Biased Supervision

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Abstract

This paper studies how an organization is affected by biased supervision. An agency relationship where a principal relies on a supervisor to obtain a verifiable report on an agent's output is considered. Depending on the output he has observed, the supervisor may either collude with the agent or with the principal, and make an uninformative report. We characterize the optimal incentive contracts in this environment and investigate how collusive activities interact. Compared with the case where the agency relationship is exposed to a single type of collusion, under the threat of two types of collusion, the configuration of the optimal preventive policy is modified, collusion between the supervisor and the agent is not harmless anymore, the expected cost of collusion prevention increases, and the supervisor more often receives a rent.

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1 Introduction

This paper investigates how an organization is affected by biased supervision. To overcome their informational disadvantage, organizations employ supervisors whose roles are to acquire and transmit information about agents.[1] Once informed, a supervisor may engage in collusive activities, conceal valuable information, and therefore produce a biased report. Since the pioneering works of Antle (1984) and Tirole (1986), a large literature has been devoted to the issue of collusion between the supervisor and the agent within principal-supervisor-agent relationships.[2]

It has recently be shown that the harmfulness of supervisor/agent collusion is very sensitive to the environment in which it may occur and that this type of collusion is often costless to deter.[3] These findings question the relevance of the literature on supervisor/agent collusion. We notably identify a cause for supervisor/agent collusion harmfulness, namely the presence of more than a single type of collusion.

Agency models of unofficial activities in hierarchies have been criticized on the ground that they focus exclusively on unofficial activities involving members of lower levels (e.g., Perrow 1986; Dow 1987). Milgrom and Roberts (1988) argue that the cost of unofficial activities by superiors is the major cost of transacting in hierarchies. The main shortcoming of the literature on collusion in hierarchical organizations is that it only focuses on collusion between the supervisor and the agent. In addition to supervisor/agent collusion, a principal-supervisor-agent organization is also exposed to collusion between the principal and the supervisor. Is this type of collusion also often harmless? Is there any interaction between the two types of collusion or, equivalently, does it matter that an organization is exposed to multiple types of collusion rather than to a single one?

[1] Gathering and transmitting information are key activities in organizations with many other dimensions (e.g., Garicano 2000; Bar-Isaac 2009).

[2] See Tirole (1992) for a non-exhaustive survey of this literature. Collusion is a tenacious feature of institutions that may occur in many environments (e.g., Lambertini and Trombetta 2002; Bajari and Summers 2002; Tan and Wang forthcoming).

[3] See, for example, Cont (2004) and references therein.

To answer these questions, we consider a general environment where a three-level organization may be exposed to both supervisor/agent and principal/supervisor collusion. The paper thus accounts for the above-mentioned criticism by considering both collusion involving members of lower and upper levels of hierarchies.

In our three-level hierarchy with moral hazard a supervisor is responsible for providing a principal with a report about an agent's output. The supervision operates an imperfect supervision technology that reveals hard (unforgeable) information/evidence about the agent's output only with a certain probability. The informed supervisor then has the possibility to conceal information and claim that supervision has not been conclusive. If supervision reveals hard evidence that the agent has produced a high output, that is, if the information obtained by the supervisor is unfavorable to the principal - in the sense that the principal then has to pay a higher wage to the agent than in the case where supervision is inconclusive - the supervisor may collude with the principal and, in exchange for a bribe, make an uninformative report. If instead supervision reveals evidence that a low output has been produced, that is, if the information obtained by the supervisor is unfavorable to the agent - in the sense that the agent then receives a lower wage than in the case where supervision is inconclusive - the supervisor may collude with the agent and, in exchange for a bribe, report that supervision has not been conclusive.

We characterize the optimal contracts in this environment. To understand the extra modifications that the simultaneous threats of supervisor/agent and principal/supervisor collusion introduce in an organization, we first characterize the optimal contracts in a three-level hierarchy exposed to a single type of collusion. It is shown that collusion may be costlessly deterred in an organization exclusively exposed to supervisor/agent collusion. In other words, the sole possibility of supervisor/agent collusion is harmless. We then show that it is costly to deter principal/supervisor collusion in an organization exclusively exposed to this type of unofficial activity. Unlike supervisor/agent collusion, collusion between the principal and the supervisor is thus harmful. We finally analyze the general case where the organization is exposed to both types of collusion and show that, compared with the case where only a single type of collusive activity may occur, in the presence of two types of collusion, the configuration of the optimal preventive policy is modified, supervisor/agent collusion is not systematically harmless anymore, the efficiency of the organization is sometimes reduced, and the supervisor more often receives a rent. These results are the consequences of the existence of negative interactions between collusive activities. Indeed, compared with the single type cases, in certain circumstances the threat of a second type of collusion increases the cost of deterring each type of collusion.

These findings prove that the multiplication of collusive activities has major impacts on an organization, and hence considering only the possibility of supervisor/agent collusion in the analysis of multi-level hierarchies is deceptive. Although supervisor/agent collusion may be costlessly deterred in an organization exposed exclusively to this type of unofficial activity, this is not the case anymore when principal/supervisor collusion is also considered. Similarly, the extra threat of supervisor/agent collusion in an organization increases the cost of deterring principal/supervisor collusion.

The remainder of this paper is organized as follows. The model is outlined in Section 2. Section 3 characterizes the optimal incentive contracts in the absence of collusion. Section 4 characterizes the optimal incentive contracts respectively when a single type of collusion is possible and when two types of collusion are possible. Section 5 concludes. Proofs are given in the Appendix.

2 The model

A risk neutral principal-supervisor-agent organization under moral hazard operates as follows. The agent is in charge of production. She has the choice between shirking, in which case her effort level is $e = 0$, and working hard by exerting $e = 1$. Working hard results in the production of a high output $x_H > 0$ with probability $\pi \in (0, 1)$ and the production of a low output $x_L \equiv 0$ with probability $1 - \pi$, whereas shirking results in the production of a low output. The agent's effort level is unobservable to the principal and the supervisor.

The principal (it) charges a supervisor (he) to acquire and transmit, through a verifiable report, hard information/evidence about the agent's output. The supervisor has access to an imperfect supervision technology that reveals hard evidence on the agent's output with probability $p \in (0, 1)$. The supervisor's report, r, thus belongs to $I = \{x_L, \emptyset, x_H\}$, where $r = \emptyset$ means that supervision has been inconclusive. Since the acquired evidence is hard, it can be concealed but not forged. Therefore, when the evidence indicates that x_L (resp. x_H) has been produced, the supervisor may report $r = x_L$ (resp. $r = x_H$) or $r = \emptyset$ but not $r = x_H$ (resp. $r = x_L$).[4] The evidence obtained by the supervisor is his private information, but, once revealed it is verifiable.[5]

The agent's and the supervisor's utility functions are, respectively, $U^A(w, e) = w - \gamma e$ and $U^S(s) = s$, where w and s are their wages and $\gamma > 0$ is the agent's disutility of effort. Reservation utilities are normalized to zero.

Given that only the supervisor's report is verifiable, contracts are contingent on this report. The agent's contract is $(w_L, w_\emptyset, w_H)$, where w_L and w_H are the wages she receives when $r = x_L$ and $r = x_H$, respectively, and $w_\emptyset$ is the wage she receives when $r = \emptyset$. Similarly, the supervisor's contract is $(s_L, s_\emptyset, s_H)$. Employees are protected by limited liability, and hence the principal cannot make negative transfers to them.

It is assumed that x_H is large enough for it to be in the principal's interest to operate

[4] As standard in the literature on collusion (see Tirole, 1992), this means that the supervisor cannot misreport the high output as low output or the low output as high output. Upon observing the agent's output, the supervisor can only conceal his information and claim that the monitoring was inconclusive, that is, $r = \emptyset$.

[5] In other words, evidence is verifiable only by the person(s) to whom the supervisor reveals it. Evidence is publicly verifiable only when the supervisor produces his report.

the firm. In this environment, the goal of the principal is thus to both motivate the agent to work hard and minimize the expected cost of the organization $C(w_L, w_\emptyset, w_H, s_L, s_\emptyset, s_H) \equiv p[\pi(w_H + s_H) + (1-\pi)(w_L + s_L)] + (1-p)(w_\emptyset + s_\emptyset)$.

Since the supervision technology is imperfect and information is hard, the supervisor has discretion to make an untruthful report. Indeed, the supervisor may engage in two types of collusive activity.[6] When the supervisor acquires evidence that the output produced is x_L, he may accept a bribe from the agent to report $r = \emptyset$ instead of $r = x_L$. If supervisor/agent collusion occurs, the agent thus receives $w_\emptyset$ instead of w_L from the principal and pays the promised bribe to the supervisor. When the supervisor acquires evidence that the output produced is x_H, he may accept a bribe from the principal to report $r = \emptyset$ instead of $r = x_H$. If principal/supervisor collusion takes place, the principal therefore pays $w_\emptyset + s_\emptyset$ to its employees (plus a bribe to the supervisor) instead of $w_H + s_H$.

We make the assumption that when engaging in collusive activities, the supervisor unofficially shows the private evidence acquired to the other involved party. Expressed differently, in line with Tirole's (1986, 1992) standard models, collusion occurs under symmetric information on evidence among involved parties.

Regarding collusion, we make the following standard assumptions (see, for example, Tirole, 1992). First, the technology used to transfer bribes, which we refer to as the side transfer technology, may be less or equally efficient to the official transfer technology (i.e., the transfer technology used by the principal to pay its employees). Unofficial income can therefore be transferred to the supervisor at a rate $k \in (0, 1]$. Formally, if $k \in (0, 1)$, z dollars unofficially transferred to the supervisor only worth kz dollars to him. This may be, for example, because collusion is costly to organize. If instead $k = 1$, the side transfer technology is totally efficient. Second, side transfers

[6] As shown in Vafaï (2002, 2010), organizations may also be exposed to other forms of unofficial activities.

are enforceable. Third, collusion is only observable to the involved parties. Fourth, the supervisor has all the bargaining power when engaging in collusion. Fifth, the supervisor does not engage in collusion when indifferent, that is, when payoffs associated with colluding and not colluding are identical.

The evolution of the three-level agency relationship is thus the following: (1) The principal offers a contract $(w_L, w_\emptyset, w_H)$ to the agent and a contract $(s_L, s_\emptyset, s_H)$ to the supervisor. (2) The agent and the supervisor decide whether to accept or refuse the contract. If either refuses, the relationship ends and they both get their reservation utility. If the agent and the supervisor accept the contracts, the relationship continues as follows. (3) Supervision takes place and the agent decides whether to work or to shirk. (4) Hard information about the output produced by the agent is obtained with probability p and decisions of whether or not to engage in collusive activities are made. (5) The supervisor produces a report. (6) Transfers and side transfers take place.

We look for a subgame perfect equilibrium of this game.

3 Unbiased supervision

This section investigates the benchmark case where the supervisor does not engage in collusive activities. This corresponds to the case where $k = 0$. The agent's incentive constraint writes $p\left[\pi w_H + (1-\pi) w_L\right] + (1-p) w_\emptyset - \gamma \geq p w_L + (1-p) w_\emptyset$, or equivalently,

$$w_H - w_L \geq \frac{\gamma}{p\pi}. \tag{1}$$

This constraint makes the agent prefer to exert effort in equilibrium.[7]

Since transfers must be nonnegative, we have

$$w_L \geq 0, w_\emptyset \geq 0, w_H \geq 0, s_L \geq 0, s_\emptyset \geq 0, s_H \geq 0. \tag{2}$$

[7] We make the standard assumption that the agent chooses to work when she is indifferent.

The agent's contract must also verify her participation constraint, $p[\pi w_H + (1-\pi)w_L] + (1-p)w_\emptyset - \gamma \geq 0$. However, given that transfers must be nonnegative, this constraint is less restrictive that the agent's incentive constraint, and hence will be disregarded in the rest of the paper.

Since supervision does not imply any cost, the supervisor will accept any contract $(s_L, s_\emptyset, s_H) \in \mathbb{R}^3_+$.

To motivate the informed supervisor to report truthfully, the principal must also set $s_L \geq s_\emptyset$ and $s_H \geq s_\emptyset$. However, given that the optimal contract offered to the supervisor in this case and in the subsequent cases automatically satisfies these constraints we will disregard them in the rest of the paper.

An organization where supervision is unbiased has the following program:

$$[P_0] \qquad \min_{w_L, w_\emptyset, w_H, s_L, s_\emptyset, s_H} \quad C(w_L, w_\emptyset, w_H, s_L, s_\emptyset, s_H)$$

$$\text{s.t. (1) and (2).}$$

Proposition 1 summarizes the solution to this program.

Proposition 1. The optimal contracts offered in an organization with a non-collusive supervisor are $(w_L, w_\emptyset, w_H) = (0, 0, \frac{\gamma}{p\pi})$ and $(s_L, s_\emptyset, s_H) = (0, 0, 0)$. The expected cost of the organization is $C^0 \equiv \gamma$.

No rent is thus provided to employees in the absence of collusion.

4 Biased supervision

This section considers respectively the cases where only a single type of collusion is possible and where both types of collusion may occur. As shown in Vafaï (2008), there is no loss of generality in restricting attention to contracts that deter collusive activities

4.1 Single type of collusion

The features of an organization exposed to a single type of collusion are the following:

4.1.1 Collusion between the supervisor and the agent

In a the three-level hierarchy exposed exclusively to supervisor/agent collusion, this type of unofficial activity may occur when supervision reveals the evidence that a low output has been produced. This is the case either when the agent shirks or when she works hard but is unlucky. In both of these cases, the agent promises to pay a bribe to the supervisor for untruthful reporting, that is, for reporting $r = \emptyset$ instead of $r = x_L$.

The prevention of supervisor/agent collusion imposes an extra constraint on the organization. Collusion occurs if it is profitable for both parties. If the agent chooses to collude with the supervisor, her utility is $w_\emptyset - b_{S/A}$ if she has shirked, and $w_\emptyset - b_{S/A} - \gamma$ if she has worked but has been unlucky, where $b_{S/A}$ is the bribe offered to the supervisor. If the agent chooses not to collude with the supervisor, her utility is w_L if she has shirked, and $w_L - \gamma$ if she has worked but has been unlucky. The agent thus finds collusion beneficial as long as $w_\emptyset - b_{S/A} \geq w_L$, that is, as long as $b_{S/A} \leq w_\emptyset - w_L$. The maximum bribe, $b^M_{S/A}$, the agent is willing to offer for information concealment is hence $b^M_{S/A} \equiv w_\emptyset - w_L$. Since the supervisor has all the bargaining power, he may extract $b^M_{S/A}$ from the agent. The supervisor will decide not to collude with the agent if his utility from reporting truthfully, s_L, exceeds his utility from reporting untruthfully, $s_\emptyset + kb^M_{S/A}$, that is, if

$$s_L \geq s_\emptyset + k(w_\emptyset - w_L), \tag{3}$$

where $w_\emptyset - w_L$ is the stake of supervisor/agent collusion. We refer to this constraint as the supervisor/agent no-collusion constraint. As explained in the Appendix, since it optimal to set $s_\emptyset = 0$, constraint (3) expresses that supervisor/agent collusion may be deterred either by creating incentive payments for the supervisor, that is, by setting $w_\emptyset > w_L$ and then by setting s_L sufficiently large, or by destroying its stake, that is, by setting $w_\emptyset = w_L$. The program of a three-level organization exposed to supervisor/agent collusion is thus program $[P_0]$ with the extra constraint (3).

As may easily be seen, the collusion-free contracts of Proposition 1 also immunize the organization against supervisor/agent collusion. We may thus state:

Proposition 2. Collusion between the supervisor and the agent is harmless.

This result is in line with the growing body of literature, mentioned in the introduction, that proves the harmfulness of supervisor/agent collusion in many environments. However, in contrast to that literature which exclusively considers supervisor/agent collusion, we show in the next sections that all types of collusion are not harmless and that in the presence of collusion between the principal and the supervisor, collusion between the supervisor and the agent is not harmless anymore.

4.1.2 Collusion between the principal and the supervisor

In a hierarchical organization exposed only to principal/supervisor collusion, this type of unofficial activity may take place when supervision reveals the evidence that the agent has produced a high output. The principal may then pay a bribe to the supervisor to report $r = \emptyset$ instead of $r = x_H$.

The wage costs for the principal associated with not colluding and colluding with the supervisor are respectively $w_H + s_H$ and $w_\emptyset + s_\emptyset + b_{P/S}$, where $b_{P/S}$ denotes the bribe paid for an untruthful report. The principal thus finds collusion beneficial as long as $w_\emptyset + s_\emptyset + b_{P/S} \leq w_H + s_H$. The maximum bribe, $b^M_{P/S}$, it is then ready to pay for an untruthful report is $b^M_{P/S} \equiv w_H + s_H - w_\emptyset - s_\emptyset$. Given that the supervisor has all the bargaining power, he may extract $b^M_{P/S}$ from the principal. The supervisor does not find collusion beneficial if $s_H \geq s_\emptyset + kb^M_{P/S}$. That is, if

$$s_H \geq s_\emptyset + \frac{k}{1-k}(w_H - w_\emptyset) \text{ for } k \in (0,1) \text{ or } w_\emptyset \geq w_H \text{ for } k = 1. \tag{4}$$

These constraints will be referred to as the principal/supervisor no-collusion constraints. Since optimally $s_\emptyset = 0$, when $k \in (0,1)$, principal/supervisor collusion may hence be deterred either by creating incentive payments for the supervisor, that is, by setting $w_H > w_\emptyset$ and then by setting s_H sufficiently large, or by destroying its stake, that is, by setting $w_H = w_\emptyset$.

The program of a hierarchical organization exposed exclusively to principal/supervisor collusion then writes:

$$[P_1] \qquad \min_{w_L, w_\emptyset, w_H, s_L, s_\emptyset, s_H} \quad C(w_L, w_\emptyset, w_H, s_L, s_\emptyset, s_H)$$

$$\text{s.t. (1), (2), and (4).}$$

Define $\widetilde{\pi} \equiv \frac{(1-k)(1-p)}{pk}$. The solution to this program is summarized in the following proposition.

Proposition 3. The optimal contracts offered in an organization exposed only to principal/supervisor collusion are : (a) $(w_L, w_\emptyset, w_H) = (0, 0, \frac{\gamma}{p\pi})$ and $(s_L, s_\emptyset, s_H) = (0, 0, \frac{k\gamma}{(1-k)p\pi})$ for $k \in (0,1)$ and (i) $p \leq 1-k$; (ii) $p > 1-k$ and $\pi \leq \widetilde{\pi}$. The expected cost of the organization is $C^1 \equiv \frac{\gamma}{1-k}$. (b) $(w_L, w_\emptyset, w_H) = (0, \frac{\gamma}{p\pi}, \frac{\gamma}{p\pi})$ and $(s_L, s_\emptyset, s_H) = (0, 0, 0)$ for (i) $k \in (0,1)$, $p > 1-k$ and $\pi > \widetilde{\pi}$; (ii) $k = 1$. The expected cost of the organization is $C^2 \equiv \frac{[1-p(1-\pi)]\gamma}{p\pi}$.

Corollary. Collusion between the principal and the supervisor is harmful.

We optimally have $w_L = 0$ $(= s_\emptyset)$, and thus $w_H = \frac{\gamma}{p\pi}$ from the agent's incentive constraint. To set $w_\emptyset$, the principal faces a trade-off. As shown in the Appendix, to cope with principal/supervisor collusion, it may either destroy the stake of this type of collusion by setting $w_\emptyset = w_H = \frac{\gamma}{p\pi}$ - and hence $s_H = 0$ - or it may conserve the stake of principal/supervisor collusion while creating incentives for the supervisor to report truthfully, that is, it may set $w_H = \frac{\gamma}{p\pi} > w_\emptyset = 0$ - and thus $s_H = \frac{k\gamma}{(1-k)p\pi}$. We refer to these policies respectively as *stake-eliminating* policy and *incentive* policy. Compared with the incentive policy, the stake eliminating policy consists in paying a higher wage $w_\emptyset$ to the agent but a lower wage s_H to the supervisor. When the incentive policy is adopted, a rent is paid to the supervisor, whereas when the stake eliminating policy is adopted a rent is paid to the agent. In other words, the incentive policy benefits the supervisor whereas the stake eliminating policy benefits the agent.

Several cases should then be distinguished for $k \in (0,1)$. If the supervision technology is strongly imperfect ($p \leq 1-k$), that is, if it is likely that supervision will be inconclusive, and hence it is likely that $w_\emptyset$ will be paid to the agent and s_H will not be paid to the supervisor, the principal's optimal policy is the incentive policy. In the alternative case where the supervision technology is weakly imperfect ($p > 1-k$), it is unlikely that supervision will be inconclusive, that is, it is unlikely that $w_\emptyset$ will be paid to the agent. The principal's policy choice then depends on the quality of the production technology. When the production technology is relatively inefficient ($\pi \leq \widetilde{\pi}$), and thus it is unlikely that s_H will be paid to the supervisor, the optimal policy is the incentive policy.

Finally, in the case where $k = 1$ the optimal policy is the stake eliminating policy. Indeed, since in this case deterring principal/supervisor collusion requires that the principal sets $w_\emptyset \geq w_H$ and since the objective function is increasing in $w_\emptyset$, we optimally have $w_\emptyset = w_H = \frac{\gamma}{p\pi}$.

4.2 Multiple types of collusion

The program of a hierarchical organization exposed to both supervisor/agent and principal/supervisor collusion is $[P_1]$ with the additional constraint (3). This extended program will be referred to as $[P_2]$.

Define $\overline{\pi} \equiv \frac{(1-k)[1-p(1-k)]}{pk(2-k)}$. Proposition 4 summarizes the solution to $[P_2]$.

Proposition 4. The optimal contracts offered in an organization exposed to supervisor/agent and principal/supervisor collusion are : (a) $(w_L, w_\emptyset, w_H) = (0, 0, \frac{\gamma}{p\pi})$ and $(s_L, s_\emptyset, s_H) = (0, 0, \frac{k\gamma}{(1-k)p\pi})$ for $k \in (0,1)$ and (i) $p \leq 1-k$; (ii) $p > 1-k$ and $\pi \leq \overline{\pi}$. The expected cost of the organization is C^1. (b) $(w_L, w_\emptyset, w_H) = (0, \frac{\gamma}{p\pi}, \frac{\gamma}{p\pi})$ and $(s_L, s_\emptyset, s_H) = (0,0,0)$ for (i) $k \in (0,1)$, $p > 1-k$ and $\pi > \overline{\pi}$; (ii) $k = 1$. The expected cost of the organization is $C^3 \equiv \frac{[1-p(1-\pi)(1-k)]\gamma}{p\pi}$.

As explained above, the principal has the choice of two policies to deter principal/supervisor collusion. However, compared with the case where the organization is only exposed to principal/supervisor collusion, in the presence of both types of collusion, the stake eliminating policy has an extra cost. Indeed, using the stake eliminating policy and setting $w_\emptyset$ above 0 then makes supervisor/agent collusion costly to deter. Expressed differently, if the principal decides to adopt the stake eliminating policy, the prevention of one type of collusion increases the cost of the prevention of the other type of collusion. In this case, collusive activities negatively interact. When both types of collusion are accounted for, the expected cost of the stake eliminating policy thus rises from $C^2 \equiv \frac{[1-p(1-\pi)]\gamma}{p\pi}$ to $C^3 \equiv \frac{[1-p(1-\pi)(1-k)]\gamma}{p\pi}$ and reaches its highest level for $k = 1$. By contrast, the expected cost of the incentive policy is not affected by the multiplication of collusive activities. When the organization adopts the incentive policy, collusion between the supervisor and the agent thus remains harmless.

Since in an organization exposed to two types of collusion there is a negative interaction between collusive activities associated with the stake eliminating policy, this policy is less often adopted than in the case where only principal/supervisor collusion is possible. Formally, we have $\overline{\pi} > \widetilde{\pi}$ for $p > 1 - k$. As explained in the previous subsection, given that when the incentive policy is adopted the supervisor receives a rent and that the incentive policy is now more often adopted, the supervisor is the one who benefits from the multiplication of collusive activities.

The following theorem summarizes how the threat of two types of collusion affects a multi-level hierarchy compared with the case where only a single type of collusion is possible.

Theorem. Compared with the case where the organization is exposed to a single type of collusion, under the threat of two types of collusion: (i) the configuration of the optimal preventive policy is modified; (ii) supervisor/agent collusion is not systematically harmless anymore; (iii) the expected cost of collusion prevention increases; (iv) the supervisor more often receives a rent, and thus is the member of the organization who benefits from the multiplication of collusive activities.

5 Conclusion

In the last two decades the literature on principal-supervisor-agent organizations has largely studied a specific type of bias in supervision, namely supervisor/agent collusion. We have extended the analysis of these hierarchies by also considering the possibility of principal/supervisor collusion. We have shown that the multiplication of collusive activities deeply affects the contractual choices of a hierarchical organization, and hence have proved that focusing on a single type of collusion in the analysis of hierarchies is deceptive.

Appendix

Proof of Proposition 1. Whether the organization is exposed or not to collusive activities, it is obviously optimal to set $s_\emptyset$ as low as allowed by the limited liability constraints, that is $s_\emptyset = 0$.

Since the expected cost of the organization is increasing in w_L and reducing this wage does not make constraints more severe (more specifically, it relaxes the agent's incentive constraint), it is optimal to set this wage as low as the limited liability constraint $w_L \geq 0$ makes it possible. We therefore have $w_L = 0$. From the same argument $w_\emptyset = s_L = s_H = 0$ and the principal optimally sets w_H as low as allowed by the agent's incentive constraint, that is, $w_H = \frac{\gamma}{p\pi}$ (since $w_L = 0$).

Proof of Proposition 3. To both soften constraints and lower the expected cost of the organization, w_L, w_H and s_L should be reduced. Given that the agent is protected by limited liability, it is optimal to set $s_L = 0$. By the same argument, $w_L = 0$ and the agent's incentive constraint becomes $w_H = \frac{\gamma}{p\pi}$. There are then two cases to consider with respect to k.

1. $k < 1$. Recalling that optimally $s_\emptyset = 0$ and substituting $w_H = \frac{\gamma}{p\pi}$ into the principal/supervisor no-collusion constraint, this constraint writes $s_H \geq \frac{k}{1-k}(\frac{\gamma}{p\pi} - w_\emptyset)$. This constraint may then be relaxed by increasing $w_\emptyset$. However, since setting $w_\emptyset > \frac{\gamma}{p\pi}$ instead of $w_\emptyset \leq \frac{\gamma}{p\pi}$ increases the expected cost of the organization (because this cost is increasing in $w_\emptyset$) without allowing to reduce s_H below 0 (because of the limited liability constraint $s_H \geq 0$), the principal sets $w_\emptyset \in \left[0, \frac{\gamma}{p\pi}\right]$. The relevant constraint on s_H is then $s_H \geq \frac{k}{1-k}(\frac{\gamma}{p\pi} - w_\emptyset)$, that is, the limited liability constraint $s_H \geq 0$ is redundant.

Given that the objective function is increasing in s_H and lowering this wage does not make the other constraints more severe, we have $s_H = \frac{k}{1-k}(\frac{\gamma}{p\pi} - w_\emptyset)$. Substituting this equation into the objective function of program $[P_1]$, this program becomes:

$$\min_{w_\emptyset} \quad \frac{\gamma}{1-k} + \frac{(1-k)(1-p)-p\pi k}{1-k} w_\emptyset$$

$$\text{s.t. } w_\emptyset \in \left[0, \frac{\gamma}{p\pi}\right].$$

Two cases have then to be distinguished.

Let $\Delta \equiv (1-k)(1-p) - p\pi k$. Then if $\Delta \geq 0$, that is, if $\pi \leq \widetilde{\pi} \equiv \frac{(1-k)(1-p)}{pk}$, the objective function is increasing in $w_\emptyset$. It is then optimal to set $w_\emptyset$ as low as possible, that is, $w_\emptyset = 0$. We then have $s_H = \frac{k\gamma}{(1-k)p\pi}$.

If instead $\Delta < 0$, that is, if $\pi > \widetilde{\pi}$, the objective function is decreasing in $w_\emptyset$, and it is hence optimal to set $w_\emptyset$ as high as possible. The principal then sets $w_\emptyset = \frac{\gamma}{p\pi}$. We then have $s_H = 0$.

Since we have $\widetilde{\pi} \geq 1$ if $p \leq 1-k$, and thus we then systematically have $\Delta \geq 0$, the optimal contracts are: (a) $(w_L, w_\emptyset, w_H) = (0, 0, \frac{\gamma}{p\pi})$ and $(s_L, s_\emptyset, s_H) = (0, 0, \frac{k\gamma}{(1-k)p\pi})$ for (i) $p \leq 1-k$; (ii) $p > 1-k$ and $\pi \leq \widetilde{\pi}$. The expected cost of the organization is then $C^1 \equiv \frac{\gamma}{1-k}$. (b) $(w_L, w_\emptyset, w_H) = (0, \frac{\gamma}{p\pi}, \frac{\gamma}{p\pi})$ and $(s_L, s_\emptyset, s_H) = (0, 0, 0)$ for $p > 1-k$ and $\pi > \widetilde{\pi}$. The expected cost of the organization is then $C^2 \equiv \frac{[1-p(1-\pi)]\gamma}{p\pi}$.

2. $k = 1$. Given that the principal's objective function is increasing in $w_\emptyset$ and s_H - and lowering these wages does not make constraints more severe in this case - we optimally have $w_\emptyset = \frac{\gamma}{p\pi} (= w_H)$ from the principal/supervisor no-collusion constraint and $s_H = 0$ from the limited liability constraints. The expected cost of the organization is then C^2.

Proof of Proposition 4. As in the previous proof, to both soften constraints and lower the expected cost of the organization, w_H should be reduced. The agent's incentive constraint thus becomes $w_H = \frac{\gamma}{p\pi} + w_L$. There are then two cases to consider with respect to k.

1. $k < 1$. Substituting $w_H = \frac{\gamma}{p\pi} + w_L$ into the principal/supervisor no-collusion constraint, this constraint writes $s_H \geq \frac{k}{1-k}(\frac{\gamma}{p\pi} + w_L - w_\emptyset)$. This constraint may be relaxed by increasing $w_\emptyset$.

However, given that setting $w_\emptyset > \frac{\gamma}{p\pi} + w_L$ instead of $w_\emptyset \leq \frac{\gamma}{p\pi} + w_L$ both increases the expected cost of the organization (because this cost is increasing in $w_\emptyset$) and makes the supervisor/agent no-collusion constraint $s_L \geq k(w_\emptyset - w_L)$ more severe without allowing to reduce s_H below 0 (because of the limited liability constraint $s_H \geq 0$), we have $w_\emptyset \leq \frac{\gamma}{p\pi} + w_L$. The relevant constraint on s_H is then $s_H \geq \frac{k}{1-k}(\frac{\gamma}{p\pi} + w_L - w_\emptyset)$. In other words, the limited liability constraint $s_H \geq 0$ may be disregarded.

Following a similar argument, we have that the relevant constraint on s_L is the supervisor/agent no-collusion constraint $s_L \geq k(w_\emptyset - w_L)$. Expressed differently, the limited liability constraint $s_L \geq 0$ may be disregarded. It is, indeed, possible to relax the supervisor/agent no-collusion constraint by increasing w_L. However, given that setting $w_L > w_\emptyset$ instead of $w_L \leq w_\emptyset$ both increases the expected cost of the organization (because this cost is increasing in w_L) and makes the principal/supervisor no-collusion constraint $s_H \geq \frac{k}{1-k}(\frac{\gamma}{p\pi} + w_L - w_\emptyset)$ more severe without allowing to reduce s_L below 0 (because of the limited liability constraint $s_L \geq 0$), we have $w_L \leq w_\emptyset$. Summarizing, we have $w_\emptyset \leq \frac{\gamma}{p\pi} + w_L$ and $w_L \leq w_\emptyset$, that is $w_\emptyset \in \left[w_L, \frac{\gamma}{p\pi} + w_L\right]$ with $w_L \geq 0$, and hence the relevant constraints on s_L and s_H are respectively $s_L \geq k(w_\emptyset - w_L)$ and $s_H \geq \frac{k}{1-k}(\frac{\gamma}{p\pi} + w_L - w_\emptyset)$.

The objective function is increasing in s_L and s_H and lowering these wages does not make the other constraints more severe. We therefore have $s_L = k(w_\emptyset - w_L)$ and $s_H = \frac{k}{1-k}(\frac{\gamma}{p\pi} + w_L - w_\emptyset)$. These equations should be substituted into the objective function of program $[P_2]$, which then becomes:

$$\min_{w_L, w_\emptyset} \quad \frac{\gamma}{1-k} + \frac{p[(1-k)[1-(1-\pi)k]+\pi k]}{1-k} w_L + \frac{(1-k)[1-p[1-(1-\pi)k]]-p\pi k}{1-k} w_\emptyset$$

$$\text{s.t. } w_L \geq 0 \text{ and } w_\emptyset \in \left[w_L, \frac{\gamma}{p\pi} + w_L\right].$$

There are two cases to distinguish.

Noting $\Theta \equiv (1-k)[1-p[1-(1-\pi)k]] - p\pi k$, if $\Theta \geq 0$, that is, if $\pi \leq \overline{\pi} \equiv \frac{(1-k)[1-p(1-k)]}{pk(2-k)}$, the objective function is increasing in $w_\emptyset$, and hence optimally $w_\emptyset = w_L$. After substituting this expression into the objective function of the above program, this function becomes $\frac{\gamma}{1-k} + w_L$. This function is increasing in w_L, and hence it is optimal to set w_L as low as possible, that is, $w_L = 0$. It follows that $w_\emptyset = 0$, and therefore $w_H = \frac{\gamma}{p\pi}$, $s_L = 0$ and $s_H = \frac{k\gamma}{(1-k)p\pi}$.

If $\Theta < 0$, that is, if $\pi > \overline{\pi}$, the objective function is decreasing in $w_\emptyset$, and $w_\emptyset$ should be set as high as possible. We then have $w_\emptyset = \frac{\gamma}{p\pi} + w_L$. After substituting this equation into the objective function of the above program, this function writes $\frac{[1-p(1-\pi)(1-k)]\gamma}{p\pi} + w_L$. Since this function is increasing in w_L, it is optimal to set w_L as low as possible, that is, $w_L = 0$. It follows that $w_H = w_\emptyset = \frac{\gamma}{p\pi}$, and thus $s_L = \frac{k\gamma}{p\pi}$ and $s_H = 0$.

Given that we have $\overline{\pi} \geq 1$ for $p \leq 1-k$, we then systematically have $\Theta \geq 0$ in this case. The optimal contracts are thus: (a) $(w_L, w_\emptyset, w_H) = (0, 0, \frac{\gamma}{p\pi})$ and $(s_L, s_\emptyset, s_H) = (0, 0, \frac{k\gamma}{(1-k)p\pi})$ for (i) $p \leq 1-k$; (ii) $p > 1-k$ and $\pi \leq \overline{\pi}$. The expected cost of the organization is then C^1. (b) $(w_L, w_\emptyset, w_H) = (0, \frac{\gamma}{p\pi}, \frac{\gamma}{p\pi})$ and $(s_L, s_\emptyset, s_H) = (\frac{k\gamma}{p\pi}, 0, 0)$ for $p > 1-k$ and $\pi > \overline{\pi}$. The expected cost of the organization is then $C^3 \equiv \frac{[1-p(1-\pi)(1-k)]\gamma}{p\pi}$.

2. $k = 1$. As above, since the organization's objective function is increasing in $w_\emptyset$ - and lowering this wage does not make constraints more severe in this case - we optimally have $w_\emptyset = w_H = \frac{\gamma}{p\pi} + w_L$ from the principal/supervisor no-collusion constraint and the agent's binding incentive constraint. Once $w_\emptyset = w_H = \frac{\gamma}{p\pi} + w_L$ is substituted into both the objective function of program $[P_2]$ and the supervisor/agent no-collusion constraint, this program writes:

$$\min_{w_L, s_L, s_H} \quad \frac{[1-p(1-\pi)]\gamma}{p\pi} + w_L + p[(1-\pi)s_L + \pi s_H]$$

$$\text{s.t. } w_L \geq 0,\ s_L \geq \frac{\gamma}{p\pi} \text{ and } s_H \geq 0.$$

Obviously $w_L = 0$, $s_L = \frac{\gamma}{p\pi}$ and $s_H = 0$. The expected cost of the organization is then $\frac{\gamma}{p\pi}$.

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