The conditions of efficiency of a PPP for public finances

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ABSTRACT

Public authorities seem increasingly to be involving the private sector in financing, building and operating new infrastructures. A lot of reasons are usually given to justify this private sector involvement but the reasons which are the most frequently mentioned relate to the ability of a private operator to manage the construction and operation of the project more efficiently. This amounts to assuming that the Internal Rate of Return (IRR) of the project is not the same depending on whether it is managed by an administration or public body or by a company which in theory keeps abreast of the progress in optimization techniques which is taking place all the time. This difference is explained in many ways: the private sector pays some categories of staff less well, is more flexible, offers faster construction times which speed up the return on investment and is also more able to resist political demands which generate additional costs.

Nevertheless, with a public or a private operator, there is a target IRR, very near the standard notion of Weighted Average Capital Cost (WACC), which is larger in the case of the private alternative because this cost must also include the operator’s profit.

Thus, if the main stake for the national government relates, for each project of public infrastructure, to the need of subsidies, the fundamental issue is the result of two opposite effects: on one hand the effect of a bigger efficiency of the private operator, on the other hand the effect of a lower WACC for the public operator.

The objective of this communication is to propose a modelling of the determination of the need of public financing which formalizes these two effects and allows analyzing the conditions under which the PPP would be advantageous for the public finances.

We propose for that a model of the mechanism of financing of the projects with a restricted number of parameters. This modelling will be confronted with real French cases of projects of toll highways in order to verify the relevance of the model and to determine the actual range of these parameters and their dispersal. An analysis will finally be proposed, according to the intrinsic profitability of the projects, the conditions under which a PPP can relieve the public spending.

This conclusion joins many other authors in arguing against a systematic choice of one or other solution and suggesting rather that the most appropriate solution will depend on the circumstances of each case. The original contribution of this communication consists of an original formalization and of econometric estimations of these circumstances.

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1 – INTRODUCTION

In 1990, the World Bank counted, in the countries where it operates, about 60 cases of public-private partnerships to finance, build and operate public facilities. This represented about $2 billion, all sectors combined. In the following 19 years, the World Bank has added, in the 81 countries eligible for its loans, over 4,500 PPP projects, representing an investment of $1,500 billion (World Bank, 2011). Simultaneously, in the industrialised countries, PPPs have developed rapidly, or rather have revived, considering the major role played by private initiative in the explosion of the railway system in the 19th century. The Private Finance Initiative voted in Britain in 1992 was major milestone of this renewal. France followed that example twelve years later, with the 2004 ruling on partnership contracts, allowing the diversification of the long-standing practice of concessions.

This growing implication of private operators in new public facilities has been fostered by two main concerns of governments. The first corresponds to an opportunistic budget strategy (Maskin and Tirole, 2008) to the extent that the commitments of public authorities are not generally part of its debt consolidation for partnership contracts (Marty, 2007). This is favourable to PPPs, particularly with a cosmetic goal for the accrual accounting of States: either private debt is guaranteed by public finances (de jure or de facto) as a last resort in case of failure of the operator, that is, when the liability of the debt cannot be covered by the commercial receipts of the project; or the partnership is solely at the risk of the private sphere, but then the subsidy usually necessary for the financial balance of the operation is increased according to the risk premiums demanded by the operator and the banks. In the first case, the debt linked to the project is not officially externalised through the PPPs; in the second, it is indeed excluded from the public debt but at the price of an increase of the public subsidy through the remuneration of the operator’s own funds and the compensation of the risk premiums.

However, in the two previous cases the pressure of overly indebted public finances goes in favour of systems favouring private financing and indebting, even if it implies retributions of capital and risk premiums likely to burden in the long term the cost of the operations for public finances. The PPP’s principal task is to “mask” public debt. We will deliberately ignore these opportunistic behaviours in what follows and consider that recourse to PPP is exclusively determined by the other main concern of governments.

That second concern is clearly explained in the aforementioned British and French laws, and is particularly present in the World Bank’s pressure in favour of PPPs: it is the perspective of a lower subsidy for the public authority, linked to the increase in economic profitability that the private operator is liable to bring in comparison with a public operator. One can indeed count on the fact that a private operator, used to being cost-effective, is able to ensure the better internal rate of return of an operation, either by saving on investment costs, or through shorter construction lead times, or by better operating cost control, or by a combination of these efforts to maintain profitability. This has been observed in a great number of activities (Dewenter and Malatesta, 2001).
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The political objective is to minimise the contribution of public finances in building and operating public facilities. We will leave aside the role of “hiding” the debt to consider exclusively this objective. The issue is therefore to know if the remuneration of private capital, in theory higher than that of public capital, can be compensated by gains in profitability that can be ensured by the private operator, so as to minimise public expense. This article proposes to formalise and analyse the conditions in which a PPP enables this minimization.

We will therefore consider projects benefitting from commercial receipts and public subsidies if necessary to their financial balance. To this end, we will use concrete examples of French projects for toll motorways requiring a public financing component. Although the results are illustrated this way by examples from transport economics, it should be borne in mind that they may concern any other sector where public financing completes commercial receipts. If we consider, for example, the question of how the financing of an opera house must be divided between spectators and taxpayers, we are in the same situation as that of the best combination between tolls and subsidies to finance a motorway project.

However, financing, building and operating this type of public facility may involve private operators to very different extents. We will not consider this extensive range of possible role distributions between the public and private spheres, which correspond to as many PPP formulas. The issue of minimising the subsidy will be reduced to a simplified alternative between two options that we will call “public option” and “private option.”

These two options will be defined in section 2. In each case, we will explain how to determine the weighted average cost of capital (or WACC). In section 3, we will describe the mathematical relation between a project’s need for subsidies and the level of the WACC, considering the parameters that determine the financial profitability of a given project. Section 4 will deal with the estimation of the orders of magnitude of these parameters for a set of concrete projects in order that the analyses proposed are located within the ranges of values that we may safely call realistic. In section 5, we will situate and analyse the conditions in which the efficiency of the private operator can compensate a WACC higher than that of the public operator, i.e. that the conditions for which a PPP may relieve public expense are united.

2 – THE SIMPLIFIED PUBLIC-PRIVATE ALTERNATIVE AND THE VALUES OF THE WEIGHTED AVERAGE COST OF CAPITAL

Our analysis gives an alternative to two deliberately contrasted solutions. This is the same as setting aside, unless indicated otherwise, the intermediate situations in which the roles of the public or private actors can be amended. The public and private options that we consider are “stylised” as follows:

- In the “public” option, the operator in charge of the project is a public entity, or a non-profit private society like Network Rail\textsuperscript{2} in Great Britain. In both cases, we will call it a “public operator.” It is not supposed to make profits, but should cover the

\textsuperscript{2} The private society Railtrack was stripped of the network’s management in 2002. It was then transferred to Network Rail, a “non-profit agency” under State control.
investment and operating costs, including the financial charges of its loans, through commercial receipts. The latter can comprise tolls paid by the users, or a shadow toll paid by the public authority. In the case of a loss-making project, it is assumed that the deficit is compensated by the public authority: a subsidy, determined on the basis of a cost-benefit analysis established ex ante, must then complete the expected receipt, so that the operator is ensured that the cost is covered.

- In the “private” option, the mechanism is the same, except that the private operator may have more expensive loan conditions than a public operator, and they must ensure the remuneration of their own capital, and therefore generate a profit.

In these conditions, the weighted average cost of capital (WACC) is basically different according to the option.

According to the rules indicated above, the public operator should carry out the project if the Internal Rate of Return (IRR) expected can cover the interest rate of the market augmented by a risk premium considering the uncertainties associated with any project’s financial evaluation. This concerns, for example, risks on investment and operating costs, as well as commercial risks due to uncertainties in the traffic and receipt forecasts. More specifically, if the long term rates are 4% on the financial market and if the risk premium is estimated at 4% as well, we consider that the WACC is 8% for the public operator. To follow the rules that are imposed on them, the public operator cannot commit to a project unless their IRR is at least 8%. For any value under that, a compensatory subsidy is required in order to reach that threshold.

The private operator will be interested in the same project only if they are able to cover the debt charge that they must commit to, augmented by a risk premium, like the public operator, but they must also ensure the remuneration of their own capital through a profit margin. For comparable conditions on the financial market, the profitability required of the project will be organised differently from the previous one.

Firstly, the share of funding for which the private operator raises a long term loan may be more expensive than for the public operator since a private company cannot benefit from the same credit rating as a public company whose debt is, in the last resort, guaranteed by the State. In the case of big European private operators, the rates can be higher by about 50 points compared to a public operator, thereby increases the rate on the market to 4.5%. Other elements in the risk premium are not taken into account in the banks’ consideration toward the entity that takes out the loan, but result from an analysis of the risks particular to the project. By experience, they are generally of the same order of magnitude as the public option. In total, for this share of funding covered by the loan, the interest rate required to suitably ensure the burden of the loan can rise from 8 to 8.5% for a private operator.

For the share of financing corresponding to the capital contributed by the private operator, the return on this commitment (which includes the risk premium) is

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3 A shadow toll corresponds to a free toll for the user, but the public authority compensates it by paying the tolls itself. The operator is therefore encouraged to satisfy demand as well as possible as soon as the shadow toll is higher than the marginal cost of use.

4 By way of example and to be able to propose later on some concrete representations of our theoretical results, we use orders of magnitude corresponding to the situation in the first semester of 2012 in a country rated AAA or AA+ and for long term loans (20 to 35 years, and even 40 years).
notably higher. The corresponding rate varies distinctly according to the economic situation and the business sector considered. It is often about double the cost of the loans contracted. That can mean, for example, a profitability ratio of about 16% for that share of funding.

If we assume that the financing of the investment includes 80% loans and 20% capital, which corresponds to current gearing, the combination of a return of 8.5% for the first and 16% for the second corresponds to a WACC of 10%. This means that for any value of the project’s IRR lower than 10%, a subsidy will be required by the private operator to ensure its financial balance.

It is noteworthy that this kind of demand from taxpayers is theoretically justified, whether the operator be public or private, through external advantages to the financial balance of the project, according to a calculation of the socio-economic profitability index. It is no longer the sole point of view of the operator and of their bottom line that is considered, but that of the entire community. The losses and advantages of all the economic agents are thus evaluated, for example, the net losses and receipts of the rival modes or the variations of additional users, or even the consequences of the project on safety or the environment. Territorial planning considerations can also justify the decision to invest. This counterpart of positive externalities of public subsidies can be considered equivalent in both hypotheses since the investment is assumed as being subject to the same specifications, whether the operator be public or private.

This means, in ordinary language, that the public authority “buys” the same thing, no matter the status of the operator. It has therefore every interest in choosing the vendor offering the lowest “price.” Based on the previous considerations on the profitability rate required according to whether the operator is public or private, and by assuming (temporarily) that they have the same economic efficiency, three situations are possible:

1) For highly profitable projects (over 10% with the orders of magnitude suggested), no public funding is required, whether the operator be public or private. The public authority should therefore maintain control of an operation plan which brings a financial surplus.

2) For moderately profitable projects (between 8 and 10%), the public operator can invest without subsidies, whereas the private operator must require a subsidy that brings the project’s financial profitability back up to 10%. The first option must be used.

3) For projects with low profitability (under 8%), a subsidy is required in both cases, but it is bigger if the operator is private, since it must in that case raise the financial profitability of the project to a higher level. The public operator still remains more interesting.

Under the assumption of equal efficiency between the public and private sectors, the orders of magnitude illustrate the fact that the “private” option is, in all cases, more expensive for the public finances than the “public” option. It is therefore clear that recourse to a PPP rests on the opposite assumption: it is justified by lower subsidy levels only if the private operator is more efficient to the point of compensating a higher WACC than that of the public operator and therefore requires a lower level of subsidy.
For each option, private or public, the IRRs are therefore different as are the resulting needs for subsidies. It is therefore the relation between the need for subsidies and the IRRs that is in question. This relation should thus be established in order to formalise the stakes of this alternative for the public finances in order to specify later on the origin of this increase in efficiency.

3 – FINANCIAL PROFITABILITY, WACC AND NEED FOR SUBSIDIES

For this relation, let us consider a project corresponding to a stylised but nonetheless classical chronological series of the costs and benefits represented in figure 1. We take into account only the commercial elements that enter into the calculation of financial profitability. If the commissioning is assumed to occur on date \( t = 0 \), the annual cost between the dates \(-d\) and \(0\) is \(c\). Starting at the commissioning date, the profit generated is assumed to take the form \((a+b.t)\).

![Figure 1: The Cost/Benefit theoretical model](image-url)

The project’s internal rate of return (IRR), which is the discount rate which cancels its financial net present value \(NPV_f\) is therefore a function of four parameters \(c, d, a\) and \(b\). It must be compared with a rate of return that an operator (public or private) is entitled to expect.

We will use the following writing:

- \(\alpha\) is the discount rate used to calculate the \(NPV_f\) of the project,
- \(\alpha_0\) is the IRR of the project, i.e. the discount rate which cancels the \(NPV_f\),
δ is the supplement of IRR that the subsidy brings to the operator,
τ is the subsidy rate, i.e. the proportion of c financed by the subsidy.

For the discount rate α, and the updated balance sheet from date −d to T, the net present value is:

\[
NPV_f = \int_{-d}^{0} -c.e^{-\alpha t} \, dt + \int_{0}^{T} (a + b.t).e^{-\alpha t} \, dt
\]

(1)

We will assume that the discount is extended to infinity, which is without consequences on the results which interest us because of the small weight of the distant future, and, especially, the convergence of the integral function in equation (1) when \( T \to \infty \). The equation becomes5:

\[
NPV_f = \frac{1}{\alpha} \left[ c(1 - e^{-\alpha d}) + a + \frac{b}{\alpha} \right]
\]

(2)

The project’s IRR, \( \alpha_0 \), is then an implicit solution of the equation:

\[
c(1 - e^{\alpha_0 \cdot d}) + a + \frac{b}{\alpha_0} = 0
\]

(3)

A subsidy rate \( \tau \) reduces the annual cost of construction \( c \) to \( c(1 - \tau) \) and brings the IRR \( \alpha_0 \) to \( (\alpha_0 + \delta) \) so that equation (3) becomes:

\[
(1 - \tau)c(1 - e^{(\alpha_0 + \delta)d}) + a + \frac{b}{\alpha_0 + \delta} = 0
\]

(4)

Of which we can deduce the expression of the subsidy rate:

\[
\tau = \frac{a(\alpha_0 + \delta) + b}{c(\alpha_0 + \delta)(e^{(\alpha_0 + \delta)d} - 1)}
\]

(5)

The relation between \( \tau \), the subsidy rate, and \( \delta \), the increase of the project’s IRR expected by the operator, depends on the parameters c, d, a, b and, of course, \( \alpha_0 \). These parameters are additionally linked with each other by equation (4) that defines the IRR \( \alpha_0 \) of the project (or what is equivalent, \( \tau = 0 \) if and only if \( \delta = 0 \)). This implies some difficulties in the study and the representation of these functions that we will be able to overcome with cross curves. Nevertheless, these cross curves must be represented with the variation of pertinent parameters, which correspond to values that have actually been observed.

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5 The details of the calculation are presented in the initial presentation of this formalization (Bonnafous, 2002).
4 – ESTIMATIONS OF THE ORDERS OF MAGNITUDE OF THE PARAMETERS

To represent these pertinent ranges of variation, available financial evaluations relating to concrete projects must be used. The previous profitability calculation having been reduced to a simplified representation along four parameters, it is enough to seek estimators. These estimators of a, b, c and d will be calculated on the basis of 17 evaluations of motorway projects (or variations of projects) that we have decided to analyse for two good reasons. Firstly, these evaluations are available in an official report concerning these projects. Secondly, and most importantly, it is one of the rare databases evaluating major motorway projects in which the results have been harmonised for the needs of this report, and for which the financial profitability and the needs for subsidies have been calculated with identical methods.

These methods are evidently classical and come under the cost-benefit analysis. They do not rely on the model developed in the previous section, but on a detailed calculation of the records of costs and benefits. To work with our own model, the numerical values of the corresponding parameters a, b, c and d must therefore be deduced from the evaluations available.

To this end, it is assumed that the linearity of the chronological series of costs and benefits represented in figure 1 is a good approximation resulting from the projects’ evaluations. This assumption is all the more reasonable as, in these evaluations, traffic is assumed to increase linearly and the tolls are assumed steady in actual price. We will also assume that the infinite discounting of profits provides an acceptable approximation of the discounting over 50 years.

To simplify the analysis, let us separate from equation (2) the discounted cost \( C^* \) of the works that is deduced from equation (1) and written as:

\[
C^* = \frac{C}{\alpha} (e^{\alpha d} - 1)
\]

This amounts to temporarily free ourselves from the variation of parameters c and d, whereas they could of course differ if the operator is public or private. We will choose to “mask” for the moment these two parameters in \( C^* \). In that case, equation (2) becomes:

\[
NPV_f = -C^* + \frac{a}{\alpha} + \frac{b}{\alpha^2}
\]

The project’s IRR, \( \alpha_0 \), is then a solution of equation (3) which becomes:

\[
-C^* + \frac{a}{\alpha_0} + \frac{b}{\alpha_0^2} = 0
\]

If the operator, whether private or public, requires a higher IRR, that is \( (\alpha_0 + \delta) \), it can be ensured by a subsidy S which verifies the equation:

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6 This report drafted by two French administrations (the General Council of Bridges and Highways and the General Inspection of Finances) is already fairly old (2003)
Equations (9) and (10) are therefore two linear equations with two unknowns, \( a \) and \( b \), when, for each of the 17 equations available, we know:

- The discounted cost of the project \( C^* \)
- The IRR of the project \( \alpha_0 \)
- The estimation of the subsidy \( S \), calculated for a target IRR \( (\alpha_0 + \delta) \), in this case 10\% for the report.

The numerical values obtained for the estimates of \( a \) and \( b \) for each of the 17 motorway projects used are presented in appendix 1. For our exercise, we will only present the mean values and the value ranges that merit exploration.

### 5 – PERTINENT VALUE RANGES OF THE PARAMETERS

For each of the 17 projects, we estimated the parameters using equations (9) and (10), \( C^* \) being fixed at 100 by convention for each evaluation. The mean values of the estimates are 3.3 and 0.11 for \( a \) and \( b \) respectively. The values obtained for the 17 projects are represented in figure 2 below.

**Figure 2**: Relation between \( a \) and \( b \) – 17 French motorway projects  
\( (C^* = 100 – d = 4) \)
It should be noted that the issue here is not crucial anymore when $a_0>8\%$. We can deduce easily from equation (9) that it corresponds to the inequality $b>-0.08a+0.64$ represented in the figure above. We observe that only one out of the 17 projects is in that case, and, therefore, that all the others require subsidies.

In the following calculations, we will explore the situations corresponding to the plausible orders of magnitude by varying the parameters between their boundaries. To define the boundaries of $a$ and $b$, we can simply use the values close to their maximum and minimum values, i.e. 2 and 6 for $a$, and 0 and 0.3 for $b$. In addition, Figure 2 above suggests that we can also vary the two parameters together, which will be proposed later on.

Of course, the private operator can also claim to be more efficient, which brings down the level of $C^*$. The possibility of such a gain in efficiency can be clearly seen in equation (7): it can result either from a faster execution of the work, i.e. a shorter lead time $d$, or better controlled costs, i.e. a lesser cost $c$. Strictly speaking, if we simulate the decrease of one of these parameters (or both simultaneously), $\alpha$ will increase, in accordance with equation (8). It will evidently be taken into account in the following calculations.

For the orders of magnitude to be chosen, $C^*$ is by convention fixed at 100 in each project, since this is how the estimates of $a$ and $b$ have been established based on the effective evaluations of each project. This cost will therefore be considered as that of the public operator. It corresponds to a construction period assumed to last 4 years. These are the values on which the simulations will be based.

The reader will have noticed that the normalisation of $C^*$ at 100 for all the projects has the advantage of giving a simple interpretation of parameters $a$ and $b$; $a$ is the classical first year rate of return (in % since it is rounded to a discounted cost of 100) and $b$ is the gradient of the evolution of the profit assumed to be linear (see fig. 1).

Concerning the IRR targeted by the operator $(\alpha_0+\delta)$, we recall, as already mentioned in section 2, the orders of magnitude of the WACCs depending on whether the operator is public (8%) or private (10%). In the present communication, we will consider the values as given and will not make them vary since it would considerably encumber the results.

Of course, the variations of the project’s IRR ($\alpha_0$) correspond to the ranges of variations chosen. For example, figure 3 shows the variations of $a$ and $b$ for $d=4$ and $C^*=100$. The 17 “actual cases” represented by specific values of $a$ and $\alpha_0$ can also be situated in this graph.
Figure 3: Influence of the variables $a$ and $b$ on the projects’ IRR 
($C^* = 100 – d = 4$)

We have distinguished on this graph the projects for which $\alpha_0 > 8\%$. We can see the only project in this case on the graph.

6 – A FEW RESULTS ON THE ISSUE OF A SWING IN FAVOUR OF EITHER OPERATOR

The aim is then to compare the values of the subsidies in two alternative situations that are either that of a public operator ($i = 1$ in what follows) or of a private operator ($i = 2$). If the economic values that characterise the project lead to equal subsidies $S_1$ and $S_2$, which corresponds to the swing point, then the respective parameters of the two situations verify:

$$S_1 = C_1^* - \frac{a_1}{0,08} - \frac{b_1}{0,08^2} = S_2 = C_2^* - \frac{a_2}{0,10} - \frac{b_2}{0,10^2}$$

(11)

This relation synthesises the advantage for the public operator that can settle for a WACC of 8% whereas the private operator must ensure 10%. Therefore the swing point in favour or one or the other operator depends on the values of $C_i$, $a_i$ and $b_i$.

The swing point value is defined by the situation where the project can be performed indifferently by a public operator (situation 1) or a PPP (situation 2). It is therefore necessary for a given project 1 ($a_1$, $b_1$, $c_1$, $d_1$ and $\alpha_1$), to find in which conditions a PPP can equalize the need for subsidy $S_i$, i.e. to define the range of
values of the parameters that ensure the best efficiency of a private contractor to build and operate the project. Optimal efficiency can be obtained by modifying parameters $a$, $b$, $c$ and $d$, i.e.:

- By reducing the construction cost: $c_2 < c_1$
- By reducing the project’s lead time: $d_2 < d_1$
- By increasing the first year rate of return: $a_2 > a_1$
- By improving the gradient of the annual benefit over time: $b_2 > b_1$

It is possible that the private operator is more efficient in all four domains, but at first, we will proceed with simulations to evaluate the effort to be made in a single parameter.

**What performance on $a$?**

$C^*$ stays fixed at 100 and $d$ at 4 years. The calculations are made with equation (11) and the ranges of values obtained for $a_1$ and $b_1$ give the values that $a_2$ must reach. The result in figure 4 shows that the values of $a_2$ are always higher than those of $a_1$.

**Figure 4: Gain in efficiency on the first year rate of return $a_2$**

Meanwhile to assess the gain of efficiency that a PPP requires, we can represent (figure 5) this gain in relative values.
Figure 5: Relative gain in efficiency on the first year rate of return \( a_2 \)

Given as a percentage, these gains in efficiency on \( a_2 \) are particularly high (30 to 150%, depending on the values of \( a_1 \) and \( b_1 \), under the current hypotheses). The calculation based on actual projects shows that in these cases, the gain in efficiency required on this one parameter would be between 40 and 70%.

As a first analysis, such a result seems unreachable. However one cannot omit the fact that \( a \) represents a difference (carried over to the discounted cost of the investment) between the receipts and the operating costs for the first year after opening. This means that a limited gain on the costs can have a significant effect on this difference.

What performance on \( b \)?

We are still under the assumptions that \( C^* \) is fixed at 100 and \( d \) at 4 years. The aim is to calculate with equation (11) and for the ranges of values of \( a_1 \) and \( b_1 \) the values that \( b_2 \) must reach. The result in figure 6 evidently shows values of \( b_2 \) noticeably higher than \( b_1 \).
Figure 6: Gain in efficiency on the growth of benefits $b_2$

It shows that in almost every case, $b$ should be doubled to reach the swing point under the current hypotheses. This strong variation comes partially from the small value of $b_1$. We will note that the value required of $b_2$ is higher when $a_1$ is high. Once again, this is a very ambitious gain in efficiency.

What performance on $c$?

We will assume here that all the parameters are equivalent except the cost $c_2$ for which we assume that the private operator is able to save costs compared to $c_1$. In figure 7 below, the savings necessary to reach the swing point are shown as relative values. In order to reach the equality of public subsidies, it appears that according to the values of $a$ and $b$, these savings vary between 8 and 32%.

It seems that once again, the challenge is relatively ambitious for the private operator. However, if we consider certain major construction projects of the same nature, cases have been observed in France in major construction projects for which the public operator has recorded a drift of over 17%\(^7\) for the costs initially anticipated whereas in the case of concession these excesses are rather rare. This is why we show on the graph a pertinence domain for the PPPs, which corresponds to this order of magnitude but obviously with a question mark.

\(^7\) In particular in the case of the Paris-Strasbourg high-speed railway line.
In this graph, we have also shown by a dotted arrow the trend of the project’s initial IRR. The lower the project’s IRR, the lower the effort of efficiency, which confirms the hypothesis that it is indeed for the least profitable projects that the PPP can be a good solution, as has been demonstrated in earlier works on the theme of the paradox of financial profitability (Bonnafous, 1999 and 2002). For example, for the 7 least profitable projects, the efforts to lower the costs remain below 13% whereas they are more than double for more profitable projects.

This result confirms the paradox according to which, contrary to what intuition suggests, recourse to a PPP has every chance of being more efficient for public finance when the financial profitability of the projects is poor.

It is noteworthy that on top of this effort on $c_2$, it is equally possible to lower the discounted cost of the investment $C^*$ by faster construction, i.e. by action on $d$.

**What performance on $d$?**

Since all the other parameters are fixed, we seek what would be the necessary reduction of the construction lead time ($d_2$). Equations (7) and (11) easily establish the explicit form of $d_2$ and simulate the swing point value of this lead time shown in figure 8. The necessary lead time reduction is between 7% and 33%, i.e. for a construction project assumed to last 4 years, i.e. a reduction of 3 to 15 months.

To start from a fixed basis, an undertaking such as the Millau viaduct was the object of a concession and was built in 3 years and two months, which is only one month less than the scheduled lead time. It is true that it was a particularly complex...
work. For certain more classical construction projects, gains of 3 to 6 months on a 4-year project are not unlikely.

Note that as for $c_2$, the higher the values of $a_1$ and $b_1$, the greater the reduction of the construction lead time must be (and therefore the higher the IRR). Once again we find an additional mark of the paradox of financial profitability.

![Figure 8: Gain in efficiency on construction lead time $d_2$](image)

The simulations show that the efforts of efficiency are considerable and often impossible when they are considered separately. The reality never corresponds to this logic of other things being equal. It is without a doubt more realistic to consider joint efforts.

The hypothesis of joint and equivalent performances on the four parameters

Even if it is a little naïve to consider that the four parameters can be lowered in the same proportions, it is this hypothesis that we have tested, still with the same set of equations. Since the joint variations of the parameters can be synthesised in the IRR, in figure 9 the abscissa represents this IRR, and the ordinate gives the gain in efficiency that corresponds to the swing point values. This gain is given in terms of percentage reduction of the parameters, with the percentage assumed to be the same for $a_2$, $b_2$, $c_2$ and $d_2$. 

$y = -0.0454x - 0.003$

$R^2 = 0.869$
We observe that a gain of about 6% on these four parameters ensures the swing in favour of PPPs for the least profitable projects whose IRR is lower than 5%. With this order of gain in efficiency required, it can be assumed that the challenge could be taken up by a private operator. This challenge becomes all the more complicated as the IRR and the parameters $a$ and $b$ increase. We can still note that the difficulty of the challenge depends on the relative efficiency of the public sector and there can be countries, public bodies or sectors for which poor performances enable considering gains in efficiency much higher than 6%.

**7 – CONCLUSION**

The first result of this investigation tends to confirm the paradox of financial profitability that demands that recourse to PPP is especially interesting for the public finances if the profitability of the projects concerned is poor.

The orders of magnitude obtained on what we have called swing point values constitute the other result which is (to our knowledge) original. It suggests that recourse to a PPP requires a relatively considerable gain in efficiency by the private operator, at least with the current WACCs that we have analysed.

To complete this exercise, it would be useful to explore the different values of these WACC that could result in significant changes on the long term financial markets, or even risk insurance accorded by the public authority for some of the private loans.
The other investigation that this work points to very naturally concerns a precise analysis of cost comparison (of construction and operation) between public and private operators. So far we have been unable to do more than draw an outline, because of the confidentiality of certain data, and especially due to the fact that such data may not exist when the operator is public. This detailed knowledge of the difference in efficiency between the public and private sectors would enable to better situate, sector by sector, the limits of the pertinence of recourse to PPPs.

REFERENCES

BONNAFOUS A. & JENSEN P. (2005), « Ranking Transport Projects by their Socio-economic Value or Financial Interest Rate of Return ? », Transport Policy, 12.
Conseil général des Ponts et Chaussées et Inspection générale des Finances (2003), Rapport d'Audit sur les Grands Projets d'Infrastructures de Transport.
DESRIEUX C. (2006), Le rôle de l'autorité publique dans la gestion des services publics locaux : Une approche par la théorie des contrats incomplets, Revue économique, 57(3).
## ANNEX: ESTIMATION OF PARAMETERS – 17 FRENCH HIGHWAY PROJECTS

<table>
<thead>
<tr>
<th>Project</th>
<th></th>
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<tr>
<td>A48 Ambérieu – Bourgoin – Jallieu</td>
<td>722.60</td>
<td>350.00</td>
<td>24.40</td>
<td>1.46</td>
<td>3.33</td>
<td>0.20</td>
<td>-374.41</td>
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<tr>
<td>A59 Balbigny – La Tour-de-Salvagny Scénario -</td>
<td>920.00</td>
<td>604.00</td>
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<td>0.33</td>
<td>2.23</td>
<td>0.04</td>
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<td>A59 Balbigny – La Tour-de-Salvagny Scénario +</td>
<td>920.00</td>
<td>625.00</td>
<td>25.03</td>
<td>0.50</td>
<td>2.72</td>
<td>0.07</td>
<td>-657.82</td>
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<td>A19 Artexay – Courtenay hpy. Basse</td>
<td>607.00</td>
<td>222.00</td>
<td>31.38</td>
<td>1.20</td>
<td>5.17</td>
<td>0.21</td>
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<tr>
<td>A19 Artexay – Courtenay hpy. Haute</td>
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<td>165.00</td>
<td>33.86</td>
<td>1.80</td>
<td>5.58</td>
<td>0.50</td>
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<tr>
<td>A585 Les Mées – Digne-les-Bains Scénario 1</td>
<td>250.10</td>
<td>139.80</td>
<td>8.50</td>
<td>0.30</td>
<td>3.44</td>
<td>0.12</td>
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<td>A531 Fontenay-le-Comte – Rochefort interdiction PL</td>
<td>560.00</td>
<td>243.00</td>
<td>25.75</td>
<td>0.76</td>
<td>4.60</td>
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<td>A531 Fontenay-le-Comte – Rochefort non-interdiction PL</td>
<td>560.00</td>
<td>243.00</td>
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<td>A41 Saint-Julien-en-Genève – Ville-la-Pelloux avec tunnel</td>
<td>692.20</td>
<td>475.00</td>
<td>18.27</td>
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<td>2.64</td>
<td>0.07</td>
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<td>277.00</td>
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