases}$$  \hspace{1cm} (2.1)

$$\pi_2 = \begin{cases} c_1 - c_2, & \text{if } c_2 < c_1; \\ 0, & \text{if } c_2 \geq c_1. \end{cases}$$  \hspace{1cm} (2.2)

If e-platform is honest, her expected payoff $ER$ equals zero. Expected public spending equal expected minimal bid $P_i$ made by the bidders:

$$EG = E\min\{P_i\}, i = 1,2.$$  

Expected social welfare equals the sum of the expected payoffs of all players without transfer $r$ to the public procurer:

$$EW = EU_{proc} + \sum E\pi_i - EG + ER, i = 1,2$$  \hspace{1cm} (3)

In the next part of the paper we will sequentially examine several cases:

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11 We assume that rent-seeking behavior does not arise at this stage of the procurement, and each company honestly reports, whether it can perform the contract or not.
1. case 1: social optimum,
2. case 2: information asymmetry,
3. case 3: information asymmetry and corruption of procurer (traditional auction),
4. case 4: information asymmetry, corruption of procurer and e-platform (e-auction).

B. Social optimum

Here we consider the case when information is symmetrically distributed among the players; they are honest and maximize social welfare. When the nature assigned characteristics \((c_i; s_i), i = 1, 2\) to bidders, the public procurer selects the conditions in order to maximize social welfare for realized characteristics of bidders:

\[
W = U_{proc} + \sum \pi_i - G, i = 1, 2,
\]

\[
W = v(s_{WIN}|s_P) + P_{WIN}(s_P) - c_{WIN}(s_P) - P_{WIN}(s_P),
\]

\[
W = v(s_{WIN}|s_P) - c_{WIN}(s_P)
\]

(4)

where \(P_{WIN}\) – the final bid that is equal to production costs of the bidder, which loses auction; \(c_{WIN}\) – production costs of the bidder, which wins the auction.

The Table 2 illustrates the timing of the game.

<table>
<thead>
<tr>
<th></th>
<th>The nature assigns characteristics ((c_i; s_i)) to each bidder.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Procurer sets conditions (s_p) maximizing social welfare under certain ((c_i; s_i)) and ((c_i; s_2)).</td>
</tr>
<tr>
<td>3</td>
<td>Each bidder registers for participation in auction, if (s_i \geq s_p, i = 1, 2).</td>
</tr>
<tr>
<td>4</td>
<td>If two bidders have registered, the procurer organizes reverse English auction.</td>
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<td>If one bidder has registered, he wins auction at price (r &gt; \bar{c}).</td>
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</tr>
<tr>
<td>5</td>
<td>The procurer buys a product.</td>
</tr>
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</table>

In other words, the public procurer compares the size of the public welfare at each pair \((c_1; s_1)\) and \((c_2; s_2)\), when it sets the restrictive conditions and when it does not set them. Each bidder can have one of four sets of production costs and ability to execute a contract with the restrictive conditions: \((\bar{c}; S), (\bar{c}; S), (\bar{c}; 0), (\bar{c}; 0)\). For each pair of these sets, the public procurer makes the following choice:

\[
s_p; \max \{v(s_{WIN}|S) - c_{WIN}(S); v(s_{WIN}|0) - c_{WIN}(0)\}
\]

(5)

If bidders have the same characteristics, differ only in production costs, or differ in two characteristics and only efficient bidder can execute a contract with the restrictive conditions, the public procurer does not restrict competition: \(s_p = 0\). If bidders differ only in their ability to execute a contract with restrictive conditions, the public procurer restricts competition: \(s_p = S\).
In all these cases, the purchase is carried out effectively, as production costs of the winner of the auction are not higher than that of the other bidder.

If bidders differ in two characteristics and only less efficient bidder can execute a contract with the restrictive conditions, the choice of the public procurer depends on the relation between the difference between high and low costs and the difference between the public procurer’s utility when the contract is with or without restrictive conditions. Let $\Delta c = \bar{c} - \zeta$ (production costs effect), $\Delta v = v(S) - v(0)$ (effect of restricting competition). Then:

1. If $\Delta c \geq \Delta v$,
   
   $s_p = 0 \Rightarrow W = v(0) - \tilde{c} + \bar{c} - \zeta = v(0) - \zeta$.
   
   $Prob(\text{efficient wins}) = 1$,

2. If $\Delta c < \Delta v$,
   
   $s_p = S \Rightarrow W = v(S) - r + r - \tilde{c} = v(S) - \bar{c}$,
   
   $Prob(\text{efficient wins}) = 0$,

where $Prob(\text{efficient wins})$ – the effectiveness of the auction, which equals the probability that this auction took place and the production costs of the winner were not higher, than production costs of the other bidder.

Then the expected social welfare and the effectiveness of auction equal:

1. If $\Delta c \geq \Delta v$,
   
   $EW = \frac{3}{8}v(0) + \frac{5}{8}v(S) - \frac{1}{4}\bar{c} - \frac{3}{4}\zeta$,
   
   $Prob(\text{efficient wins}) = 1$.

2. If $\Delta c < \Delta v$,
   
   $EW = \frac{1}{4}v(0) + \frac{3}{4}v(S) - \frac{3}{8}\bar{c} - \frac{5}{8}\zeta$,
   
   $Prob(\text{efficient wins}) = \frac{7}{8}$.

Thus, if the production costs effect exceeds the effect of restricting competition, the choice of the public procurer is effective and optimal for the society. If the production costs effect is lower than the effect of restricting competition, there is a conflict between the effectiveness of auction and the maximum social welfare.

C. Information asymmetry

Let us now assume that information is asymmetrically distributed among the players: a bidder $i$, $i = 1, 2$ has information about his characteristics $(c_i; s_i)$, the public procurer knows only the distribution of them. Bidders do not unveil their characteristics to the public procurer, because it can reduce their informational rent. Hence, the public procurer chooses the conditions
maximizing its expected utility on the basis of expectations about bidders’ characteristics (compare with the formula 5):

\[ s_p \text{: max}\{Ev(s_{WIN}|S); Ev(s_{WIN}|0)\} \] (6)

In contrast to the social optimum, the utility of the public procurer depends only on whether the auction winner can execute the contract with the restrictive conditions and does not depend on his production costs. Table 3 illustrates the timing of the game.

**Table 3. Timing**

<table>
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<td>2</td>
<td>Procurer sets conditions ( s_p ) maximizing its expected utility under certain expectations about ( (c_1; s_1) ) and ( (c_2; s_2) ).</td>
</tr>
<tr>
<td>3</td>
<td>Each bidder registers for participation in auction, if ( s_i \geq s_p, i = 1,2 ).</td>
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When the nature assigns characteristics \( (c_i; s_i) \) to each bidder, the public procurer sets contract conditions \( s_p \). Bidders register for participation in auction, which goes the same, as in the case of the social optimum. If the public procurer does not restrict competition, both bidders participate in the auction. The bidder that can execute the contract with restrictive conditions will be the winner with probability 0.5, and the bidder that cannot execute this contract - with probability 0.5. If the public procurer restricts competition, no bidder participates in the auction with probability 0.25, and the bidder that can execute the contract with restrictive conditions will be the winner with probability 0.75. The public procurer chooses \( s_p \) that gives it higher expected utility:

1. If \( \frac{v(S)}{v(0)} < 2 \),

\[
EU_{proc}(s_p = S) < EU_{proc}(s_p = 0) \Rightarrow s_p = 0,
\]

\[
EW = \frac{1}{2} v(0) + \frac{1}{2} v(S) - \frac{1}{4} c - \frac{3}{4} c.
\]

\[
Prob(\text{efficient wins}) = 1.
\]

2. If \( \frac{v(S)}{v(0)} \geq 2 \),

\[
EU_{proc}(s_p = S) \geq EU_{proc}(s_p = 0) \Rightarrow s_p = S,
\]

\[
EW = \frac{3}{4} v(S) - \frac{5}{16} c - \frac{7}{16} c.
\]

\[
Prob(\text{efficient wins}) = \frac{5}{8}.
\]
Thus, if the information is asymmetrically distributed, the social welfare is maximum and the auction is effective under two following two conditions:

\[ \Delta c \geq \Delta v \] – production costs effect exceeds the effect of restricting competition,

\[ \frac{v(S)}{v(0)} < 2 \] – small difference between the public procurer’s utilities if the winner can or cannot execute the contract with restrictive conditions.

In other situations, information asymmetry reduces social welfare in comparison to the social optimum.

D. Corruption in traditional auction

In traditional auction the public procurer can be corrupt and favor one bidder over the other one. Let us assume that the public procurer has long-term relations with the bidder 1 (the preferred bidder) based on their previous experience or informal connections and is not familiar with the bidder 2. Each bidder must incur non-zero costs of organizing a corrupt deal (see Laffont & Tirole, 1991b). For instance, a bidder must convince the public procurer that he wants to give a bribe, agree on the amount of bribe and secretly give a bribe after the contract is made.

In order to give 1 € bribe to the public procurer, a bidder has to spend \( (1 + \alpha_i) \), where \( \alpha_i \) – is the value of organizational costs of a bidder \( i, i = 1, 2 \). Since the public procurer has different relations with the bidder 1 and the bidder 2, the bidder 2 should carry out much more organizational costs in order to organize a corrupt deal with the public procurer. Therefore, the organizational costs of bidders equal \( \alpha_1 = \alpha, \alpha \in [0; 1], \alpha_2 = +\infty \), and the bidder 2 cannot give a bribe to the public procurer.

The public procurer can abuse its discretionary power taking a bribe \( B \) in exchange for setting contract conditions that the bidder 1 surely meets\(^{12}\). Information about the utility of the public procurer and the requirements for the contract conditions, that it is going to set, is public. Hence, the bidder 1 gives a bribe to the public procurer only if the requirements for the contract conditions, which the public procurer is going to set without a bribe, and the requirements that the bidder 1 meets, do not coincide: \( s_p \neq s_1 \).

If the bidder 1 can execute a contract with restrictive requirements, he can give the public procurer a bribe for setting restrictive requirements and preventing the bidder 1 from participation in auction with probability 0.5. In contrast, if the bidder 1 cannot execute a contract with restrictive requirements, he can give the public procurer a bribe for setting no requirements. So the bidder 1 will be able to participate in the auction.

\(^{12}\) We model such type of bribery as a rollback, because it is widely spread in public procurement. The bidder pays the public procurer pre-arranged amount of the bribe, if it wins the auction.
The Table 4 illustrates the timing of the game. Unlike the previous case, the public procurer and the bidder 1 can make a corrupt deal; hence, the public procurer can receive the information about characteristics of the bidder 1 and sets contract conditions which he meets.

**Table 4.** Timing

<table>
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<td>2</td>
<td>The bidder 1 can offer a corrupt deal to the public procurer (Nash bargaining).</td>
</tr>
<tr>
<td>3</td>
<td>If the corrupt deal has been made, the public procurer sets contract requirements (s_p), which the bidder 1 can execute. Otherwise public procurer sets contract requirements (s_p), which maximize its expected utility without the bribe.</td>
</tr>
<tr>
<td>4</td>
<td>Each bidder registers for participation in auction, if (s_i \geq s_p), (i = 1,2).</td>
</tr>
<tr>
<td>5</td>
<td>If two bidders have registered, the procurer organizes reverse English auction. If one bidder has registered, he wins auction at price (r &gt; c). If nobody has registered, all players receive zero payoffs.</td>
</tr>
<tr>
<td>6</td>
<td>The procurer buys a product and receives a bribe if it has made a corrupt deal with the bidder 1.</td>
</tr>
</tbody>
</table>

The game runs as follows. First, the nature assigns characteristics \((c_i; s_i)\) for each bidder. The bidder 1 realizes his characteristics and decides whether it is beneficial to give a bribe and change expected contract conditions or not. If the bidder 1 offers a bribe to the public procurer, the size of the bribe \(B\) is the result of Nash bargaining (Nash bargaining solution). For simplicity, we assume that the bargaining powers of the public procurer and the bidder 1 are equal. Then they maximize the product of their expected payoffs for particular characteristics of the bidder 1 \((c_1; s_1)\):

\[
F = E\pi_1 \cdot EU_{proc} \rightarrow \max_B ,
\]

where

\[
E\pi_1 = p_1 (c_{2m} - c_{1m} - B_1 (1 + \alpha)),
\]

\[
EU_{proc} = E\upsilon(s) + p_1 B_1,
\]

\[
p_1 \text{ – the probability that the bidder 1 wins auction.}
\]

Under conditions:

\[
\begin{align*}
E\pi_1 (s_p = s_1; B^*) & \geq E\pi_1 (s_p \neq s_1; B = 0) \\
EU_{proc} (s_p = s_1; B^*) & \geq EU_{proc} (s_p \neq s_1; B = 0).
\end{align*}
\]

\[
B^* > 0
\]

First-order condition: \(\frac{\partial F}{\partial B} = 0\).

The effectiveness of the auction and conditions of favoritism depend on characteristics of the bidder 1. For instance, when the bidder 1 has high costs and cannot execute a contract with the restrictive conditions, favoritism does not arise. If the public procurer and the bidder make a
corrupt deal, the public procurer will not set restrictive conditions, so the bidder 2 will participate in auction and make a bid equal to his production costs. The bid of the bidder 1 will equal the sum of high production costs and a bribe and exceed the bid of the bidder 2. Hence, the bidder 1 will always lose auction for any non-zero bribe. Since the size of the bribe is strictly greater than zero, the bidder 1 cannot make a corrupt deal with the public procurer and favoritism will not arise. The Appendix 2 contains detailed solutions to all problems.

As in the previous cases, after the public procurer sets conditions $s_p$, bidders register for participation, if $s_i \geq s_p, i = 1,2$. If no one registers, all players receive zero payoffs. If one bidder registers for participation, he wins auction at the reserve price $r$. If both bidders register for participation, the public procurer organizes reverse English auction.

Table 5 presents the main results of traditional auction depending on characteristics of the bidder 1.

**Table 5.** Results of traditional auction with corruption

<table>
<thead>
<tr>
<th>Characteristics of bidder 1</th>
<th>Bribe to the public procurer</th>
<th>Conditions of favoritism</th>
<th>Probability of effective auction $^{13}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\xi; S)$</td>
<td>$B = \frac{r-\xi}{2(1+\alpha)} - v(S)$</td>
<td>$v(S) &lt; \frac{r-\xi}{2(1+\alpha)}$</td>
<td>$\text{Prob}(\text{efficient wins}) = \frac{3}{4}$</td>
</tr>
</tbody>
</table>
| $(c; S)$                    | $B = \frac{2c+r-3c}{6(1+\alpha)} - \frac{2}{3}v(S)$ | \[
\begin{align*}
    v(S) &\geq \frac{-2r+3\xi-c}{4(1+\alpha)} \\
v(S) &\geq \frac{2r-5\xi+3c}{4(1+\alpha)} \\
v(S) &\leq \frac{2c+r-3c}{4(1+\alpha)}
\end{align*}
\] | $\text{Prob}(\text{efficient wins}) = 1$ |
| $(\xi; 0)$                  | $B = \frac{r-\xi}{2(1+\alpha)} - v(S)$ | \[
\begin{align*}
    v(S) &\geq \frac{-r+2\xi-c}{2(1+\alpha)} \\
v(S) &\leq \frac{r-2\xi+c}{2(1+\alpha)}
\end{align*}
\] | $\text{Prob}(\text{efficient wins}) = \frac{3}{4}$ |
| $(c; 0)$                    | $B = \frac{c+\xi}{2(1+\alpha)} - \frac{3}{4}v(0) - \frac{1}{4}v(S)$ | $\frac{5}{4}v(S) - \frac{3}{4}v(0) < \frac{r-c}{2(1+\alpha)}$ | $\text{Prob}(\text{efficient wins}) = 1$ |

In traditional auction favoritism of the public procurer increases public spending, but leads to more effective auction. As the public procurer sets requirements to bidders, that its preferred bidder must meet, at least one bidder always can participate in auction. So traditional auction always takes place and the public procurer buys a product, in contrast to the case with asymmetric information without favoritism. Therefore, reducing informational asymmetry, favoritism leads to higher effectiveness of the auction. Two factors reduce the social welfare. Firstly, the preferred bidder carries out non-zero organizational costs of favoritism, and the more payoff is redistributed from the government to the public procurer and its preferred bidder, the

$^{13}$ In case of favoritism.
lower the social welfare is. Secondly, the society suffers losses, because the public procurer sets requirements maximizing its expected utility, rather than the social welfare.

E. **Corruption in e-auction**

An e-platform is a new player in e-auction, which may be corrupt. Let us assume that the e-platform sets a fixed bribe $b$ for assistance in e-auction\(^{14}\). By analogy with a corrupt deal between a bidder and the public procurer, bidders have to carry out non-zero organizational costs $\beta_i, i = 1, 2$ in order to give a bribe to the e-platform. The e-platform is indifferent to which bidder will give it a bribe, therefore their costs are identical and equal $\beta_1 = \beta_2 = \beta, \beta \in [0; 1]$. Bidder pays a bribe only if he wins auction. Two bidders separately and independently decide whether to agree to give a bribe to the e-platform or not. If one bidder agrees to give a bribe, and the other bidder refuses to do it, the e-platform blocks the bid of the latter bidder and the former wins auction at the reserve price. If two bidders agree to give a bribe, the e-platform chooses the winner randomly and takes a bribe from this bidder. If both bidders refuse to give bribes, the e-platform blocks their bids and auction does not take place.

The e-platform does not know the values and distribution of production costs of bidders, therefore we consider the size of a bribe as exogenous. The effect of the bribe is similar to the effect of participation costs in auction:

1. if the bribe is low, both bidders can give it;
2. if the bribe is in the certain interval, the bidder with low costs can give it, while the bidder with high costs cannot do it,
3. If the bribe is high, no one can give it.

The Appendix 2 contains detailed calculations for all sub-cases. Further we will concentrate on the first sub-case, because it mostly reflexes “market corruption” and arises when the e-platform sets low bribe. The Table 6 presents the timing of the game.

**Table 6. Timing**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The nature assigns characteristics $(c_i; s_i)$ to each bidder.</td>
</tr>
<tr>
<td>2</td>
<td>The bidder 1 can offer a corrupt deal to the public procurer (Nash bargaining).</td>
</tr>
</tbody>
</table>

\(^{14}\) Drawing parallels with Russian public procurement we identify blocking as a situation when an employee of the e-platform prevents bidders from submitting bids in e-auction. According to point 22 article 68 44-FL, these actions are prohibited, because the e-platform must ensure the continuity of e-auction. However these situations are not seldom in practice that is confirmed by complaints to the regulator (Federal antimonopoly agency) and anecdotal evidence. For instance, one bidder could not make a bid because of some technical problems with a key of the electronic digital signature. He complained about it to the regulator (the case №K-635/10). There was only one bidder in e-auction except for the one who complained. This bidder had no technical problems and won e-auction decreasing the reserve price by 1% (to 1’119’690 rubles). The regulator stated that the e-platform restricted competition. The reason for this situation may be real technical problems, as well as corruption of the e-platform’s employees, which we model in this paper.
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>If the corrupt deal has been made, the public procurer sets contract conditions $s_p$, which the bidder 1 can execute. Otherwise public procurer sets contract conditions $s_p$, which maximize its expected utility.</td>
</tr>
<tr>
<td>4</td>
<td>Each bidder registers for participation in auction, if $s_i \geq s_p$, $i = 1,2$.</td>
</tr>
<tr>
<td>5</td>
<td>If two bidders have registered, the e-platform offers them a corrupt deal.</td>
</tr>
<tr>
<td>6</td>
<td>If no one agrees to give a bribe to the e-platform, all players receive zero payoffs. If one bidder agrees to give a bribe to the e-platform, he wins auction at the reserve price $r &gt; \bar{c}$; the other bidder cannot make a bid and receives zero profit. If both bidders agree to give a bribe to the e-platform, the e-platform chooses the winner randomly.</td>
</tr>
<tr>
<td>7</td>
<td>The procurer buys a product and receives a bribe if it has made a corrupt deal with the bidder 1. The e-platform receives a bribe if she has made a corrupt deal with any bidder.</td>
</tr>
</tbody>
</table>

The game runs as follows. First of all, the nature assigns characteristics $(c_i; s_i)$ for each bidder. The bidder 1 realizes his characteristics and decides whether it is beneficial to give any bribe. If the bidder 1 offers a bribe to the public procurer, the size of the bribe $B$ is the result of Nash bargaining (Nash bargaining solution). Unlike the previous case, the expected profit of the bidder 1 may also contain a bribe to the e-platform. The bidder 1 chooses one out of four sets of strategies: to give a bribe to the public procurer, give a bribe to the e-platform, give bribes to both public procurer and e-platform or give a bribe to nobody.

A corrupt e-platform has different impact on the effectiveness of auction and conditions of favoritism depending on characteristics of the bidder 1. For instance, a corrupt e-platform stimulates favoritism when the bidder 1 carries out high costs and cannot execute a contract with the restrictive conditions. Unlike traditional auction, where the bidder 1 never gives a bribe to the public procurer under these circumstances, in e-auction the bidder 1 wins with probability 0.5, if he agrees to give a bribe to the e-platform. Also the bidder 1 must give a bribe to the public procurer, if the effect of restricting competition ($\Delta \nu = \nu(S) - \nu(0)$) is high, because he cannot win auction without registering in it. Therefore if the e-platform sets small bribe, the bidder 1 can give bribes to both public procurer and e-platform.

In contrast, if the bidder 1 carries out high costs and can execute a contract with the restrictive conditions, corruption of the e-platform may replace favoritism of the public procurer. It will be less profitable for the bidder 1 to give a bribe only to the public procurer, because his expected profit is lower, than in situation when he gives bribes to the public procurer and the e-platform (see Appendix 3). Hence, depending on the size of the bribe to the e-platform the bidder
I will give a bribe only to the e-platform (hence, favoritism will not occur) or to both considered players (hence, the bribe to the public procurer will decrease and conditions of favoritism will be less wide). The Appendix 3 contains detailed solutions to all problems.

As in the previous cases, after the public procurer sets requirements $s_p$, bidders register for participation, if $s_i \geq s_p, i = 1,2$. If no one registers for participation, all players receive zero payoffs. If one bidder registers for participation, he wins auction at the reserve price $r$. If both bidders register for participation, the e-platform offers them to give her a bribe. If no one agrees to pay a bribe, all players receive zero payoffs. If one bidder agrees to pay a bribe, he wins auction at the reserve price $r$ and the other bidder is blocked and receives zero payoff. If both bidders agree to give bribes to the e-platform, the e-platform chooses the winner randomly.

If the purchase was organized successfully, the public procurer signs the contract with the winner of auction (or only registered bidder) and receives the bribe, if it has made a corrupt deal with the bidder 1. Table 7 presents the main results of e-auction depending on characteristics of the bidder 1.

**Table 7. Results of e-auction with corruption**

<table>
<thead>
<tr>
<th>Characteristics of bidder 1</th>
<th>Brine to the public procurer</th>
<th>Conditions of favoritism</th>
<th>Probability of effective auction</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E; S)</td>
<td>$B = \frac{r - \tau}{2(1+\alpha)} - \frac{2}{3} v(S) - \frac{(1+\beta)}{6(1+\alpha)} b$</td>
<td>${ v(S) &lt; \frac{3(1-\tau)}{4(1+\alpha)} - \frac{(1+\beta)}{4(1+\alpha)} b }$</td>
<td>$Prob(\text{efficient wins}) = \frac{7}{8}$</td>
</tr>
<tr>
<td>(C; S)</td>
<td>$B = \frac{r - \tau}{2(1+\alpha)} - \frac{2}{3} v(S) - \frac{(1+\beta)}{6(1+\alpha)} b$</td>
<td>${ v(S) &lt; \frac{3(1-\tau)}{4(1+\alpha)} - \frac{(1+\beta)}{4(1+\alpha)} b }$</td>
<td>$Prob(\text{efficient wins}) = \frac{7}{8}$</td>
</tr>
<tr>
<td>(E; 0)</td>
<td>$B = \frac{r - \tau}{2(1+\alpha)} - \frac{3}{4} v(0) - \frac{1}{4} v(S) - \frac{(1+\beta)}{2(1+\alpha)} b$</td>
<td>$b &lt; \frac{r - \tau}{(1+\beta)} \frac{3(1+\alpha)}{2(1+\beta)} v(0) - \frac{(1+\alpha)}{2(1+\beta)} v(S)$</td>
<td>$Prob(\text{efficient wins}) = \frac{3}{4}$</td>
</tr>
<tr>
<td>(C; 0)</td>
<td>$B = \frac{r - \tau}{2(1+\alpha)} - \frac{3}{4} v(0) - \frac{1}{4} v(S) - \frac{(1+\beta)}{2(1+\alpha)} b$</td>
<td>$b &lt; \frac{r - \tau}{(1+\beta)} \frac{3(1+\alpha)}{2(1+\beta)} v(0) - \frac{(1+\alpha)}{2(1+\beta)} v(S)$</td>
<td>$Prob(\text{efficient wins}) = \frac{3}{4}$</td>
</tr>
</tbody>
</table>

In e-auction each bidder has a non-zero probability to win the auction giving a bribe to the e-platform regardless of his production costs. Hence, the bidder with high costs wins e-auction more frequently than traditional auction, and the effectiveness of e-auction under favoritism decreases.

Depending on characteristics of the preferred bidder corruption of e-platform can either displace corruption of the public procurer, or stimulate it and raise the size of the bribe to the public procurer. For instance, a preferred bidder still has incentives to bribe the public procurer when he does not meet the strict conditions of the public procurer. If the public procurer set
restrictive conditions of the contract, the preferred bidder cannot participate in the auction. However the preferred bidder always gives a bribe to the e-platform regardless of his characteristics, as it increases his expected profit under given constraints on the size of the bribe. In e-auction the preferred bidder can choose new variants of strategies (the opportunity to give a bribe to the e-platform, or both the public procurer and the e-platform). As far as corruption of the e-platform may be less costly to the preferred bidder, than corruption of the public procurer, the scope of favoritism may decrease.

5. Conclusion and discussion

This paper examines the effects of a corrupt intermediary on favoritism and effectiveness of the auction. Based on the prevalence of favoritism in Russia, we assume that only the preferred bidder can give a bribe to the public procurer. Unlike the majority of articles, we model the corruption between the public procurer and the preferred bidder before the auction, rather than during it. Corrupt public procurer manipulates the conditions of the contract increasing the expected profit of the preferred bidder. Corruption of the intermediary (e-platform) occurs after that, just before e-auction starts. E-platform offers each bidder that has registered to participate in e-auction to pay a fixed bribe. Other types of rent-seeking behavior, in particular, horizontal collusion and collusion between the public procurer and e-platform, are beyond the scope of this paper.

We make a theoretical model with two bidders that are characterized by high or low production costs and different abilities to execute the proposed contract. The main contribution of our work is that a corrupt intermediary has different effect on favoritism depending on characteristics of the preferred bidder and the size of the bribe to the intermediary. Corrupt e-platform can eliminate favoritism of the public procurer, if the preferred bidder has high costs and can execute a contract with the restrictive conditions. In this case in order to win e-auction the preferred bidder should give a bribe to the e-platform, but does not have to give a bribe to the public procurer, as he meets its restrictive requirements. If the bribe to the e-platform is high, favoritism becomes less beneficial for the preferred bidder and the bribe to the public procurer decreases.

Corruption of e-platform stimulates the favoritism of the public procurer, if the preferred bidder carries out high costs and cannot execute a contract with the restrictive conditions. E-platform can block the actions of bidders during e-auction, so unlike the situation in traditional auction, the preferred provider has a chance to win e-auction and get a positive profit. As the preferred bidder cannot execute a contract with the restrictive conditions, he has to pay a bribe to the public procurer to participate in the auction. If the preferred bidder has low costs, corruption
of e-platform increases the size of the bribe to the public procurer compared to the traditional auction under the given constraints on the size of the bribe to the e-platform.

In the further work we will relax the assumption about strict constraints on the size of the bribe to the e-platform to analyze a wider class of situations. Also we are going to add to the model the assumption that the level of restriction that the public procurer sets is positively related to the production cost of bidders. The introduction of this assumption will only affect the probability that bidders have a specific set of characteristics, so we think that the direction of the results will be the same. Also it will strengthen the main result of our work: e-auction may reduce favoritism and lead to a higher level of social welfare despite the possible corruption of e-platform.
6. References


• Li, M. J., Juan, J. S. T., & Tsai, J. H. C. (2011). Practical E-auction Scheme with Strong Anonymity and Bidding Privacy. *Information Sciences, 181*(12), 2576-2586.


Appendix 1

Corruption in traditional auction: calculations

1. Type of the bidder 1 ($\bar{c}; S$)
If the procurer sets restrictive conditions $S$, with the probability 0.5 the bidder 2 does not meet them and the bidder 1 wins auction at the reserve price; with the probability 0.5 the bidder 2 meets them and the public procurer organizes auction, which the bidder 2 will win, because $c_1 + (1 + \alpha)B > c_2$. The expected utility of the public procurer equals the sum of the expected bribe and utility of the contract with the restrictions of competition.

$$EU_{PROC}(S; B) = v(S) + \frac{1}{2}B,$$
$$E\pi_1(S; B) = \frac{1}{2} (r - \bar{c}) - \frac{1}{2} (1 + \alpha)B,$$
$$F = E\pi_1(S; B) \cdot EU_{PROC}(S; B) = \frac{1}{4} [2v(S) + B][(r - \bar{c}) - (1 + \alpha)B] \rightarrow \max_B,$$
$$B = \frac{r - \bar{c}}{2(1+\alpha)} - v(S).$$

Under conditions:
$$\{EU_{PROC}(S; B) \geq EU_{PROC}(0; 0)$$
$$E\pi_1(S; B) \geq E\pi_1(0; 0)$$
$$B > 0$$

Conditions of favoritism:
$$v(S) < \frac{r - \bar{c}}{2(1+\alpha)}$$

2. Type of the bidder 1 ($c; S$)
If the procurer sets restrictive conditions $S$, with the probability 0.5 the bidder 2 does not meet them and the bidder 1 wins auction at the reserve price; with the probability 0.5 the bidder 2 meets them. In the latter case the win of the bidder 1 depends on the relation between low and high production costs: the bidder 1 wins the auction only if $\bar{c} \geq c + (1 + \alpha)B$ and the bidder 2 carries out high costs. Otherwise the bidder 1 loses the auction and does not give a bribe to the public procurer. The expected utility of the public procurer equals the sum of the expected bribe and utility from the contract with the restrictions of competition.

a) If $\bar{c} > c + (1 + \alpha)B$

$$EU_{PROC}(S; B) = v(S) + \frac{3}{4}B,$$
$$E\pi_1(S; B) = \frac{1}{4} (2r + \bar{c} - 3c) - \frac{3}{4} (1 + \alpha)B,$$
$$F = E\pi_1(S; B) \cdot EU_{PROC}(S; B) = \frac{9}{16} [\frac{4}{3}v(S) + B][\frac{(2r + \bar{c} - 3c)}{3} - (1 + \alpha)B] \rightarrow \max_B,$$
$$B = \frac{2r + \bar{c} - 3c}{6(1+\alpha)} - \frac{2}{3}v(S).$$

Under conditions:
$$\{EU_{PROC}(S; B) \geq EU_{PROC}(0; 0)$$
$$E\pi_1(S; B) \geq E\pi_1(0; 0)$$
$$B > 0$$
$$\bar{c} > c + (1 + \alpha)B$$

Conditions of favoritism:
$$\left\{ \begin{array}{l}
\quad v(S) \geq \frac{2r + 3c - \bar{c}}{4(1+\alpha)} \\
\quad v(S) \leq \frac{2r + \bar{c} - 3c}{4(1+\alpha)} \\
\quad v(S) \geq \frac{2r - 5\bar{c} + 3c}{4(1+\alpha)}
\end{array} \right.$$

b) If $\bar{c} \leq c + (1 + \alpha)B$
\( EU_{PROC}(S; B) = v(S) + \frac{1}{2}B, \)
\( E\pi_1(S; B) = \frac{1}{2}(r - c) - \frac{1}{2}(1 + \alpha)B, \)
\( F = E\pi_1(S; B) \cdot EU_{PROC}(S; B) = \frac{1}{4}[2v(S) + B][r - c - (1 + \alpha)B] \rightarrow \max_B \)
\[ B = \frac{r - c}{2(1 + \alpha)} - v(S). \]

Under conditions:
\[ \begin{aligned}
& EU_{PROC}(S; B) \geq EU_{PROC}(0; 0) \\
& E\pi_1(S; B) \geq E\pi_1(0; 0) \\
& B > 0 \\
& \bar{c} \leq c + (1 + \alpha)B \\
\end{aligned} \]

Conditions of favoritism:
\[ \begin{aligned}
& v(S) \geq \frac{-r + 2\bar{c} - c}{2(1 + \alpha)} \\
& v(S) < \frac{r - c}{2(1 + \alpha)} \\
& v(S) \leq \frac{r - 2\bar{c} + c}{2(1 + \alpha)} \\
\end{aligned} \]

3. **Type of the bidder 1 (\( \bar{c}; 0 \))**

The bidder 1 always loses the auction, as \( c_1 + (1 + \alpha)B > c_2 \). The bribe equals zero and the public procurer chooses the conditions maximizing its expected profit.

Favoritism does not arise; \( \pi_1 = 0 \).

4. **Type of the bidder 1 (\( c; 0 \))**

In this case the public procurer does not restrict competition, in order to give the bidder 1 a chance to participate and win the auction. Hence, the bidder 2 always participates in the auction. The win of the bidder 1 depends on the relation between low and high production costs: the bidder 1 wins the auction only if \( \bar{c} \geq c + (1 + \alpha)B \) and the bidder 2 carries out high costs. Otherwise the bidder 1 loses the auction and does not give a bribe to the public procurer.

**a) If \( \bar{c} \geq c + (1 + \alpha)B \)**

\( EU_{PROC}(0; B) = \frac{3}{4}v(0) + \frac{1}{4}v(S) + \frac{1}{2}B, \)
\( E\pi_1(0; B) = \frac{1}{2}(\bar{c} - c) - \frac{1}{2}(1 + \alpha)B, \)
\( F = E\pi_1(0; B) \cdot EU_{PROC}(0; B) = \frac{1}{4}\left[\frac{3}{2}v(0) + \frac{1}{2}v(S) + B][\bar{c} - c - (1 + \alpha)B] \rightarrow \max_B \right. \)
\[ B = \frac{\bar{c} - c}{2(1 + \alpha)} - \frac{3}{4}v(0) - \frac{1}{4}v(S). \]

При условиях:
\[ \begin{aligned}
& EU_{PROC}(0; B) \geq EU_{PROC}(S; 0) \\
& E\pi_1(0; B) \geq E\pi_1(S; 0) \\
& B > 0 \\
& \bar{c} \geq c + (1 + \alpha)B \\
\end{aligned} \]

As the public procurer initially was ready to set requirements \( S, v(S) > 2v(0) \), hence \( \frac{5}{4}v(S) - \frac{3}{4}v(0) > \frac{1}{4}v(S) + \frac{3}{4}v(0). \)

Conditions of favoritism:
\[ \frac{3}{4}v(S) - \frac{3}{4}v(0) < \frac{\bar{c} - c}{2(1 + \alpha)} \]

**b) If \( \bar{c} < c + (1 + \alpha)B \)**

Favoritism does not arise because the bidder 1 always loses the auction; \( \pi_1 = 0 \).
Appendix 2

The size of the bribe to the e-platform

Let us find how the size of the bribe, which the e-platform sets, influences the behavior of bidders.

Lemma 1

I. If \( b \in (0; \frac{(r-\varepsilon)}{(1+\beta)}) \), bidders can give a bribe to the e-platform independently of the size of costs.

II. If \( b \in \left(\frac{(r-\varepsilon)}{(1+\beta)}, \frac{(r-\varepsilon)}{(1+\beta)}\right) \), only a bidder with low costs can give a bribe to the e-platform.

III. If \( b \geq \frac{(r-\varepsilon)}{(1+\beta)} \), no bidder can give a bribe to the e-platform.

The Picture 1 presents areas I, II and III.

Picture 1

![Diagram](image)

Proof

We analyze the behavior of bidders 1 and 2, which can choose one of the following strategies: to give a bribe to the e-platform or refuse to give it. First of all, we find bidders’ expected profits. If both bidders give bribes, each of them wins the auction with probability 0.5, and receives the expected profit

\[ E\pi_1 = \frac{r-c_i}{2} - \frac{(1+\beta)}{2} b, \quad i = 1,2. \]

If the bidder 1 gives a bribe to the e-platform and the bidder 2 refuses, the bidder 1 wins e-auction with probability 1 and the bidder 2 gets zero profit. Hence, their expected profits equal: \( E\pi_1 = r - c_1 - (1 + \beta)b, E\pi_2 = 0.\)

Analogically, if the bidder 2 gives a bribe to the e-platform and the bidder 1 refuses to give it, their expected profits equal:

\[ E\pi_1 = 0, E\pi_2 = r - c_2 - (1 + \beta)b. \]

If both bidders do not give bribes to the e-platform, \( E\pi_1 = E\pi_2 = 0.\)

Let us assume that if the bidder is indifferent whether to give a bribe or not, he does not give it. Then we can find Nash equilibria and analyze what has impact on the strategy that each bidder chooses.

Matrix 1: corruption of the e-platform (general form)

<table>
<thead>
<tr>
<th>The bidder 1</th>
<th>The bidder 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Give a bribe</td>
<td>Refuse</td>
</tr>
<tr>
<td>( \frac{r-c_1}{2} - \frac{(1+\beta)}{2} b; \frac{r-c_2}{2} - \frac{(1+\beta)}{2} b )</td>
<td>( r - c_1 - (1 + \beta)b; 0 )</td>
</tr>
<tr>
<td>Refuse</td>
<td>0; ( r - c_2 - (1 + \beta)b )</td>
</tr>
</tbody>
</table>

Each bidder carries out high or low costs with probability 0.5, hence, four different cases may arise, each with probability 0.25:

- \( c_1 = c_2 = \varepsilon \),
- \( c_1 = \varepsilon, c_2 = \overline{c} \).
\( b \in c, c_2 = c, \)
\( c_1 = c_2 = \bar{c}. \)

Let us consider the first case.

**Matrix 2:** corruption of the e-platform (case 1)

<table>
<thead>
<tr>
<th>The bidder 1 ( c_1 = \bar{c} )</th>
<th>The bidder 2 ( c_2 = \bar{c} )</th>
<th>Give a bribe</th>
<th>Refuse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Give a bribe ( \frac{r - \bar{c}}{2} - \frac{(1 + \beta)}{2} b; \frac{r - \bar{c}}{2} - \frac{(1 + \beta)}{2} b )</td>
<td>( r - \bar{c} - (1 + \beta) b; 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Refuse ( 0; r - \bar{c} - (1 + \beta) b )</td>
<td>0;0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If \( b \in (0; \frac{(r - \bar{c})}{(1 + \beta)}), \) both bidders get positive profit if they give bribes to the e-platform. Nash equilibrium in pure strategies is \{ give a bribe; give a bribe \}.

If \( b \geq \frac{(r - \bar{c})}{(1 + \beta)} \), both bidders get zero or negative profit if they give bribes to the e-platform. Previously we assumed that if the bidder is indifferent about giving a bribe to the e-platform, he will not give it. Then Nash equilibrium in pure strategies is \{ refuse; refuse \}.

**Matrix 3:** corruption of the e-platform (case 2)

<table>
<thead>
<tr>
<th>The bidder 1 ( c_1 = \bar{c} )</th>
<th>The bidder 2 ( c_2 = \bar{c} )</th>
<th>Give a bribe</th>
<th>Refuse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Give a bribe ( \frac{r - \bar{c}}{2} - \frac{(1 + \beta)}{2} b; \frac{r - \bar{c}}{2} - \frac{(1 + \beta)}{2} b )</td>
<td>( r - \bar{c} - (1 + \beta) b; 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Refuse ( 0; r - \bar{c} - (1 + \beta) b )</td>
<td>0;0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If \( b \in (0; \frac{(r - \bar{c})}{(1 + \beta)}), \) both bidders get positive profit if they give bribes to the e-platform. Nash equilibrium in pure strategies is \{ give a bribe; give a bribe \}.

If \( b \in [\frac{(r - \bar{c})}{(1 + \beta)}; \frac{(r - \bar{c})}{(1 + \beta)}], \) the bidder 1 gets positive profit and the bidder 2 gets zero or negative profit if they give bribes to the e-platform. Nash equilibrium in pure strategies is \{ give a bribe; refuse \}.

If \( b \geq \frac{(r - \bar{c})}{(1 + \beta)} \), both bidders get zero or negative profit if they give bribes to the e-platform. Nash equilibrium in pure strategies is \{ refuse; refuse \}.

Case 3 is analogous to case 2.

**Matrix 4:** corruption of the e-platform (case 4)

<table>
<thead>
<tr>
<th>The bidder 1 ( c_1 = \bar{c} )</th>
<th>The bidder 2 ( c_2 = \bar{c} )</th>
<th>Give a bribe</th>
<th>Refuse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Give a bribe ( \frac{r - \bar{c}}{2} - \frac{(1 + \beta)}{2} b; \frac{r - \bar{c}}{2} - \frac{(1 + \beta)}{2} b )</td>
<td>( r - \bar{c} - (1 + \beta) b; 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Refuse ( 0; r - \bar{c} - (1 + \beta) b )</td>
<td>0;0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If \( b \in (0; \frac{(r - \bar{c})}{(1 + \beta)}), \) both bidders get positive profit if they give bribes to the e-platform. Nash equilibrium in pure strategies is \{ give a bribe; give a bribe \}.

If \( b \geq \frac{(r - \bar{c})}{(1 + \beta)} \), both bidders get zero or negative profit if they give bribes to the e-platform. Nash equilibrium in pure strategies is \{ refuse; refuse \}.

Finally we get:

If \( b \in (0; \frac{(r - \bar{c})}{(1 + \beta)}), \) bidders can give a bribe to the e-platform independently of the size of costs.
If \( b \in \left( \frac{r_c}{1+\beta}, \frac{r_c}{1+\beta} \right) \), only a bidder with low costs can give a bribe to the e-platform.

If \( b \geq \frac{r_c}{1+\beta} \), no bidder can give a bribe to the e-platform.

QED.
Appendix 3
Corruption in e-auction: calculations

1. Type of the bidder 1 (\( \tilde{c}; S \))
The bidder 1 chooses which set of strategies will give him the highest profit:

- Give a bribe to nobody,
- Give a bribe to the public procurer,
- Give a bribe to the e-platform,
- Give a bribe to public procurer and the e-platform.

If the bidder 1 does not give a bribe, the public procurer will no set restrictive conditions, therefore both bidders register for participation in e-auction. The e-platform will offer them a corrupt deal; as the bidder 1 will refuse to make it, he will not be able to make bids in e-auction and get zero payoff:

\[
E \pi_1 (B = 0; b = 0) = 0.
\]

If the bidder 1 gives a bribe only to the public procurer, the public procurer will set conditions \( S \), so with probability 0.5 the bidder 2 will not meet them and the bidder 1 will win auction at the reserve price. With probability 0.5 the bidder 2 will meet them and both bidders will register for participation. Then the e-platform will offer them a corrupt deal; as the bidder 1 will refuse to make it, he will not be able to make a bid and his expected profit will be equal to:

\[
E \pi_1 (B > 0; b = 0) = \frac{1}{2} (r - \bar{c}) - \frac{1}{2} (1 + \alpha)B.
\]

If the bidder 1 gives a bribe only to the e-platform, the public procurer will not set restrictive conditions, so both bidders will register for participation. The e-platform will offer them a corrupt deal; as the bidder 1 will agree to make it, he will win e-auction at the reserve price with probability 0.5. So his expected profit will be equal to:

\[
E \pi_1 (B = 0; b > 0) = \frac{1}{2} (r - \bar{c}) - \frac{1}{2} (1 + \beta) b.
\]

If the bidder 1 gives a bribe to the public procurer and the e-platform, the public procurer will set conditions \( S \), so with probability 0.5 the bidder 2 will not meet them and the bidder 1 will win auction at the reserve price. With probability 0.5 the bidder 2 will meet them and both bidders will register for participation. Then the e-platform will offer them a corrupt deal; as the bidder 1 will agree to make it, he will win e-auction at the reserve price with probability 0.5. So his expected profit will be equal to:

\[
E \pi_1 (B > 0; b > 0) = \frac{1}{2} (r - \bar{c}) - \frac{3}{4} (1 + \alpha)B - \frac{1}{4} (1 + \beta) B.
\]

In order to compare expected profits of the bidder 1, we will find bribes to the public procurer. Let the bribe to public procurer be equal to \( \tilde{B} \), if the bidder 1 gives a bribe only to the public procurer, and let the bribe to public procurer be equal to \( \tilde{B} \), if the bidder 1 gives a bribe to the public procurer and the e-platform.

First of all we will find \( \tilde{B} \):

\[
\tilde{B} = \arg \max_S E \pi_1 (S; B; b = 0) \cdot EU_{PROC} (S; \tilde{B}).
\]

\[
E \pi_1 (S; \tilde{B}; b = 0) = \frac{1}{2} (r - \bar{c}) - \frac{1}{2} (1 + \alpha) \tilde{B},
\]

\[
EU_{PROC} (S; \tilde{B}) = v(S) + \frac{1}{2} \tilde{B},
\]

\[
F = E \pi_1 (S; \tilde{B}; b = 0) \cdot EU_{PROC} (S; \tilde{B}) = \frac{1}{4} [2v(S) + \tilde{B}] [(r - \bar{c}) - (1 + \alpha) \tilde{B}] \to \max_{\tilde{B}}.
\]
\[ \bar{B} = \frac{r^{-\frac{\varepsilon}{2}}}{2(1+\alpha)} - v(S). \]

Under conditions:
\[
\begin{align*}
\bar{B} > 0 & \quad \text{if } EU_{\text{PROC}}(S; \bar{B}) \geq EU_{\text{PROC}}(0; 0) \\
E\pi_1(S; \bar{B}; b = 0) & \geq E\pi_1(0; 0) \\
E\pi_1(S; \bar{B}; b > 0) & \geq E\pi_1(S; \bar{B}; b = 0) \\
E\pi_1(S; \bar{B}; b > 0) & \geq E\pi_1(0; b > 0) \\
E\pi_1(S; \bar{B}; b > 0) & \geq E\pi_1(0; b > 0)
\end{align*}
\]

Then
\[ E\pi_1(S; \bar{B}; b = 0) = \frac{1}{4} (r - \bar{c}) + \frac{(1+\beta)}{2} v(S). \]

Further we will find \( \bar{B} \):
\[ \bar{B} = \arg \max E\pi_1(S; \bar{B}; b > 0) \cdot EU_{\text{PROC}}(S; \bar{B}). \]
\[ E\pi_1(S; \bar{B}; b > 0) = \frac{3}{4} (r - \bar{c}) - \frac{3}{4} (1 + \alpha) \bar{B} - \frac{1}{4} (1 + \beta)b, \]
\[ EU_{\text{PROC}}(S; \bar{B}) = v(S) + \frac{3}{4} \bar{B}. \]
\[ F = E\pi_1(S; \bar{B}; b > 0) \cdot EU_{\text{PROC}}(S; \bar{B}) = \frac{9}{16} \left[ v(S) + \bar{B} \right] [(r - \bar{c}) - \frac{1}{3} (1 + \beta)b - (1 + \alpha)\bar{B}] \rightarrow \max \bar{B}, \]
\[ \bar{B} = \frac{r^{-\frac{\varepsilon}{2}}}{2(1+\alpha)} - \frac{2}{3} \frac{(1+\beta)}{6(1+\alpha)} v(S). \]

Under conditions:
\[
\begin{align*}
\bar{B} > 0 & \quad \text{if } EU_{\text{PROC}}(S; \bar{B}) \geq EU_{\text{PROC}}(0; 0) \\
E\pi_1(S; \bar{B}; b = 0) & \geq E\pi_1(0; 0) \\
E\pi_1(S; \bar{B}; b > 0) & \geq E\pi_1(0; b = 0) \\
E\pi_1(S; \bar{B}; b > 0) & \geq E\pi_1(0; b > 0)
\end{align*}
\]

Then
\[ E\pi_1(S; \bar{B}; b > 0) = \frac{3}{8} (r - \bar{c}) + \frac{(1+\alpha)}{2} v(S) - \frac{(1+\beta)}{8} b. \]

Then we will compare values of expected profits. Earlier we have assumed that \( b \in (0, \frac{(r-\bar{c})}{(1+\beta)}) \).

Under this assumption \( E\pi_1(S; \bar{B}; b > 0) > E\pi_1(S; \bar{B}; b = 0) \). Thereby the bidder 1 will not give a bribe only to the public procurer. The size of a bribe and conditions of favoritism are the following:
- If \( b < \frac{(r-\bar{c})}{3(1+\beta)} \frac{4(1+\alpha)}{3(1+\beta)} v(S) \), the bidder 1 will give a bribe only to the e-platform, and favoritism will not occur.
- If \( v(S) < \frac{3(r-\bar{c})}{4(1+\alpha)} - \frac{(1+\beta)}{4(1+\alpha)} b \) and \( b \in \left( \frac{(r-\bar{c})}{3(1+\beta)} - \frac{4(1+\alpha)}{3(1+\beta)} v(S); \frac{(r-\bar{c})}{(1+\beta)} \right) \), the bidder 1 will give a bribe to the e-platform and the public procurer. A bribe to procurer will decrease in comparison with traditional auction if \( b > \frac{2(1+\alpha)}{(1+\beta)} v(S) \).
- Otherwise the bidder 1 will not give any bribe.

2. **Type of the bidder 1 (c; S)**
The bidder 1 chooses which set of strategies will give him the highest profit:
• Give a bribe to nobody,
• Give a bribe to the public procurer,
• Give a bribe to the e-platform,
• Give a bribe to the public procurer and the e-platform.

By analogy with the previous situation, first we find the expected profit of the bidder 1 when he chooses different sets of strategies.
If the bidder 1 does not give bribes, he will get zero profit:
\[ E\pi_1(B = 0; b = 0) = 0. \]

If the bidder 1 gives a bribe only to the public procurer, his expected profit will equal:
\[ E\pi_1(B > 0; b = 0) = \frac{1}{2} (r - c) - \frac{1}{2} (1 + \alpha) B. \]

If the bidder 1 gives a bribe only to the e-platform, his expected profit will equal:
\[ E\pi_1(B = 0; b > 0) = \frac{1}{2} (r - c) - \frac{1}{2} (1 + \beta) b. \]

If the bidder 1 gives a bribe to the public procurer and the e-platform, his expected profit will equal:
\[ E\pi_1(B > 0; b > 0) = \frac{3}{4} (r - c) - \frac{3}{4} (1 + \alpha) B - \frac{1}{4} (1 + \beta) b. \]

In order to compare expected profits of the bidder 1, we will find bribes to the public procurer. As we have done above, the bribe to public procurer equals \( \bar{B} \), if the bidder 1 gives a bribe only to the public procurer, and the bribe to public procurer equals \( \hat{B} \), if the bidder 1 gives a bribe to the public procurer and the e-platform.

Firstly, we will find \( \bar{B} \):
\[ \bar{B} = \text{arg}\max E\pi_1(S; \bar{B}; b = 0) \cdot EU_{PROC}(S; \bar{B}), \]
\[ E\pi_1(S; \bar{B}; b = 0) = \frac{1}{2} (r - c) - \frac{1}{2} (1 + \alpha) \bar{B}, \]
\[ EU_{PROC}(S; \bar{B}) = v(S) + \frac{1}{2} \bar{B}. \]

\[ F = E\pi_1(S; \bar{B}; b = 0) \cdot EU_{PROC}(S; \bar{B}) = \frac{1}{4} [2v(S) + \bar{B}][(r - c) - (1 + \alpha)\bar{B}] \]
\[ \Rightarrow \bar{B} = \frac{r - c}{2(1 + \alpha)} - v(S). \]

Under conditions:
\[ \begin{cases} EU_{PROC}(S; \bar{B}) \geq EU_{PROC}(0; 0) \\ E\pi_1(S; \bar{B}; b = 0) \geq E\pi_1(0; 0) \\ E\pi_1(S; \bar{B}; b > 0) \geq E\pi_1(0; 0; b > 0) \end{cases} \]
\[ \Rightarrow v(S) \leq \frac{r - c}{2(1 + \alpha)}. \]

Then
\[ E\pi_1(S; \bar{B}; b = 0) = \frac{1}{4} (r - c) + \frac{(1 + \alpha)}{2} v(S). \]

Secondly, we will find \( \hat{B} \):
\[ \hat{B} = \text{arg}\max E\pi_1(S; \hat{B}; b > 0) \cdot EU_{PROC}(S; \hat{B}), \]
\[ E\pi_1(S; \hat{B}; b > 0) = \frac{3}{4} (r - c) - \frac{3}{4} (1 + \alpha) \hat{B} - \frac{1}{4} (1 + \beta) b, \]
\[ EU_{PROC}(S; \hat{B}) = v(S) + \frac{3}{4} \hat{B}. \]
\( F = E\pi_1(S; \hat{B}; b > 0) \cdot EU_{PROC}(S; \hat{B}) = \frac{9}{16} \left[ \frac{4}{3} v(S) + \hat{B} \right] \left[ (r - c) - \frac{1}{3}(1 + \beta)b - (1 + \alpha)\hat{B} \right]\) to \\
\[
\hat{B} = \left. \frac{r - c}{2(1 + \alpha)} - \frac{2}{3} v(S) \right| \left. -\frac{(1 + \beta)}{6(1 + \alpha)} b \right. \\
\] 
Under conditions:

\[
\begin{align*}
& \hat{B} > 0 \\
& EU_{PROC}(S; \hat{B}) \geq EU_{PROC}(0; 0) \\
& E\pi_1(S; \hat{B}; b > 0) \geq E\pi_1(0; 0) \\
& E\pi_1(S; \hat{B}; b > 0) \geq E\pi_1(S; \hat{B}; b = 0) \\
& E\pi_1(S; \hat{B}; b > 0) \geq E\pi_1(0; 0; b > 0)
\end{align*}
\]

Then \( E\pi_1(S; \hat{B}; b > 0) = \frac{3}{8} (r - c) + \frac{1 + \alpha}{2} v(S) - \frac{(1 + \beta)}{8} b \).

Then we will compare expected profits. Earlier we assumed that \( b \in (0, \frac{(r - c)}{(1 + \beta)}) \). Under this assumption \( E\pi_1(S; \hat{B}; b > 0) > E\pi_1(S; \hat{B}; b = 0) \). Thereby the bidder 1 will not give a bribe only to the public procurer. The size of a bribe and conditions of favoritism are following:

- If \( b < \frac{(r - c)}{3(1 + \beta)} v(S) \), the bidder 1 will give a bribe only to the e-platform and favoritism will not arise.
  
- If \( b \in \left( \frac{(r - c)}{3(1 + \beta)}, \frac{4(1 + \alpha)}{3(1 + \beta)} v(S) \right] \), the bidder 1 will give bribes to the public procurer and the e-platform. A bribe to the public procurer will increase in comparison with traditional auction.
- Otherwise the bidder 1 will not give any bribe.

3. **Type of the bidder 1 (c; 0)**

The bidder 1 chooses which set of strategies will give her the highest profit. As the bidder 1 cannot execute the contract with restrictive conditions, he can participate in auction only if he gives a bribe to the public procurer, which will not set restrictive conditions. Hence, the bidder 1 cannot give a bribe the e-platform without giving a bribe to the public procurer. He chooses among the following sets of strategies:

- Give a bribe to nobody,
- Give a bribe to the public procurer,
- Give a bribe to the public procurer and the e-platform.

As in the previous situations, first we find the expected profit of the bidder 1 when he chooses different sets of strategies.

If the bidder 1 does not give bribes, he will get zero profit:

\[ E\pi_1(B = 0; b = 0) = 0. \]
If the bidder 1 gives a bribe only to the public procurer, the public procurer will not set restrictive conditions \( S \), so and both bidders will register for participation. Then the e-platform will offer them a corrupt deal; as the bidder 1 will refuse to make it, he will not be able to make a bid and gets zero profit. Hence, the bidder 1 will never win and will not be able to give a bribe to the public procurer.

If the bidder 1 gives a bribe to the public procurer and the e-platform, the public procurer will set conditions 0, so and both bidders will register for participation. Then the e-platform will offer them a corrupt deal; as the bidder 1 will agree to make it, he will win auction at the reserve price with probability 0.5 and his expected profit will equal:

\[
E\pi_1(B > 0; b > 0) = \frac{1}{2} (r - \bar{c}) - \frac{1}{2} (1 + \alpha)\bar{B} - \frac{1}{2} (1 + \beta) b.
\]

In order to compare expected profits of the bidder 1, we will find a bribe \( \hat{B} \) to the public procurer when the bidder 1 gives a bribe to the public procurer and the e-platform.

\[
\hat{B} = \arg\max E\pi_1(0; \hat{B}; b > 0) \cdot EU_{PROC}(0; \hat{B}).
\]

\[
E\pi_1(0; \hat{B}; b > 0) = \frac{1}{2} (r - \bar{c}) - \frac{1}{2} (1 + \alpha)\hat{B} - \frac{1}{2} (1 + \beta) b,
\]

\[
EU_{PROC}(0; \hat{B}) = \frac{3}{4} v(0) + \frac{1}{4} v(S) + \frac{1}{2} \hat{B},
\]

\[
F = E\pi_1(0; \hat{B}; b > 0) \cdot EU_{PROC}(0; \hat{B}) = \frac{1}{4} \left[ \frac{3}{2} v(0) + \frac{1}{2} v(S) + \hat{B} \right] (r - \bar{c}) - (1 + \beta) b - (1 + \alpha) \hat{B} \rightarrow \max \hat{B}.
\]

Under conditions:

\[
\begin{cases}
\hat{B} > 0 \\
EU_{PROC}(0; \hat{B}) \geq EU_{PROC}(S; 0) \\
E\pi_1(0; \hat{B}; b > 0) \geq 0
\end{cases}
\]

Then we will compare values of expected profits. The size of a bribe and conditions of favoritism depend on the following conditions:

- If \( b < \frac{r - \bar{c}}{(1 + \beta)} - \frac{3(1 + \alpha)}{2(1 + \beta)} v(0) - \frac{(1 + \alpha)}{2(1 + \beta)} v(S) \), i.e. when the e-platform sets small bribe, the bidder 1 will give a bribe to the e-platform and the public procurer.
- Otherwise the bidder 1 will not give any bribe.

Consequently, in contrast to traditional auction, favoritism may arise in e-auction if the bidder 1 carries out high costs and cannot execute a contract with restrictive conditions.

4. **Type of the bidder 1 \( (c; 0) \)**

The bidder 1 chooses which set of strategies will give her the highest profit. As in the previous situation, the bidder 1 cannot give a bribe to the e-platform without giving a bribe to the public procurer, so his available strategies are the following:

- Give a bribe to nobody,
- Give a bribe to the public procurer,
- Give a bribe to the public procurer and the e-platform.

If the bidder 1 does not give any bribe, he will get zero profit:
\[ E\pi_1(B = 0; b = 0) = 0. \]

If the bidder 1 gives a bribe only to the public procurer, the public procurer will set no restrictive conditions, so both bidders will register for participation. Then the e-platform will offer them a corrupt deal; as the bidder 1 will refuse to make it, he will not be able to make bids in e-auction. Hence, the bidder 1 never wins e-auction and will not be able to give a bribe to the public procurer.

If the bidder 1 gives a bribe to the public procurer and the e-platform, the public procurer set no restrictive conditions, so both bidders will register for participation. Then the e-platform will offer them a corrupt deal; as the bidder 1 will agree to make it, he will win e-auction at the reserve price with probability 0.5. So his expected profit will be equal to:

\[ E\pi_1(B > 0; b > 0) = \frac{1}{2} (r - \epsilon) - \frac{1}{2} (1 + \alpha)B - \frac{1}{2} (1 + \beta)b. \]

In order to compare expected profits of the bidder 1, we will find a bribe \( \hat{B} \) to the public procurer when the bidder 1 gives bribes to the public procurer and the e-platform:

\[ \hat{B} = \arg \max_{B} E\pi_1(0; \hat{B}; b > 0) \cdot EU_{PROC}(0; \hat{B}), \]

\[ E\pi_1(0; \hat{B}; b > 0) = \frac{1}{2} (r - \epsilon) - \frac{1}{2} (1 + \alpha)\hat{B} - \frac{1}{2} (1 + \beta)b, \]

\[ EU_{PROC}(0; \hat{B}) = \frac{3}{4} v(0) + \frac{1}{4} v(S) + \frac{1}{2} \hat{B}, \]

\[ F = E\pi_1(0; \hat{B}; b > 0) \cdot EU_{PROC}(0; \hat{B}) = \frac{1}{4} [\frac{3}{2} v(0) + \frac{1}{2} v(S) + \hat{B}] [(r - \epsilon) - (1 + \beta)b - (1 + \alpha)\hat{B}] \rightarrow \max_{\hat{B}}. \]

\[ \hat{B} = \frac{r - \epsilon}{2(1 + \alpha)} - \frac{3}{4} v(0) - \frac{1}{4} v(S) - \frac{(1 + \beta)}{2(1 + \alpha)} b. \]

Under conditions:

\[ \begin{align*}
\hat{B} & > 0 \\
EU_{PROC}(0; \hat{B}) & \geq EU_{PROC}(S; 0); \\
E\pi_1(0; \hat{B}; b > 0) & \geq 0
\end{align*} \]

Then

\[ E\pi_1(0; \hat{B}; b > 0) = \frac{1}{4} (r - \epsilon) + (1 + \alpha) \frac{3}{8} v(0) + \frac{1}{8} v(S) - \frac{(1 + \beta)}{4(1 + \alpha)} b. \]

Then we will compare expected profits. The size of a bribe and conditions of favoritism depend on the following factors:

- If \( \begin{align*}
b & < \frac{r - \epsilon}{(1 + \beta)} - \frac{3(1 + \alpha)}{2(1 + \beta)} v(0) - \frac{(1 + \alpha)}{2(1 + \beta)} v(S), \\
b & < \frac{r - \epsilon}{(1 + \beta)} + \frac{3(1 + \alpha)}{2(1 + \beta)} v(0) - \frac{5(1 + \alpha)}{2(1 + \beta)} v(S),
\end{align*} \)

bidder 1 will give bribes to the public procurer and the e-platform.

- Otherwise the bidder 1 will not give any bribe, and favoritism will not arise.